ESD-TR-84-035

(12)

AD-A146 885

Technical Report
699

Tristatic Tracking Filter Used by the Multistatic Measurement System

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12 September 1984

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Prepared for the Department of the Army
under Electronic Systems Division Contract F19628-80-C-0002.

The work reported in this document was performed at Lincoln Laboratory, a center for research operated by Massachusetts Institute of Technology. The work is sponsored by the Kwajalein Missile Range Directorate of the Ballistic Missile Defense System Command, under Air Force Contract F19528-80-C-0002.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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USED BY THE MULTISTATIC MEASUREMENT SYSTEM

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Group 33

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ABSTRACT

This report describes the mathematical models and algorithms that comprise the ballistic tristatic tracking filter used by the Multistatic Measurement System at the Kiernan Re-entry Measurement Site.

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1. INTRODUCTION

This report describes the mathematical models and algorithms that comprise the tristatic tracking filter used by the Multistatic Measurement System (MMS) at Kiernan Re-entry Measurement Site (KREMS). The rest of this section gives a brief description of MMS. Section 2 discusses the basic physics and mathematical model upon which the tristatic filter is based. Section 3 details the implementation of this Kalman filter. Section 4 compares the tracking accuracy of the tristatic filter to the accuracy of the TRADEX monostatic tracker. Some mathematical details have been collected in the appendixes.

1.1. Basic Concept of the Multistatic Measurement System

The Multistatic Measurement System consists of the monostatic radar TRADEX and two remote sites on the islands of Gellinam and Illeginni.

A microwave link provides real time communication between the three sites via a repeater station at Gagan. See Fig. 1.

The equipment at each remote site includes:

(1) A radar antenna and receiver.

(2) Microwave communications hardware.

(3) A clock used to time tag the radar returns.

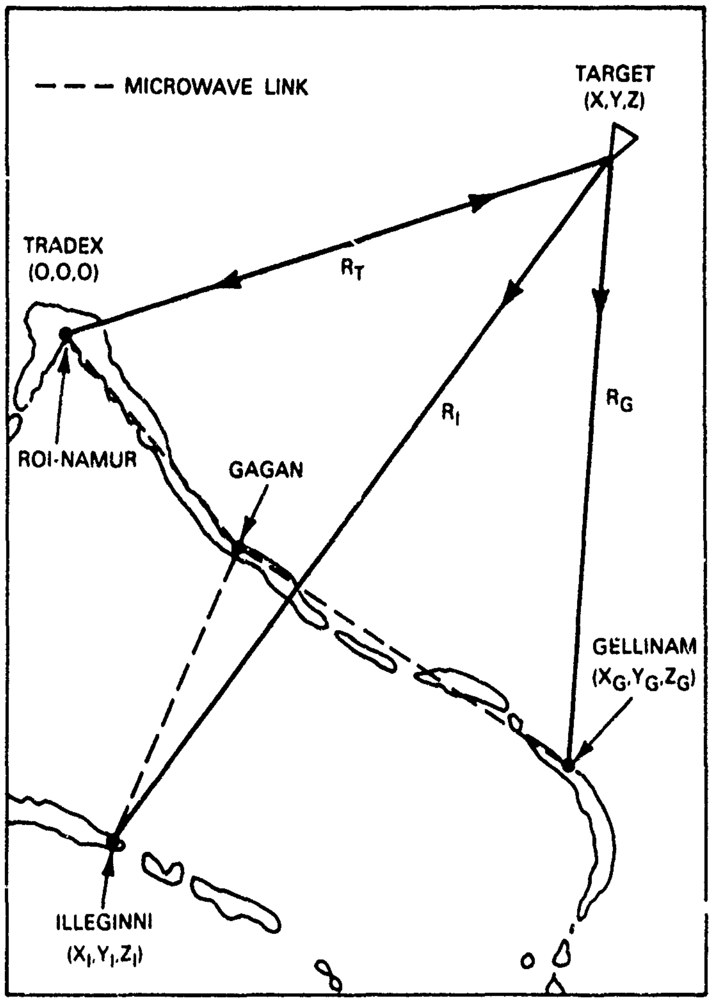

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Fig. 1. Configuration of Multistatic Measurement System.

