Thermal Evaporation in Blast Waves: A Model for Aneurism Formation in the NRL LASER-HANE Simulation Experiment

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I. INTRODUCTION

During the past two years the Plasma Physics Division at the Naval Research Laboratory has undertaken a program to develop a set of experiments which would simulate certain physical processes that are characteristic of a high altitude nuclear event (HANE). The program is under the auspices of DNA's Division of Atmospheric Effects and has the primary goal of a better understanding of the degradation of radar and communication transmissions through an atmosphere seeded by nuclear bursts. At the present time the experiment is initiated by irradiating a thin target foil with a neodymium laser. The subsequent rapid heating and disintegration of the target is followed by an expanding shock wave once the target debris couples to the background gas. Typical experimental parameters used thus far consist of an Al-foil less than 1 mm in diameter with a total mass of several tenths of a µgm, a 3.4 nsec laser pulse of total energy 4-400 Joules, and a mostly N\textsubscript{2} background gas of pressure 0.001-10.0 Torr threaded by a uniform magnetic field of either 0 or 800 Gauss strength.\textsuperscript{1}

An important diagnostic has been dark-field shadowgraphy which gives a visual impression of the coupling region (or blast wave) as it expands. This process picks out steep density gradients in the image plane and permits multiple-time recordings on a single photograph. Stamper et al.\textsuperscript{2} present several photographs which depict the coupling region at 52 and 96 nsec after the initial laser pulse, when the blast wave has expanded to about 1 cm from the target position. An unusual feature seen in the high ambient pressure shots is the development of an aneurism in the coupling shell: an outward protrusion from the nearly spherical blast wave toward the direction of the laser. The purpose of this report is to present a physical mechanism which may account for the development of this aneurism. Since this feature seems to occur only in shots with a high ambient pressure (several Torr), the present theoretical model may have relevance for HANE phenomena at altitudes of 100 to 200 km.

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II. OVERVIEW OF THE PHYSICAL MECHANISM

In this section we develop a schematic picture of the proposed physical mechanism responsible for the aneurism. A detailed account of the specific model equations will be given in the following section. Ripin et al. have used the dark-field shadowgraphs to show that the radius of the coupling shell, $R$, expands as an adiabatic blast wave. The Taylor-von Neumann-Sedov self-similar solution shows that $R = R_0 (E_o/p_o)_{1/5} t^{2/5}$ where $E_o$ is the total laser energy, $p_o$ is the ambient density, $t$ is the time since the laser pulse and $R_0$ is a constant of order unity. The basic physics of the blast wave can be readily found by assuming that all of the swept-up mass resides in a thin shell moving with the leading strong shock front while being pushed by the pressure inside the enclosed, hot cavity.

Basically, the proposed model complicates this simple picture by including a thermal conduction front which forms on the inside edge of a small segment of the swept-up shell. The remainder of the shell is thermally insulated due to the magnetic fields which are generated at the target during laser burning and are subsequently carried out with the expanding plasma. Within the conduction front the electron heat flux from the hot cavity to the cooler shell is balanced by mass evaporation off the shell with the flow directed into the cavity. The mass unloading of the evaporating segment results in an acceleration relative to the insulated segment, assuming the cavity pressure is nearly uniform. Furthermore, the evaporating segment is accelerated by a rocket-like effect; the inward momentum carried off by the evaporative mass flow is balanced by forward moving pressure waves driven into the swept-up shell. Of course, as the evaporating segment is accelerated forward, it sweeps up relatively more material and suffers an increase in the opposing ram pressure. Under the proper conditions we will show that this evaporating segment moves ahead of the remaining shell resulting in an aneurism-like feature.

A. Thermal Conduction

We first justify the model by estimating some relevant physical quantities. The effect of thermal conduction is central in the proposed model, but how important is thermal conduction in the experiment? Consider the total energy equation for the cavity,
\[
\frac{3}{\partial t} \left( \frac{1}{2} \rho v^2 + \frac{1}{\gamma - 1} p \right) + \frac{\partial}{\partial x} \left( \frac{1}{2} \rho v^2 + \frac{\gamma}{\gamma - 1} p \right) + q = 0
\]  

(1)

We can neglect radiative losses since the analysis\(^3\) of the present experiment indicates that the total energy with the blast wave is constant. The heat flux is predominantly carried by the electrons, so

\[ q = -\kappa_e (T) \frac{\partial T}{\partial x} \]

where \( \kappa_e (T) = KT^{5/2} \) and \( K = 1.4 \times 10^8 \) ergs cm\(^{-1}\) sec\(^{-1}\) ev\(^{-7/2}\). The constant \( K \) was calculated from Braginskii\(^5\) for a Coulomb logarithm of 10 and a charge per ion in the cavity \( Z_c = 5 \). Let \( L_T \) be the temperature scale height, \( L_T = T/|\partial T/\partial x| \), then

\[ |q_e| \sim 6.5 \times 10^{16} \frac{T_c}{300\text{eV}}^{7/2} \left( \frac{\text{cm}}{L_T} \right) \frac{\text{ergs}}{\text{cm} \cdot \text{sec}} \]  

(2)

where \( T_c \) is a characteristic cavity temperature. The 300 eV estimate for \( T_c \) was based on the following considerations. The high sound speed within the cavity implies a nearly uniform cavity pressure \( p_c \) and the conservation of total energy, together with an estimate of the postshock gas velocity, leads to the relation \( p_c \sim p_s/2 \), where \( p_s \) is the shell pressure\(^4\). Hence,

\[
\frac{T_c}{T_s} \sim \frac{1}{2} \frac{p_s}{p_c} \frac{1 + Z_s}{1 + Z_c}.
\]  

(3)

where \( Z_s \) is the mean charge per ion in the shell. The mean mass per particle, \( \mu \), has been replaced by \( m_i/(1 + Z) \), with \( m_i \) the mass of a Nitrogen ion. The measured\(^6\) values at the time of aneurism formation are \( T_s \sim 15\text{eV}, Z_s \sim 3 \) and \( Y_s \sim 1.2 \) implying \( p_s/p_o \sim 11 \), where \( p_o \) is the ambient density. Since most of the swept up mass resides in the shell, \( p_s/p_c \gg 11 \). In our calculations we will consider the set of values \( T_c/T_s = (10,20,30) \) with the middle one corresponding \( T_c = 300\) eV and \( p_s/p_c = 60 \).

