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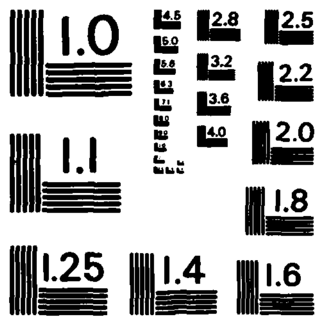

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During this period the investigator worked on the development of parallel algorithms to be used in the following linear algebra areas --- stability and inertia problems, controllability and observability problems, pole assignment problems, and matrix equations problems (Sylvester, Lyapunov, Riccati, etc). In particular, algorithms have been developed which require O (n log n) steps for solution on O (n^2) processors. Several presentations on these results were given, including a talk at the SIAM Fall Meeting. Three papers have been accepted for publication and several more are in preparation.

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Interim Report on

"Sequential and Parallel Matrix Computations"

by

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Presented to Air Force Office of Scientific Research

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The system of differential equations

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1.1}$$

and its discrete counter part

$$\dot{x}_{k+1}(t) = Ax_k(t) + Bu(t) \tag{1.2}$$

Where A and B are constant matrices of appropriate dimensions and x and us are time dependent vectors, arise in a wide variety of practical situations. These include design and analyses of industrial complexes such as chemical plants, electro-mechanical machines, like motor cars, aircraft, spaceships, economics structures of countries, etc. Associated with these sytems are many interesting Linear Algebra problems. The very well-knowns are

STABILITY AND INERTIA PROBLEMS

CONTROLLABILITY AND OBSERVABILITY PROBLEMS

POLE ASSIGNMENT PROBLEMS

MATRIX EQUATIONS PROBLEMS (SYLVESTER, LYAPUNOV, RICCATI, etc.)

In addition, there are many other subproblems such as (1) evaluating e^{At} (2) determining Relative Primeness of two polynoials or matrices (3) finding the Cauchy index of rational functions, etc.

Computer solutions of these problems are of utmost importance. Unfortunately, only very few viable sequential methods are available for solutions of these problems. Moreover, FAST ($O(n)$ or $O(n\log_2 n)$ steps) PARALLEL algorithms for these problems do not exist at all (as far as these investigators are aware).

The major objectives of this project are to develop fast computational algorithms, both sequential and parallel, for the problems mentioned above and, to study parallel arithmetic omplexities of these problems, i.e. how far these problems can be solved assuming that sufficient number of processors are available [8, 43].

Up to date, the following accomplishments have been made:

(A) We have developed FAST parallel algorithms ($O(n \log n)$ steps - $O(n^2)$ processors algorithms) for (i) determinining controllability of the pair (A,B) where A is a companion matrix and B is full (ii) single-input pole assignment problem (iii) stability problems for polynomials and (iv) problems for determining relative primeness of two polynomials and the Cauchy index of a rational function. These algorithms have parallel efficiency of $O(\frac{1}{\log n})$ in most cases.

A desirable feature of these algorithms is that they use only linear algebraic operations for which parallel algorithms already do exist.

(B) We have also been able to show (sometimes reconstructing the proposed $O(n \log n)$ steps - $O(n^2)$ processors algorithms, sometimes using entirely different technqiues) that parallel arithmetic complexities of all the control problems mentioned above are upper bounded by $O(\log^2 n)$ parallel steps.

(C) We have developed a FAST sequential algorithm (two to three times faster then the best known methods) for solving the symmetric positive semidefinite Lyapunov matrix equation $XA + A^TX = C^TC$.

(D) A FAST direct sequential method for finding the eigenvalue distribution of a matrix inside and outside various regions of the complex plane including half planes, shifted half planes, hyperbolas, sectors, quadrants, imaginary axis, regions contained within two straight lines that pass through the origin etc.

(E) We have just developed a parallel algorithm for computing the zeros of a polynomial whose zeros are all real and distinct. This algorithm requires

BIBLIOGRAPHY

1. R. Bartels and G.W. Stewart, A solution of the equation AX + XB = C, Commun. ACM 15 (1972), 820-826.

2. S. Barnett, Matrices in Control Theory, Clarendon Press, Oxford, 1975.

3. A. Berman and R. Plemmons, Nonnegative Matrices in the Mathematical Sciences, Academic Press, NY, 1979.

4. S. P. Bhattacharyya and E. De Souza, Pole assignment via Sylvester's equations, Systems, and Control Letters 1 (1982), 261-263.

5. Daniel Lucius Boley, Computing the controllability/observability decomposition of a linear time-invariant dynamic System, A Numerical Approach, Ph.D. Dissertation, Stanford University, California, 1981.

6. D. Carlson and B. N. Datta, On the effective computation of the inertia of a non-hermitian matrix, Numerische Mathematik, 33 (1979), 315-322.

7. D. Carlson and B. N. Datta, The Lyapunov matrix equation SA + A*S = S*B*BS, Linear Alg. Appl. (dedicated to A. Householder) 28 (1979), 43-52.

8. L. Csanky, Fast parallel matrix inversion algorithms, SIAM J. Computer, 5 (1976), 618-623.

9. B. N. Datta and Karabi Datta, Parallel computations of some linear Algebra problems in Control Theory, an INVITED TALK given at SIAM Fall Meeting, November, 1983, Norfolk, Virginia (Manuscript Enclosed).