Control signals are sent between TRADEX and the remote sites via the microwave link to synchronize the remote site clocks with the one at TRADEX. When TRADEX transmits a radar pulse the signal reflects from the target to the TRADEX receiver and to each remote site receiver. The propagation times from the TRADEX transmitter to each of the three receivers measure the TRADEX range R_T, the Gellinam bistatic range sum R_T+R_G, and the Illeginni bistatic range sum R_T+R_I. The remote sites use the microwave link to report these measurements to the KREMS Data Center (KDC) in real time. TRADEX also reports its range R_T, azimuth and elevation to KDC in real time.

The KDC computer provides two major functions for MMS. It controls the equipment at the remote sites via the microwave link, and it executes the tristatic filter. The results from this filter can be used to direct the remote sites.

2. GENERAL MATHEMATICAL MODEL OF MMS

This section presents the mathematical model used in the filter for the target motion between measurements and the formulae relating target position to the MMS measurements.

2.1. Coordinate System

The coordinate system used in this filter is the KDC reference system. This is a cartesian system with origin at the TRADEX antenna oriented so that the x, y, and z axes are pointing eastward, northward, and upward respectively. This choice of coordinate system allows for easy computation of ranges from the three antennas using their coordinates. It is also a convenient coordinate system in which to express target motion.

2.2. Equations of Motion

The equations of motion discussed in this section are used in the filter to predict the motion of the target between measurements.

The equations of motion can be written as a system of first order differential equations using the state vector $x = (x, y, z, x, y, z)^T$. In vector form, these equations are

$$\frac{d\vec{x}}{dt} = \vec{f}\,(\vec{x})$$

where

$$\vec{f}(\vec{x}) = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -g_x - A_{cx} - Dvx \\ -g_y - A_{cy} - Dvy \\ -g_z - A_{cz} - Dvz \end{bmatrix}$$

and target velocity is $v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$.

The first three components of this equation merely state that the derivative of target position is its velocity. The bottom three components state that the derivative of velocity is acceleration.

This acceleration is expressed in three parts. The acceleration due to gravity is modeled in the vector $(g_x, g_y, g_z)^T$. This gravity model includes the second harmonic of the gravity potential. Since the coordinate system is moving with the earth, there is an apparent Coriolis acceleration $(A_{cx}, A_{cy}, A_{cz})^T$ in the equation. The third term in each acceleration component of $\vec{f}$ is for aerodynamic drag on the target. The factor D in the drag terms is used to model the effects of atmospheric pressure and the ballistic coefficient of the target. It is computed in real time as a function of altitude from a table of quadratic polynomials. Each polynomial is used in a specified altitude interval. This table of polynomials and intervals is prepared from a nominal trajectory set up prior to a mission.

For more mathematical details on these equations of motion see Appendix A.

2.3. Measurements Used by the Filter

The measurements used by the filter are estimates of TRADEX range, azimuth, elevation, the Gellinam bistatic range sum, and the Illeginni bistatic range sum. The three range measurements are assumed to have been calibrated and corrected for all significant distinctions between the apparent propagation times and the geometric distances between the target and the three sites.

Currently the real time program corrects the range measurements for tropospheric refraction and for the delay induced by range-doppler coupling in the pulse compression network.

The time of validity of the five measurements is the estimated time of incidence of the radar pulse on the target. Since TRADEX measures its azimuth and elevation at a slightly different time, the real time program extrapolates these angle measurements to the time of incidence using the angular rates and accelerations reported to it by TRADEX.

The filter adjusts its estimate of target position and velocity on the basis of the difference between the calibrated and corrected measurements described above and corresponding values computed from the predicted position. This predicted position is determined from previous measurements. The formulas for these computed values are given in Appendix C.

The tristatic filter updates its estimate of the target position and velocity once every 100 millisecond cycle of the KDC real time program. During each cycle, any subset of the five corrected measurements described above may be processed by the filter. Typically this subset consists of TRADEX range and both bistatic range sums. The choice of subset may be altered for each update cycle. This capability allows the real time program to avoid processing bad measurements and to start the filter with TRADEX range and angle measurements before the remote sites acquire the target.