Let us now estimate the enthalpy flux \( \gamma \nu p/(\gamma - 1) \) in the cavity. The kinetic energy flux \( \nu v^3/2 \) in the energy equation (1) is negligible due to the low cavity density. Again using \( p_c \sim p_s/2 \)

\[
Y \frac{\nu p}{\gamma - 1} \sim 7.2 \times 10^{14} \frac{1 + Z_s}{Z_s} \left( \frac{n_e}{3 \times 10^{18} \text{cm}^{-3}} \right) \left( \frac{T_s}{15\text{eV}} \right) \left( \frac{\nu c}{10^7 \text{cm/sec}} \right) \frac{\text{ergs}}{\text{cm} \cdot \text{sec}}.
\]  

(4)
where $\gamma = \gamma_c = 5/3$ and $3 \times 10^{18} \text{ cm}^{-3}$ and $10^7 \text{ cm sec}^{-1}$ are a typical electron density and shell velocity, respectively, for the high pressure shots$^6$. Clearly the approximately measured enthalpy flux, Eq. (4), is much less than the estimated heat flux, Eq. (2) for any reasonable $L_T (\ll \text{cm})$. A similar result holds, though to a lesser degree, even if the heat flux is saturated. Saturation of the heat flux will be discussed more fully in section III. Thus the expansion of the cavity should be more nearly isothermal rather than adiabatic as in the Taylor-von Neumann-Sedov self-similar solution.

3. Magnetic Fields

Since the thermal conductivity perpendicular to the magnetic field can be smaller than along the field it is important to investigate the topology and strength of the magnetic field in the experiment. Consider the case where there is no ambient magnetic field. According to Kacenjar et al.$^7$ a magnetic field of 100 to 200 Gauss was still detected as the shell crossed the first magnetic probe located 1 cm from the symmetry axis and $R = 1 \text{ cm}$ from the target. A second measurement at the same offset, but $R = 3 \text{ cm}$ from the target found that the field strength near the shell had decreased to less than 20 Gauss. The orientation of the magnetic probe indicated that the field was azimuthal, i.e., field lines circled the symmetry axis. The presence of this field agrees with the theoretical model of magnetic field generation through the thermoelectric effect at the target surface during laser burning$^8\text{-}^{10}$. If this self-generated magnetic field is carried outward by the debris, it would decrease as $1/R^3$, consistent with the above measurements. This suggests that the magnetic field strength is $\geq 1$ kilogauss when $R \geq 1 \text{ cm}$, where the aneurism appears to start growing. Actually, this estimate is a lower limit since the magnetic probe may not resolve the magnetic field structure and the probe, which is encased in quartz tube to prevent space charging effects, requires the diffusion of the field out of the gas for a positive detection.

Thus, we adopt the estimate of 1 kG for the off-axis self generated magnetic field strength when $R < 1 \text{ cm}$ and an azimuthal orientation. The topology of such a field requires that it vanish along the symmetry axis. (A schematic diagram of the evolution of the blast wave is shown in Figure
1). In the off-axis portion of the shell the heat flux is perpendicular to the field and is thereby reduced relative to that in the segment of the shell along the symmetry axis where there is no field. We can quantitatively estimate this effect by using the approximation \( \kappa_\perp = \kappa_\parallel / [1 + (\omega T)^2] \), where \( \omega_\perp \) is the electron cyclotron frequency and \( \tau_\parallel \) is the electron collision time. The correct relation is more complicated, but the above simple one does have the proper dependence in the extreme limits. Using the previous estimates for the shell parameters one finds

\[
(\omega T)_{e,s} \sim 0.01 \frac{B(kG)}{Z_s \left( \frac{T_s}{15 \text{ ev}} \right)^{3/2} \left( \frac{3 \times 10^{18} \text{ cm}^{-3}}{n_{e,s}} \right)},
\]

which means that in the shell \( \kappa_\perp \sim \kappa_\parallel \).

However, the field cannot remain in the shell. The magnetic Reynolds number of the shell is

\[
R_{M,s} = \frac{4\pi}{c^2} \frac{\sigma_\perp L_M}{Z_s} \left( \frac{L_M}{Z_s \text{ cm}^{-1}} \right) \left( \frac{v_s}{10^7 \text{ cm sec}^{-1}} \right) \left( \frac{T_s}{15 \text{ ev}} \right)^{3/2},
\]

where \( \sigma_\perp \) is the perpendicular electrical conductivity and \( L_M \) is the magnetic length scale. Using the shell thickness \( L_s \ll 0.03 \text{ cm} \) for \( L_M \approx Z_s \sim 3 \), one finds \( R_{M,s} < 1 \). This implies that the field will diffuse out of the shell. Ahead of the shell \( R_M \) remains \( < 1 \) due to the small temperature and velocity, and a broad zone with a gentle compression of the magnetic field is formed. On the other hand, as the field diffuses back into the cavity it experiences a rapid increase in the temperature. The scale length of the diffusion region in the cavity where \( R_{M,c} < 1 \) is given by

\[
L_M < 1.5 \times 10^{-4} \frac{Z_c \left( \frac{10^7 \text{ cm sec}^{-1}}{v_c} \right)}{T_c} \left( \frac{300 \text{ ev}}{T_c} \right) \text{ cm},
\]

which is much less than the radius of the shell. Hence the field will stop diffusing once it moves back into the cavity while it will readily move ahead of the blast wave (see Figure 1c). This picture is verified by the temporal behavior of the field strength as measured by a stationary probe.
in the experiments with an ambient magnetic field\(^7\). For the field in the cavity one finds

\[
(\omega t)_{e,c} \sim 70.0 \frac{B(\text{kg})}{Z_c} \frac{T_c}{\text{300 eV}} \frac{C}{\text{5 x 10}^{16} \text{cm}^{-3}}. \quad (8)
\]

Thus away from the symmetry axis, the field resides just inside the shell and \(A_{<i} >> \gamma\), while along the axis \(B \sim 0\) and \(A_{<i} \). The effects of an ambient magnetic field will be briefly discussed in the last section.