10. B. N. Datta and Karabi Datta, A fast solution method for positive semi-definite Lyapunov matrix equation, an INVITED TALK given at Latin American Conference in Applied Mathematics, Rio, Brazil, December, 1983.

11. B. N. Datta, Matrix Methods for assignment of Canonical forms and eigenvalues, In preparation.

12. B. N. Datta and Karabi Datta, on finding eigenvalue distribution of a matrix in several regions of the complex plane, to be presented at 23rd IEEE Conference on Decision and Control, Las Vegas, December, 1984.

13. B. N. Datta, A New Criterion of Controllability to appear in May, 1984 issue of IEEE Trans. Automatic Control.

14. B. N. Datta and Karabi Datta, The matrix equation $XA = A^TX$ and an associated algorithm for solving the inertia problem, to appear in Numerische Mathematik.

15. Karabi Datta, The Use of Matrix determinant of Krnecker product in matrix parallel computations, submitted to SIAM J. Computing.

16. B. N. Datta, A solution of the Unit Circle problem via Schwarz Canonical Form, IEEE Transaction Automatic Control, June, 1982.

17. B. N. Datta, Stability and Controllability, Int. J. Control, 38 (1983), 1013-1022.

18. Karabi Datta, An algorithm to determine if two matrices have common eigenvalues, IEEE Trans. Auto Control, October 1982.

19. Karabi Datta, Algorithmos Paralelos em Algebra Linear (parallel algorithms in linear algebra), Ph.D. thesis, Universidade Estudual de Campinas, Campinas, S.P., Brazil, 1982.

20. G. Golub, S. Nash, and C. VanLoan, A Hessenberg-Schur method for the problem AX + XB = C, IEEE Trans. Automatic Control (1979), 909-913.

21. G. J. Golub and C. Reinsch, Singular value decomposition and least square solutions, Numer. Math. 14 (1970), 403-420.

22. J. Grcar and A. Sameh, On certain parallel Toeplitz linear system solvers, SIAM J. Sci. Stat. Comp. (1981), 238-256.

23. Max D. Gunzburger and R. J. Plemmons, Energy conserving norms for the solutions of hyperbolic system of partial equations, Mathematics of Computation, 33 (1979) 1-10.

24. S. Gutman and E. I. Jury, A general theory for matrix root-clustering in subregions of the complex plane, IEEE Trans. Auto Control, AC-26 (1981), 853-863.

25. M. L. J. Hautus, Controllability and observability conditions for linear autonomous systems, Nederl. Alcad. Wetensch. Proc. Ser. A72 (1969), 443-448.

26. E. I. Jury, Theory and Applications of the Z-transform Method, John Wiley, New York, 1964.

27. E. I. Jury, Inners approach to some problems of systems theory, IEEE Trans. Auto Control AC-16 (1971), 233-240.

28. T. Kailath, Linear Systems, Prentice Hall, Englewood Cliffs, NJ, 1980.

29. D. W. Kammler, Numerical evaluation of exp (tA) when A is a companion matrix, SIAM J. Numer. Anal. 15 (1978) 1077-1102.

30. G. Miminis and C. C. Paige, An algorithm for pole assignment of time invariant linear systems, Inter. J. Control, vol. 35, (1981), 341-354.

31. A. Ostrowski and H. Schneider, Some theorems on the inertia of general matrices, J. Math. Anal. Appl. 4 (1962), 72-84.

32. C. Paige, Properties of numerical algorithms related to computing controllability, IEEE Trans. Auto. Control, AC-26 (1981), 130-138.

33. A. H. Sameh and D. J. Kuck, On stable parallel linear system solvers, Journal of the Association for Computing Machinery 25 (1978), 81-91.

34. A. H. Sameh and D. J. Kuck, A parallel QR algorithm for symmetric tridiagonal matrices, IEEE Trans. Computers C-26 (1977), 147-153.

35. H. Schwarz, Ein Verfahren Zur Stabilitatsfrag bei Matrizen-Eigenwerte-Problemn, Z. Angun. Math. Phys. (1956), 473-500.

36. D. L. Slotnick and A. H. Sameh, Numerical calculation and computer design, Comp. & Maths. with Appls. 3 (1978), 201-210.

37. O. Taussky, A generalization of a theorem of Lyapunov, J. Soc. Ind. Appl. Math. 9 (1961), 640-643.

38. O. Taussky, Positive definite matrices and their role in the study of characteristic roots of general matrices, Advances in Math., 2 (1968), 175-186.

39. O. Taussky and H. Zassenhaus, On the similarity between a matrix and its transpose, Pacific J. Mathematics 9, (1959), 893-896.

40. R. Varga, Matrix Iterative Analysis, Prentice Hall, 1962.

41. J. H. Wilkenson, The Algebraic Eigenvalue Problem, Clarendon Press, Oxford, 1965.

42. D. Pierce, A computational comparison of four methods for finding the inertia of a matrix, M.S.C. thesis, Northern Illinois University, DeKalb, Illinois, 1983.

43. A. Borodin and I. Munro, Computational complexity of Algebraic and numeric problems, American Elsevier, 1975.

44. B. N. Datta and K. Datta, On parallel arithmetic complexities of some linear algebra problems in control theory - Part II, (Under preparation - to be published in Contemporary Mathematics).

45. K. Datta, The matrix equation LA - BL = R and its applications (under preparation).

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