3. ALGORITHMS AND EQUATIONS USED IN THE FILTER

This section describes the algorithms and equations used to implement the tristatic filter based on the mathematical model described in Section 2. The next subsection gives an overview of the structure of the filter.

3.1. Overview of the Filter

The general organization of the tristatic filter is shown in Fig. 2. During each 100 millisecond cycle of the real time program, each block is executed once.

The Raw Measurements consist of the TRADEX range, azimuth and elevation as well as the propagation times for the bistatic measurement to each remote site. The "Apply Corrections" block calibrates the Raw Measurements and makes the adjustments discussed in Section 2.3.

The resulting Corrected Measurements are estimates of the geometric TRADEX range, azimuth, elevation, Gellinam bistatic range sum, and Illeginni bistatic range sum at time of target illumination. These five corrected estimates are termed geometric ones because they correspond to the values derived from the locations of the sites and target using geometry.

The Prefilter block in the diagram checks the Corrected Measurements for reasonableness by comparing them with the previous measurements. The algorithm for this comparison is discussed in Section 3.2.

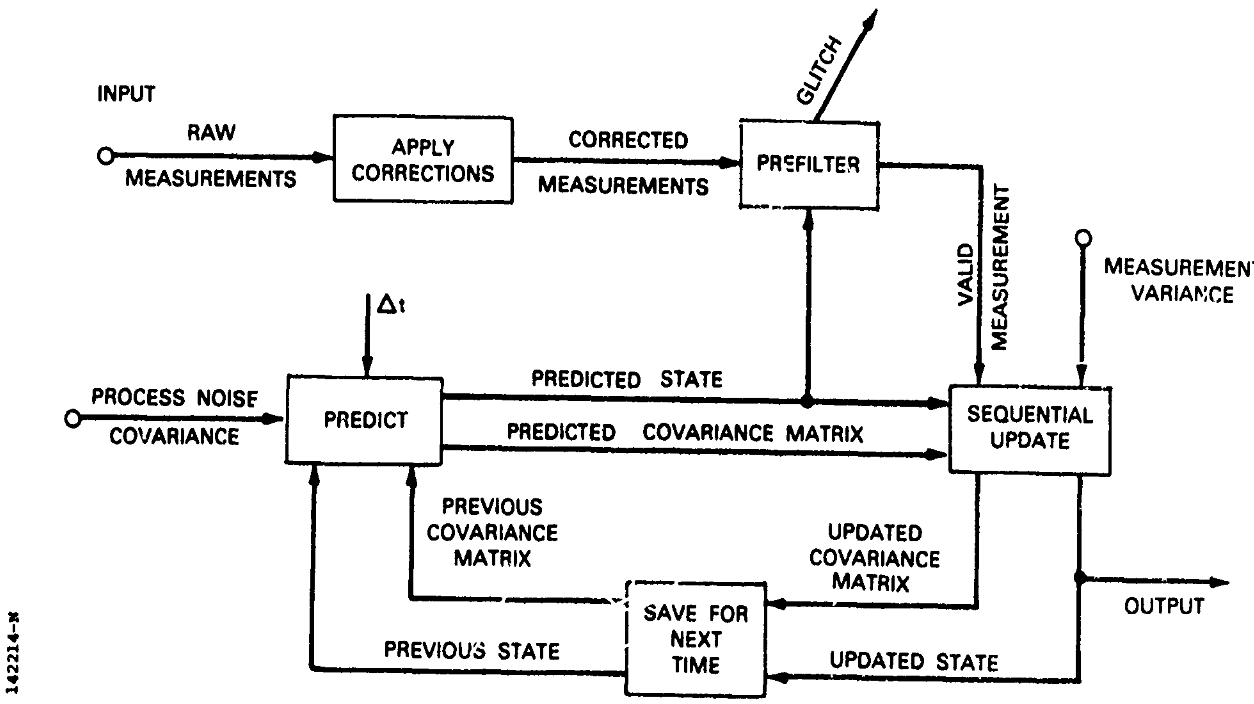

Fig. 2. Structure of tristatic filter.