C. Evaporative Mass Loss

Given that thermal evaporation is confined to a small segment of the shell the final question is how significant is the evaporative mass loss. We model the evaporative process as a quasi-steady thin, i.e., planar, conduction front at the rear of the dense swept-up shell. Let \(v\) be the gas velocity relative to the conduction front. For subsonic flow one obtains from Eq. (1),

\[
(\rho v) = \frac{\gamma}{\gamma - 1} \frac{T_c}{\mu_c} (e^{(T_c / \gamma) c} - 1) \frac{dT}{dr} \quad (9)
\]

where \(\mu_c = m_i / (1 + Z_c)\) is the mean mass per particle in the cavity. Following the nomenclature prior to Eq. (2) we compare

\[
\text{evaporative mass loss rate} = \rho v \frac{\gamma - 1}{\gamma} \frac{m_i}{\mu_c} \frac{KT_c^{5/2}}{\rho R} \frac{L_T}{L_T},
\]

\[
= 6.6 \left(\frac{10^{18} \text{cm}^{-3}}{n_0}\right) \left(\frac{10^{7} \text{cm sec}^{-1}}{R}\right) \left(\frac{T_c}{\text{300 eV}}\right)^{5/2} \text{cm}^{5/2}, \quad (10)
\]

where \(R\) is the velocity of the shell. This result means that evaporation can significantly reduce the mass in the conductive segment of the shell resulting in a protuberance (see Figure 1d). A more realistic estimate of the ratio will depend on the additional effects of a saturated heat flux, an increase in the mass gain rate as the shell segment accelerates forward, and a temporal decrease in the cavity's temperature.
III. MODEL EQUATIONS

A. Geometry and Nomenclature

Now that the general scenario of the proposed physical mechanism for the development of the aneurism has been presented, we will develop a set of more detailed equations to model the evaporative effects only. The topology and strength of the magnetic field is not calculated here; we assume it has the topology described in section II. The specific geometry is given in Figure 2. Thermal evaporation occurs within a sector $20^\circ$, while outside of this sector the shell is assumed to be insulated by the azimuthal self-generated magnetic field. We will use the "thin-shell" approximation with $R_s'$ being the radius of the insulated shell segment, and $R_s$ the radius of the evaporating segment. Primed quantities will always refer to the former segment and a subscript $s$ denotes shell referenced values. We will neglect the detailed shape of the protrusion and consider the evaporating segment to be part of spherical surface. This of course greatly simplifies the calculation by reducing the model equations to one spatial dimension; however, it allows one to study the nonlinear physical processes acting to generate the protrusion. $M_s$ and $M_s'$ are the masses of the respective sections of the shell, and $v_s$ and $v_s'$ the gas velocities in the lab frame. The shock front on the outside edge of the swept-up shell will be taken to be a strong adiabatic shock. Hence we can use the relations

$$\frac{p_s'}{p_o} = \frac{\gamma_s + 1}{\gamma_s - 1}, 
\frac{v_s}{v_o} = \frac{2}{\gamma_s + 1} R_s', 
\frac{p_s'}{\mu_s'} = \frac{\gamma_s - 1}{(\gamma_s + 1)^2} R_s'^2$$

where $\dot{R}_s = dR_s/dt$, $\gamma_s$ and $\mu_s$ are the specific heat ratio and mean mass per particle in the shell, respectively, and $p_o$ is the ambient density.

B. Dimensional Form

In our model the total energy $E_o$ is conserved, i.e., there are no radiative losses, and the cavity temperature and shell pressure are approximated as uniform. The total energy is the sum of the cavity's internal energy and the kinetic as well as internal energy in the two shell segments:
\begin{equation}
E_0 = \frac{1}{\gamma_c - 1} \frac{4\pi}{3} \left[ R_s^{-3} \cos^2 \left( \frac{1}{2} \theta \right) + R_s^{-3} \sin^2 \left( \frac{1}{2} \theta \right) \right] p_c + \frac{1}{2} M_s v_s^2 + \frac{1}{2} M_{\cdot} v_{\cdot}^2 + \frac{1}{\gamma_s - 1} 4\pi R_s^{-2} \Delta R p_s + \frac{1}{\gamma_s - 1} 4\pi R_s^{-2} \Delta R^* p_{\cdot}^* .
\end{equation}

The kinetic energy of the cavity material can be neglected due to its low density and small velocity. The presence of a conduction front does not alter the total energy equation since it only acts to exchange energy from internal to kinetic. The internal energy of the shell is typically neglected\(^4\), though actually it equals the shell's kinetic energy. The equality follows from the thin-shell approximation which gives \( M_s = 4\pi R_s^{-2} \Delta R p_s \), and Eq. (11). The resulting total energy equation can then be written as

\begin{equation}
E_0 = \frac{1}{\gamma_c - 1} \frac{4\pi}{3} \left[ R_s^{-3} \cos^2 \left( \frac{1}{2} \theta \right) + R_s^{-3} \sin^2 \left( \frac{1}{2} \theta \right) \right] p_c + M_s v_s^2 + M_{\cdot} v_{\cdot}^2 .
\end{equation}

The total momentum equation represents a balance between the rate of change of the momentum of the shell and the pressure force supplied by the cavity. Since there are two shell segments one has

\begin{equation}
\frac{d}{dt} \left( M_s v_s \right) = 4\pi R_s^{-2} \cos^2 \left( \frac{1}{2} \theta \right) p_c .
\end{equation}

and

\begin{equation}
\frac{d}{dt} \left( M_{\cdot} v_{\cdot} \right) = 4\pi R_s^{-2} \sin^2 \left( \frac{1}{2} \theta \right) \left( p_c + p_{evp} \right) .
\end{equation}

The additional pressure \( p_{evp} \) in Eq. (15) is due to the rocket-like effect as mass evaporates off the shell segment and flows backward into the cavity. Its value will be developed when we discuss the equations describing the conduction front.

The total masses in the different segments of the swept-up shell are

\begin{equation}
M_s = \frac{4\pi}{3} R_s^{-3} \rho_o \cos^2 \left( \frac{1}{2} \theta \right) ,
\end{equation}

\begin{equation}
M_{\cdot} = \frac{4\pi}{3} R_s^{-3} \rho_o \sin^2 \left( \frac{1}{2} \theta \right) .
\end{equation}
and

\[ \dot{M}_s = \frac{4\pi}{3} R^3 \rho_s \sin^2 \left( \frac{\theta}{2} \right) - \dot{M}_{\text{evp}}, \quad (17) \]

where \( \dot{M}_{\text{evp}} \) is the total mass which has been evaporated from the conducting segment of the shell. Again its value will be discussed below.

Let us pause in the development of the model equations and consider the solutions in the purely adiabatic case, i.e., \( \theta = 0 \). The results will be required as initial conditions prior to the onset of thermal evaporation over a segment of the shell. Combining Eqs. (11), (13), (14), and (16), one readily finds

\[ R_s = \xi_0 \left( \frac{\rho_o}{\rho} \right)^{1/5} t^{2/5}, \quad (18) \]

\[ \xi_0 = \frac{75}{16\pi} \frac{(\gamma - 1)(\gamma_s + 1)^2}{[\gamma_s + 1 + 4(\gamma_s - 1)]}, \quad (19) \]

and

\[ p_c = \frac{1}{\gamma + 1} \rho_o \frac{R_s^2}{2} = \frac{1}{2} p_s \quad (20) \]

These results agree with Ripin et al.\(^3\). If the shell's internal energy had been neglected the quantity \( 4(\gamma - 1) \) in Eq. (19) would be replaced by \( 2(\gamma - 1) \). Then for \( \gamma_s = \gamma_c \) one recovers the formula given in Zel’dovich and Raizer\(^4\).

In order to include the effects of thermal evaporation we need an equation for the cavity temperature \( T_c \), which is assumed to be uniform except in the conduction front. The total energy equation will be used to evaluate the cavity pressure \( p_c \) and by the equation of state for an ideal gas,

\[ p_c = \frac{3(M_{c, \text{ad}} + \dot{M}_{\text{evp}})}{4\pi [R^3 \cos^2 \left( \frac{\theta}{2} \right) + R_s \sin^2 \left( \frac{\theta}{2} \right)]} T_c \quad (21) \]

In the absence of evaporative mass gain to the cavity, \( M_{c, \text{ad}} \) is the total mass in the cavity under adiabatic conditions. This mass is negligible.
compared to that in the swept-up shell, but it cannot be zero since the cavity must have a finite temperature. The Taylor-von Neumann-Sedov solution gives a cavity density which depends only on the self-similar variable, $r/R_s'$. Hence the total mass in the cavity increases as $R_s'^3$, i.e., $M_{c,ad}/p_o R_s'^3$ is constant, and the proportionality factor depends on the artificial demarcation in $r/R$ separating the cavity from the shell. We can avoid this problem noting that for adiabatic expansion

$$\frac{p_c}{p_s} = \frac{3 M_{c,ad}}{(4\pi \rho_o R_s'^3)} \frac{\gamma_s - 1}{\gamma_s + 1} \frac{\mu_s T}{\mu_{c,s}}$$

from Eqs. (11) and (21). During this phase, $p_c/p_s$ is fixed by the energy equation (Eq. [20]) and a choice of the parameter $T_c/T_s$ gives an effective value for $M_{c,ad}/p_o R_s'^3$. We will assume that this ratio remains constant after evaporation begins. This approximation is reasonable in the case that $\theta$ is a small angle, resulting in a small change to the adiabatic solution, and is irrelevant in the case that the whole shell is subject to evaporation, since $M_{evp}$ quickly exceeds $M_{c,ad}$.

Finally, we need to formulate a set of equations for the conduction front so that $M_{evp}$ and $p_{evp}$ can be explicitly evaluated. We use the approach given in section II, i.e., the conduction front is a thin, planar shell at the inside edge of the swept-up shell and in quasi-steady equilibrium. Let $\dot{\rho_v}$ be the mass flow rate through the conduction front with $\dot{v}$ the gas velocity relative to the front. Then setting

$$j = \frac{\dot{\rho_v}}{\rho_o R_s}$$

we have

$$M_{evp} = j 4\pi R_s^2 \rho_o \sin^2(\frac{\theta}{2})$$

Conservation of momentum through the conduction front is simply,

$$\dot{\rho_v}^2 + p = p_s + \rho_s \dot{v}_s^2$$

with $\rho_s = \rho_o (\gamma_s + 1)/(\gamma_s - 1)$ since the leading shock is taken everywhere to be a strong adiabatic one. By momentum conservation the additional
pressure on the evaporating segment of the shell due to the rocket-like effect is

$$p_{evp} = \rho s \mu s = j^2 \frac{\gamma_s - 1}{\gamma_s - 1} \rho_0 R^2. \quad (26)$$

This pressure prevents the conduction front from overrunning the leading shock.

For the energy equation we balance the heat flux into swept-up shell against the energy carried by the mass flux off the shell. From Eq. (1) we have

$$\frac{1}{2} \dot{\rho}^2 + (\dot{\rho v}) \frac{\gamma T}{\gamma - 1} = -q = \frac{\kappa e(T)}{1 + \frac{\kappa e(T)}{q_{sat}} \frac{dT}{dz}}, \quad (27)$$

where \( z = 0 \) at the interface between the conduction front and the swept-up shell and increases toward the cavity. The generalized heat flux \( q \) in eq. (27) is an approximate representation for the limitation on the heat flux when the electron mean free path \( \ell_e \) is on the order of the temperature length \( L_T \). Under this condition the electron distribution function is far from Maxwellian, the classical relation \( \dot{q} = -\kappa e(T) \dot{T} \) breaks down, and the maximum, i.e., saturated, heat flux is that given by the free-streaming limit:

$$q_{sat} = a n e T_{m e}^{1/2}. \quad (28)$$

The parameter \( a \) has been determined to be \( \sim 0.06 \) from Fokker-Planck simulations and experiments. It is instructive to express the conductivity coefficient in terms of the electron mean free path:

$$\kappa e(T) = \frac{f(z)}{\sqrt{3}} n_e (\frac{T}{m_e})^{1/2} \ell_e, \quad (29)$$

and \( f(z) \) is a function tabulated by Braginskii. Rewriting Eq. (27) via Eqs. (28) and (29) gives
which explicitly displays the dependence of the heat flux on the different length scales.