The Valid Measurements are sent on to the Sequential Update block along with an indication of which measurements have been selected for use in the update process.

Prior to executing the Sequential Update block, the filter predicts the target position and velocity (Predicted State in Fig. 2) using the Previous State vector derived from previous measurements. This is done by integrating the equations of motion of the target over the time interval between the time of validity of the previous measurements and the time of validity of the current measurements. The accuracy of the Predicted State is given in terms of the Predicted Covariance Matrix, which is determined from the Previous Covariance Matrix, the Process Noise Covariance Q, and the equations of motion. The initial conditions for this integration are the target position and velocity that constitute the Previous State in Fig. 2. The "Predict" block also extrapolates the Previous Covariance Matrix to determine the Covariance Matrix for the Predicted State.

The Sequential Update combines each Valid Measurement with the Predicted State vector to improve the accuracy of the target position and velocity estimates. These estimates constitute the Updated State. The Updated Covariance Matrix indicates the accuracy of the Updated State. Measurement Variances are used to control the weights applied to the observation to obtain the Updated State. If the Prefilter selects no measurements for use in the Sequential Update, then the Predicted State and covariance matrix become the updated ones. This effects a coast of approximately 200 milliseconds that might be extended during the processing of the next measurement.

To get started, the filter needs the following things supplied to it.

(1) Target position and velocity to use as a "Previous State" vector.

(2) A Covariance Matrix for the error in this "Previous State" vector.

(3) The Process Noise Covariance Matrix Q.

(4) The measurement variances used in the Sequential Update.

The "Previous State" vector is taken from an already existing track file, such as the TRADEX monostatic track file. The other items in the above list are supplied to the filter by operators when the real time operations start.

3.2. The Prefilter

The job of the prefilter is to detect glitches in the data so that only valid measurements are fed to the Sequential Update. The difference is computed between each actual range measurement and the corresponding value computed from the predicted target position for the same time. The average and standard deviation are computed for the previous 30 such residuals, and the residual for the current measurement is compared to them. A detailed statement of the glitch-catching algorithm for TRADEX range follows.

Let t_k be the time of validity of the k-th TRADEX range in the sequence of range values being processed.

Let $R_p(k) = R_T(\vec{x}(t_k \mid t_{k-1}))$, the predicted TRADEX range computed from the predicted state vector $\vec{x}(t_k \mid t_{k-1})$.

Let $R_M(k)$ = the measured TRADEX range at time t_k.

For k = 1 to 30 define

$$d(k) = R_M(k) - R_p(k)$$

For each $k \geq 31$ define the average $\bar{d}(k)$ and standard deviation $\sigma(k)$ of the previous 30 residuals by

$$\bar{d}(k) = \frac{1}{30} \sum_{\ell=1}^{30} d(k-\ell)$$

$$\sigma^2(k) = \frac{1}{30} \sum_{\ell=1}^{30} (d(k-\ell) - d(k))^2$$

If

$$\left| R_M(k) - R_p(k) - \bar{d}(k) \right| < 5\,\sigma(k)$$

then

(1) Use $R_M(k)$ in Sequential Update for time t_k

(i.e. $R_M(k)$ is Valid Measurement)

(2) Define $d(k) = R_M(k) - R_p(k)$

Otherwise

(1) Declare a glitch and do not use R_M (k) in Sequential Update

(2) Define d(k) by

$$d(k) = \begin{cases} -500 & \text{if} \quad R_M(k) - R_p(k) < -500 \\ R_M(k) - R_p(k) & \text{if} \quad \left| R_M(k) - R_p(k) \right| \leq 500 \\ 500 & \text{if} \quad R_M(k) - R_p(k) > 500 \end{cases}$$

The above algorithm is written for TRADEX range.

To monitor the Gellinam bistatic range sum measurement, a similiar algorithm is used. In this version the measured bistatic range sum replaces the measured TRADEX range for each time t_k, and $S_G(\vec{x}(t_k \mid t_{k-1}))$ replaces $R_T(\vec{x}(t_k \mid t_{k-1}))$. A similiar algorithm is also used for the Illeginni bistatic range sum.