C. Non-dimensionalization of the Dynamic Equations

It is convenient to write the equations in dimensionless form prior to obtaining solutions. For the total mass, momentum and energy equations we choose a fudicial time \( t_1 \) prior to which the expansion is entirely adiabatic. The value of \( t_1 \) is determined by the solutions for the structure of the conduction front. At time \( t_1 \) one has from Eqs. (18), (19), and (20),

\[
R_1 = \xi_0 \left( \frac{E_0}{\rho_0} \right)^{1/5} t_1^{2/5}, \quad R^* = \frac{2}{3} \frac{R_1}{t_1},
\]

\[
M_1 = \frac{4}{3} \pi R_1^3 \rho_0, \quad p_{c,1} = \frac{4}{25(\gamma_s + 1)} \rho_0 \left( \frac{R_1}{t_1} \right)^2
\]

We now define the dimensionless variables as

\[
t = t t_1, \quad R_s = \lambda R_1, \quad R^* = \lambda^* R_1, \quad p_c = n p_{c,1}
\]

\[
M_{\text{evp}} = \omega M_1 \sin^2(\frac{1}{2} \theta), \quad M_{c,ad} = \varepsilon M_1 \lambda^3
\]

\[
T = \zeta T_s, \quad n_e = n n_{e,s}, \quad \rho v = j \rho_0 \frac{R^*}{s}
\]

Note that the last three definitions are written in terms of the evaporating shell variables. They will be used to analyze the conduction front. Also recall from Eq. (22) that \( \varepsilon_M \) is taken to be a constant determined by an initial choice of

\[
(T_c/T_s)_1 = (T_c'/T_s')_1 = \xi_{c,1},
\]
where

\[ \frac{1}{2} = \frac{\gamma_s - 1}{\gamma_s + 1 + \frac{Z_s}{\gamma_s}} \zeta_c. \]  

(33)

Equation (32) gives in turn

\[ M_s = M_s \cos^2 \left( \frac{1}{2} \theta \right) \lambda^3, \quad M_s = M_s \sin^2 \left( \frac{1}{2} \theta \right) \left( \lambda^3 - \omega \right), \]

\[ v_s' = \frac{2}{\gamma_s + 1} \frac{R_1 d\lambda'}{d\tau}, \quad v_s = \frac{2}{\gamma_s + 1} \frac{R_1 d\lambda}{d\tau}. \]  

(34)

The equations for energy (Eq. [13]) and momentum (Eqs. [14] and [15])
can now be written, respectively, as

\[ \frac{2}{25} \left( \frac{\gamma_s + 4 \gamma_c - 3}{(\gamma_c - 1)} \right) - \lambda^3 \left( \frac{d\lambda'}{d\tau} \right)^2 \cos^2 \left( \frac{1}{2} \theta \right) - \left( \lambda^3 - \omega \right) \left( \frac{d\lambda}{d\tau} \right)^2 \sin^2 \left( \frac{1}{2} \theta \right) \]

\[ - \frac{2}{25} \left( \frac{\gamma_s + 1}{(\gamma_c - 1)} \right) \left[ \lambda^3 \cos^2 \left( \frac{1}{2} \theta \right) + \lambda^3 \sin^2 \left( \frac{1}{2} \theta \right) \right] \eta, \]  

(35)

\[ \lambda^3 \frac{d^2 \lambda'}{d\tau^2} = \left[ \frac{6}{25} \eta + \frac{3}{2} \gamma_s - 1 \right] \lambda^2 \left( \frac{d\lambda}{d\tau} \right)^2 - 3 \left( \frac{d\lambda}{d\tau} \right)^2 \lambda^2 + \frac{d\omega}{d\tau} \frac{d\lambda}{d\tau}. \]  

(36)

and

\[ (\lambda^3 - \omega) \frac{d^2 \lambda}{d\tau^2} = \left[ \frac{6}{25} \eta + \frac{3}{2} \gamma_s - 1 \right] \lambda^2 \left( \frac{d\lambda}{d\tau} \right)^2 - 3 \left( \frac{d\lambda}{d\tau} \right)^2 \lambda^2 + \frac{d\omega}{d\tau} \frac{d\lambda}{d\tau}. \]  

(37)

In the last equation we have used Eqs (26) and (31). Equation (24) for the
rate of change of the evaporated mass transforms to

\[ \frac{d\omega}{d\tau} = 3j \lambda^2 \frac{d\lambda}{d\tau}. \]  

(38)

Finally using Eqs. (11), (31), (32) the relation between the cavity
pressure and temperature (Eq. [21]) becomes

\[ \eta = \frac{25}{2} \gamma_s - 1 \left( \frac{1}{\gamma_s + 1} + \frac{Z_s}{\gamma_s} \right) \frac{\epsilon M \lambda^3 + \omega \sin^2 \left( \frac{1}{2} \theta \right)}{\lambda^3 \cos^2 \left( \frac{1}{2} \theta \right) + \lambda^3 \sin^2 \left( \frac{1}{2} \theta \right)} \left( \frac{d\lambda}{d\tau} \right)^2 \zeta_c. \]  

(39)
The set of parameters is \{y_s, \gamma_c, Z_s, Z_c, \theta, \xi_c, j\} and the set of unknowns is \{\lambda, \lambda^c, \partial \lambda / \partial \tau, \partial \lambda^c / \partial \tau, \omega, j\}. Subject to the initial conditions at \(\tau = 1\),

\[
\lambda = \lambda^c = 1, \frac{d\lambda}{d\tau} = \frac{d\lambda^c}{d\tau} = \frac{2}{5}, \omega = j = 0,
\]

the system of equations (36), (37), and (38) can be integrated forward in time (\(\tau\)), using Eq. (35) to determine the cavity pressure (\(\eta\)), once an expression for the evaporated mass loss rate (\(j\)) has been given. To do so we will need Eq. (39) and a solution of the structure of the conduction front.

IV. RESULTS

A. Solution for the Evaporative Mass Loss Rate

Let us begin by multiplying the momentum conservation Eq. (25) by \(p/p_s^2\) to obtain

\[
\frac{j^2 (\rho o_s^*)^2}{p_s^2} \frac{d^2}{d\tau^2} \frac{D}{p} + (\frac{D}{p})^2 = (\frac{D}{p}) [1 + j^2 (\frac{\rho o_s^*}{p_s})^2].
\]

From Eq. (11), \((\rho o_s^*)^2/p_s p_s = (\gamma_s - 1)/2\), and one can now solve the quadratic equation for the pressure ratio:

\[
\frac{p}{p_s} = \frac{1}{2} \left[ 1 + \frac{j^2 (\gamma_s - 1)}{2} \pm \left( (1 + \frac{j^2 (\gamma_s - 1)}{2})^2 - 4 j^2 \frac{(\gamma_s - 1)}{2} \left( \frac{T}{T_s} \right) \right)^{1/2} \right].
\]

Since \(j\) is limited to about 1 and \(\gamma_s = 1.2\) from the experimental analysis\(^3\), \(\frac{1}{2} j^2 (\gamma_s - 1) \ll 1\). The positive sign is then the proper choice to satisfy the boundary condition at the front of the conduction front where \(\mu_s T/\mu T_s = 1\). Clearly, Eq. (41) constrains the mass flux giving approximately

\[
j \leq \frac{1}{2} \frac{\gamma_s - 1}{1 + \frac{Z_c}{Z_s} \frac{T}{T_s}} \frac{1}{j^{1/2}}.
\]

When the equality holds, the maximum value for the mass loss rate \(j_{\text{max}}\) corresponds to the maximum pressure drop through the conduction front. Note that although the temperature at the rear of the conduction front is
that of the cavity \( T_c \), the pressure at this point \( p_2 \) is larger than \( p_c \); once the evaporating segment accelerates relative to the adiabatic segment \( p_s > p'' \) giving \( p_c - p''/2 < p_s/2 < p'' \). We surmise that the conduction front will adjust to maximize the pressure drop and hence also \( j \). In the terminology of fluid combustion\(^{12} \), the choice of the positive sign in Eq. (41) means that our conduction front is analogous to a deflagration reaction. The composite structure of a conduction front and a preceding shock wave can be viewed as a deflagration initiated by a shock i.e., a detonation wave. Our assumption that \( j = j_{\text{max}} \) is equivalent to the Chapman-Jouguet hypothesis.

The size of the conduction front is given by solving the energy equation through the conduction front, Eq. (30). To do this first scale \( z \) by the radius \( R_s \) \((z = y R_s)\) and then combine Eqs. (29) and (32) to get

\[
\frac{\frac{\gamma}{\Delta} \frac{dT}{dz}}{\frac{T_s^2}{\gamma_e s^2 \ln h}} \frac{\zeta}{v_c} \frac{dy}{dz} = \frac{3}{4} \frac{3}{2\pi} \frac{\gamma_e s^2 \ln h}{\gamma_e s^2 \ln h} \zeta \frac{dy}{dz}
\]

(43)

Except for the variation in the Coulomb logarithm and \( Z \), the term in the curly braces is the ratio of the electron mean free path in the swept-up shell to \( R_s \). Let us then rewrite Eq. (43) as

\[
\frac{\frac{\gamma}{\Delta} \frac{dT}{dz}}{\frac{T_s^2}{\gamma_e s^2 \ln h}} \frac{\zeta}{v_c} \frac{dy}{dz} = \frac{\gamma_e s^2 \ln h}{\gamma_e s^2 \ln h} \zeta \frac{dy}{dz}
\]

(44)

By using this equation and Eqs. (11) and (32), the energy equation (30) can be transformed to

\[
(1 + \frac{\gamma - 1}{2} \zeta^2) \frac{dy}{dz} = \frac{\gamma_e s^2 \ln h}{\gamma_e s^2 \ln h} \zeta \frac{dy}{dz}
\]

(45)

where

\[
a = \frac{Z_e \sigma(Z)}{(1 + Z)^3} \frac{\gamma - 1}{\gamma} \left( \frac{2m_e}{m_e (\gamma_s - 1)(1 + Z_s)} \right)^{1/2},
\]

\[
b = \frac{f(Z)}{m_e^2},
\]

(46)
and $M$ is the adiabatic Mach number of the flow relative to the conduction front. We make several approximations in order to integrate equation (45). First we neglect the variation of $Z$ through the conduction front and set $Z$ equal to $Z_c$ in Eqs. (42), (43), and (46). Second, we set $\gamma = \gamma_c = 5/3$. It is then reasonable to neglect the term $(\gamma - 1)M^2/2$ compared to unity. The Chapman-Jouguet condition for the conduction front has $M$ reaching a maximum of one at the rear of the front and so this term is $< 1/3$. Finally we can write

$$n_{e, s} T Z \frac{1}{(1 + Z_s)} \frac{p}{p_s} = \frac{n_e T}{n_{e, s, T}} \frac{Z(1 + Z_s)}{Z_s (1 + Z)} \frac{p}{p_s}. \quad (47)$$

Before we noted that $p/p_s$ drops from 1 to $\sim 1/2$ through the front and from Eq. (41) most of the change occurs near the rear. Thus we fix $\nu \zeta = 1$. The solution to Eq. (45) is now readily found to be

$$y = \left( \frac{e_{s, s}}{R_s} \right) \frac{2a}{5} \left( \zeta^{5/2} - 1 \right) - \frac{b}{3} \left( \zeta^3 - 1 \right). \quad (48)$$