3.3. Predicting Target Position and Velocity

The prediction of the target position and its velocity are based on numerical integration of the equations of motion described in Section 2. The numerical integration consists of one step of the Runge-Kutta method from the time of the previous measurement t_{k-1} to the time of the current measurement t_k. The initial condition for the integration is the result of the previous Sequential Update, namely $\vec{x}(t_{k-1} \mid t_{k-1})$.

The estimate of the state vector at time t_k is

$$\vec{x}(t_k \mid t_{k-1}) = \vec{x}(t_{k-1} \mid t_{k-1}) + (\vec{\zeta}_1 + 2\vec{\zeta}_2 + 2\vec{\zeta}_3 + \vec{\zeta}_4)/6$$

where

$$\vec{\zeta}_1 = \Delta t \, \vec{f} \, (\vec{x} \, (t_{k-1} \mid t_{k-1}))$$
$$\vec{\zeta}_2 = \Delta t \, \vec{f} \, (\vec{x} \, (t_{k-1} \mid t_{k-1}) + 1/2 \, \vec{\zeta}_1)$$
$$\vec{\zeta}_3 = \Delta t \, \vec{f} \, (\vec{x} \, (t_{k-1} \mid t_{k-1}) + 1/2 \, \vec{\zeta}_2)$$
$$\vec{\zeta}_4 = \Delta t \, \vec{f} \, (\vec{x} \, (t_{k-1} \mid t_{k-1}) + \vec{\zeta}_3)$$

3.4. Prediction of the Covariance Matrix

The computation of the covariance matrix associated with $\vec{x} \, (t_k \mid t_{k-1})$, namely $P \, (t_k \mid t_{k-1})$ is based on the equation

$$P \, (t_k \mid t_{k-1}) = \Phi(t_k) \, P \, (t_{k-1} \mid t_{k-1}) \, \Phi(t_k)^T + \Phi(t_k) \, Q \, \Phi(t_k)^T.$$

In this equation $\Phi(t_k)$ is an approximate transition matrix given by

$$\Phi(t_k) = I + \Delta t F \, (\vec{x}(t_k \mid t_{k-1}))$$

where
$$F \, (\vec{x}) = \frac{\partial \vec{f}}{\partial \vec{x}} \, (\vec{x}),$$

is the Jacobian of the vector function in the right hand side of the equations of motion. Using this formula for the transition matrix, we can express the predicted covariance matrix as

$$P(t_k \mid t_{k-1}) = \tilde{P} + \Delta t(F\tilde{P} + (F\tilde{P})^T + \Delta t\, FPF^T)$$

where $\tilde{P} = P(t_k \mid t_k) + Q,$

$F = F(\vec{x}(t_k \mid t_k)),$

and $\Delta t = t_k - t_{k-1}.$

The filter currently uses this formula for $P(t_k \mid t_{k-1})$ with a simplified version of the Jacobian F, instead of the exact Jacobian matrix.

3.5. Using the Simplified Jacobian to Predict the Covariance Matrix

This section discusses the simplifications applied to the Jacobian matrix $\frac{\partial \vec{f}}{\partial \vec{x}}$ and the algorithm that takes advantage of the them.

One simplification is to replace the nine partial derivatives of f_4, f_5, and f_6 with respect to x, y, and z in the Jacobian matrix with zeroes. Another simplication occurs in the expressions for $\frac{\partial f_4}{\partial y}$, $\frac{\partial f_4}{\partial z}$, $\frac{\partial f_5}{\partial x}$, and $\frac{\partial f_6}{\partial x}$ where the rotation rate of the earth ω is taken to be zero. These simplications eliminate all the terms in the partial derivatives of acceleration except those involving drag.

The simplified Jacobian can be written

$$F(\vec{x}) = \begin{bmatrix} O & I \\ O & A \end{bmatrix} \qquad (1)$$

where each block matrix is 3 by 3 and the drag matrix A is given by

$$A = -\frac{D}{V}\begin{bmatrix} v^2 + x^2 & xy & xz \\ xy & v^2 + y^2 & yz \\ xz & yz & v^2 + z^2 \end{bmatrix} \tag{2}$$

The algorithm reduces the amount of computation required to predict the covariance matrix by taking advantage of the blocks of zeros and the identity matrix in the simplified Jacobian matrix as described below.

Partition the covariance matrix P into 3 by 3 blocks.

$$\tilde{P} = \begin{bmatrix} B & C \\ C^T & E \end{bmatrix} \tag{3}$$

where B and E are symmetric matrices.

The needed matrix products are given by:

$$F\,\tilde{P} = \begin{bmatrix} C^T & E \\ AC^T & AE \end{bmatrix} \tag{4}$$

$$F\,\tilde{P}\,F^T = \begin{bmatrix} E & EA^T \\ AE & AEA^T \end{bmatrix} \tag{5}$$

Let $M = AC^T$, $N = AE$ and $K = AEA^T$. Then we have

$$\begin{bmatrix} M & N \end{bmatrix} = A \begin{bmatrix} C^T & E \end{bmatrix} \tag{6}$$

and

$$K = N \; A^T$$

Using this notation the martrix products are:

$$F \; \tilde{P} = \begin{bmatrix} C^T & E \\ M & N \end{bmatrix} \tag{7}$$

$$(F \; \tilde{P})^T = \begin{bmatrix} C & M^T \\ E & N^T \end{bmatrix} \tag{8}$$

$$F \; \tilde{P} \; F^T = \begin{bmatrix} E & N^T \\ N & K \end{bmatrix} \tag{9}$$

The algorithm consists of the following steps:

(1) Compute the drag term A using equation 2.

(2) Compute the 3 by 6 matrix $[\; M \; N \;]$ using equation 6.

(3) Compute $K = N \; A^T$.

(4) Construct the matrix products using equations 7, 8 and 9 and the entries in M, N and K.

(5) Compute the Predicted Covariance Matrix $P(t_k \mid t_{k-1})$, from last equation in previous section.

3.6. The Sequential Update

The Sequential Update combines the MMS measurements and the predicted target position and velocity to improve the estimates of position and velocity.

The Sequential Update consists of a sequence of Kalman updates, each of which combines a single scalar measurement with target position and velocity estimates. This approach presumes that the MMS measurements at any one time are uncorrelated.

The measurement selection in the prefilter determines which scalar updates are done during a 100 millisecond cycle of the real time program. Fig. 3 shows all five scalar updates that are possible and their inter-connections. Fig. 3 assumes that all five measurements have been selected for processing. More typically only the three MMS measurements are selected, in which case the scalar updates for TRADEX azimuth and elevation are deleted from the Sequential Update as shown in Fig. 4. Another useful configuration for the sequential update is one which processes only TRADEX data. This is shown in Fig. 5. Such a configuration could be used to start the filter before the remote sites acquire the target.

The configurations for the Sequential Update shown in the figures presume that the prefilter has not detected a glitch in

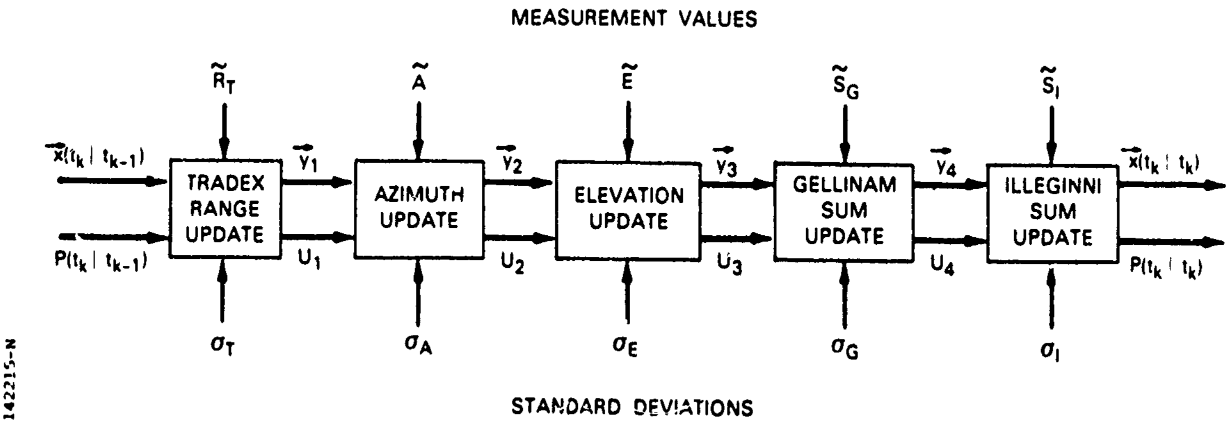

Fig. 3. Sequential update using all five measurements.