The first property to note from this result is that at $\zeta = \zeta_c$, $y$ must be less than unity in order to fit the conduction front within the blast wave radius $R_s$. Initially $\zeta$ has the constant value $\zeta_c, 1$, while $e_{s, s}/R$ decreases. The evaporation process can begin once

$$\left( \frac{e_{s, s}}{R_s} \right) < \left( \frac{2a}{5} \right) \left( \zeta_c, 1 \right)^{5/2} - 1 - \frac{b}{3} (\zeta_c, 1 - 1)^{-1}. \quad (49)$$

By a series of manipulations involving Eqs. (11) and (31) one finds

$$\frac{e_{s, s}}{R} = \left( \frac{3}{2\pi} \right)^{3/5} \frac{m_1^3}{e^4} \frac{1}{n \Lambda} Z_c^2 \left( 1 + Z_s \right)^2 \frac{(y - 1)^3}{(y + 1)} \frac{\zeta_0^3 E_0}{\rho_0 \beta^3 \xi_1 \tau_{1/4}^{1/5} \lambda^4 d\tau}. \quad (50)$$

We now identify $t_1$ with the turn-on time for evaporation. At this time $\lambda = 1$, $d\lambda/d\tau = 2/5$ and by using the relation $\rho_0 (\text{gr cm}^{-3}) = 1.64 \times 10^{-6} \rho_0$ (Torr) we can convert Eq. (50) to practical units and explicitly define the initial time as

$$t_1(\text{nsec}) = 1.1 \times 10^2 \frac{1}{Z_c^2 \left( 1 + Z_s \right)^2} \frac{(y - 1)^3}{(y + 1)^5} \frac{5/14}{x}$$

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The corresponding length is

\[
R_1 (\text{cm}) = 0.60 \left[ \frac{1}{Z_c^{-2}(1 + Z_s^{-2}) (Y_s + 1)^5} \right]^{1/7} \times \\
\frac{10}{\xi_0} E_o (\text{Joules})^{2/7} \frac{R_s}{p_0 \text{(Torr)}^{3/7}} (\xi_{e,s})^{1/7}. 
\]

We can further use Eq. (48) to determine a turn-off condition for evaporation. After evaporation begins, the size of the conduction front \( y_c \), given by setting \( \zeta = \xi_c \) in Eq. (48), decreases rapidly. This is due to the decrease in \( T_c/T_s \) as the evaporating shell segment accelerates relative to the insulated segment. Since there is only a small mass addition into the cavity for small \( \theta \), \( T_c/T_s \) remains nearly constant, \( T_s/T_s \) increases, and hence \( T_c/T_s \) decreases. Furthermore \( \xi_{e,s}/R_s \) decreases as evidenced by the form of Eq. (50). At some point \( y_c \) can become on the order of the electron mean free path in the cavity. The latter quantity is given by

\[
\frac{\xi_{e,c}}{R_s} = \frac{\xi_{e,s}}{R_s} \frac{\zeta_c^2}{V_c}. 
\]

We have rather arbitrarily cut off conduction when \( y_c \) is twice this value. The aspect represents the fact that at this point electrons can freely stream from the cavity right into the swept-up shell. The resulting large anisotropy in the electron distribution is not accounted for in the present simple model for saturated conduction.

To summarize, the value of the cavity to shell temperature \( \zeta_c \) is given by Eq. (39). The value of the mass evaporation rate (\( j \)) can then be computed using the equality in Eq. (42). If the size of the conduction front is less than \( R_s (y_c < 1) \), the calculated value of \( j \) is used in eq. (38). However, if the size of the front is less than twice the electron mean free path in the cavity, the value of \( j \) is set to zero. We note from eq. (42) that \( j \propto \zeta_c^{-1/2} \). This is consistent with the form of equation (45) since the heat flux becomes more strongly saturated as the expansion proceeds.
B. Solution for the Evolution of the Aneurism

In Figure 3 we present the results for three calculations. For all angels we found that the expansion of the insulated segment of the shell was close to the purely adiabatic law, \( \lambda = r^{2/5} \). This relation is shown as a solid line in the graph. The position of the evaporating segment, i.e., the aneurism, is represented by the different symbols corresponding to the different choices for the parameters which are listed in Table 1. The time and length units, \( t_1 \) and \( R_1 \), are calculated from Eqs. (49), (51), and (52) for different initial choices of \( \zeta_{c,1} = (T_c/T_s)_1 \). These units are normalized to \( E_0 = 10 \) Joules and \( p_0 = 5 \) Torr, which are typical values for the high pressure experiments.

The calculations do not indicate a significant angle dependence of the expansion of the protuberance for \( \theta \) within a factor of two of 30°. However, changes in the parameters \( Z_c \) and/or \( \gamma_c \) do result in a wide variation in the shape of the solution curve for \( R_s/R_1 \), as well as changes in the time and length units.

V. SUMMARY AND DISCUSSION

In this report we have suggested a new physical mechanism for the development of the aneurism observed in the laser-Hane experiment. The essential feature of the model is the formation of a thermal conduction front on the inside surface of the swept-up blast wave shell. The conduction front is limited to a small segment about the symmetry axis of the incoming laser beam due to an azimuthal magnetic field carried outward by the shell. The field is generated at the target during the laser burning. Within the conduction front the thermal heat flux from the cavity into the shell is balanced by an evaporative mass flow from the shell back into the cavity. The major analysis of this report used the thin-shell approximation to model the rocket-like acceleration of the evaporating segment ahead of the insulated part of the blast wave shell.

Below are a number of points on the implication and limitations of the present model for the development of the aneurism.

1.) Figure 3 and Table 1 clearly shows that the growth of the protuberance is faster the smaller the value of \( (T_c/T_s)_1 \). This feature is contrary to one's expectation in a mechanism where thermal conduction is
important. However this expectation is based on the use of the classical thermal conductivity coefficient. Let us use Eq. (48) to determine \( j \):

\[
\frac{2}{5} a \left( \frac{e_s}{R_s} \right) \left( \zeta_c^{5/2} - 1 \right) \]

\[
j = \frac{\frac{2}{3} b \left( \frac{e_s}{R_s} \right) \left( \zeta_c^{3} - 1 \right)}{y_c + \frac{2}{3} b \left( \frac{e_s}{R_s} \right) \left( \zeta_c^{3} - 1 \right)}.
\]

(54)

We can effectively keep the heat flux from strongly saturating by maximizing the size of the conduction front, i.e., setting \( y_c = 1 \). Then as \( \left( \frac{e_s}{R_s} \right) \) decreases (roughly as \( \tau^{-14/5} \) from Eq. [50], the conduction front would become less and less saturated and the evaporation rate would have the expected behavior on the temperature ratio \( j = \zeta_c^{5/2} \). The resulting value of \( j \) would be very small after \( \tau \) reaches a few. However, this classical model is inconsistent. The conduction front, which would extend over the whole cavity, has a negligible pressure drop, contrary to the known relation \( p_c \sim 1/2 p_s \). In the pressure model, this pressure drop enforces the Chapman-Jouguet condition for the conduction front. This condition is equivalent to the equality in Eq. (42), which gives rise to the calculated dependence in \( T_c/T_s \). As a consequence, Eq. (54) shows that \( y_c \ll 1 \) and the heat flux is highly saturated through the front.

2.) As a test of the present model we solved the system of equations with \( 2\theta = 2\pi \), i.e., we modeled an "isothermal" blast wave. Independent of the parameter \( (T_c/T_s) \), the results were that after the evaporation process commenced, the radius of the shell \( R_s \) was 1.09 times larger than the radius of the Taylor-von Neumann-Sedov, purely adiabatic, blast wave, \( R_{ad} \). This compares favorably with the detailed self-similar solution13,14 for an isothermal blast wave where \( R_s = 1.08 R_{ad} \).

3.) We note that the time scale for turn-on of the evaporation process (Eq. [51]) scales as \( E_0^{3/14} \rho_{0}^{-8/14} \). This is consistent with the experimental observation that the aneurism does not occur, or is delayed, in high energy, low pressure shots2. The parameters in Table 1 for the calculations in Figure 3 give time scales for the appearance of the aneurism that are within a factor of two of the experimental values. However the experiment shows a wide variety in the structure of the aneurism; some grow, some grow then decay, while others appear turbulent. Undoubtedly there are a large number of factors which effect the dynamics
of the aneurism. The model proposed here has only attempted to analyze a single potential mechanism in the nonlinear regime.

4.) One of the most serious drawbacks of the present calculation is that it is limited to one dimension. The two dimensional aspects of tangential flow in the aneurism and oblique shock could significantly alter the evolution of the proturbation in our model.

5.) A second drawback in the model is the neglect of the evolution of the topology and magnitude of the self-generated magnetic field. We have also ignored the presence of an ambient magnetic field. For the parameters of the present experiment, the ambient field would have no dynamic effect on the growth of the aneurism; the total magnetic energy of a 800 Gauss field in a sphere of radius 1 cm is \(0.01\) Joules which is far less than the several Joules imparted by the laser to the target. This conclusion is consistent with the claim of the experimental group that the aneurism shows no dramatic change whether the ambient field is on or off (though see Brecht\(^15\). However, one interesting feature also noted by the group was that the aneurism sometimes appears to form slightly of the symmetry axis. A possible explanation for this feature in the context of the present model is as follows. When the ambient field was on it was directed into or out of the plane depicted in Figure 1. As the shell sweeps over some of the ambient field, a cancellation with the self-generated field could result in a negligible magnetic field at an off-axis position thereby generating an off-axis aneurism. This position would depend on the orientation of the ambient field. A review of the present data might be relevant in this matter. We also suggest that future runs of the experiment be made with the ambient magnetic field in the plane of Figure 1 to see if, and how, the aneurism is affected.

6.) A final limitation in the model was the approximate calculation for the structure of the conduction front. We assumed a quasi-steady state, neglecting variations in the ionization level \(Z\), and ignored other physical aspects such as temperature non-equipartition between the ions and electrons. Despite these limitations we can make two general comments. If evaporation is acting to drive the observed aneurism there would result a mass flow off the shell into the region behind the aneurism. This mass flow is much larger than the flow off the shell from the adiabatic, i.e., not evaporating, segments. One might expect to find a higher gas density.
behind the aneurism than behind the other parts of the blast wave. This is in fact the observed case, at least for the electrons in the single shot where an interferogram was obtained\textsuperscript{16}. This model would also predict that the gas near the aneurism would have a higher temperature than the gas in the other segments of the blast wave. This is due to the relative acceleration of the evaporating segment leading to a higher temperature in the shell and to the heating within the conduction front. It might be possible in future experiments to measure the temperature near the aneurism using spectroscopy.

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Fig. 1) Schematic cartoon of the proposed physical mechanism leading to the development of an aneurism in the blast wave. The crosses and dots represent the magnetic field into and out of the plane of the diagram. A heat flux $q$ acts along the thermally unisulated section of the blast wave shell. The evaporative mass flux $\rho v$ then leads to the aneurism development. Further aspects are explained in section II of the text.
Fig. 2) Geometrical definitions used in the thin-shell approximation for the dynamic equations.
Fig. 3) Plot of the dimensionless radius $R/R_1$ of the evaporating segment, i.e., the aneurism (given by the different symbols), and the radius of the insulated segment (solid line) as a function of the dimensionless time $\tau/t_1$. The scaling and parameters for the different solutions are listed in Table 1.
Table 1 — Data for Figure 3

\[ t_1 = a \left( \frac{E}{10 \text{ Joules}} \right)^{3/14} \left( \frac{5 \text{ Torr}}{P_o} \right)^{8/14} \text{nsec} \]

\[ R_1 = b \left( \frac{E}{10 \text{ Joules}} \right)^{2/7} \left( \frac{5 \text{ Torr}}{P_o} \right)^{3/7} \text{cm} \]

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01CY ATTN DEPT 60-12
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01CY ATTN J. B. CLADIS DEPT 52-12

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