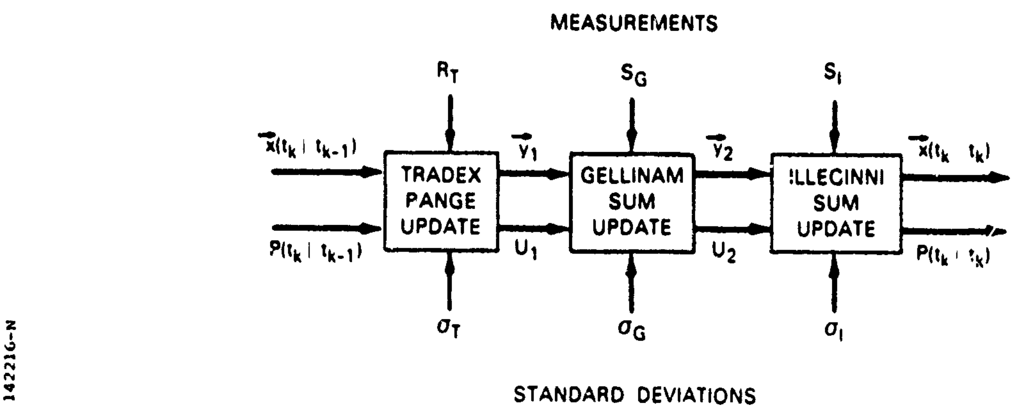

Fig. 4. Sequential update using MMS data.

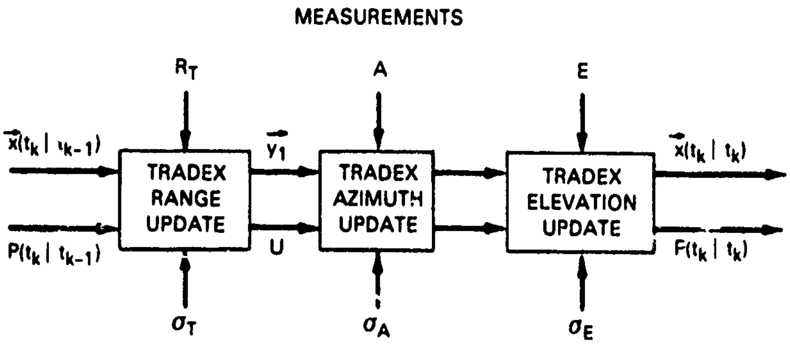

Fig. 5. Sequential update using TRADEX data.

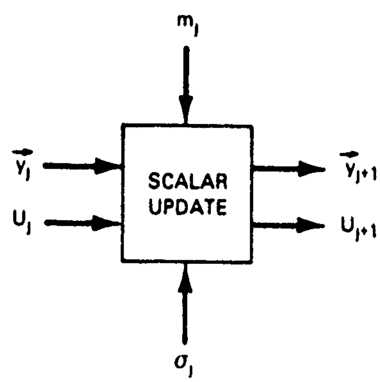

Fig. 6. Generic form of scalar update.

the MMS data. If a glitch is detected by the Prefilter, the measurement selection passed to the sequential update block in Fig. 2 will exclude the bad scalar measurement. In reaction tc this deletion, the sequential update will also exclude the corresponding scalar update from the configuration chosen a priori. Unless the Prefilter detects a second glitch in the following 100 millisecond cycle, the associated scalar update is put back into the sequence of scalar updates.

Each box in Figs. 3, 4, and 5 represents a scalar update based on the Kalman update equations. A generic version of this scalar update is shown in Fig. 6. In terms of the notation in Fig. 6, the scalar update equations are:

$$y_{j+1} = y_j + K_j \, (m_j - h_j \, (y_j))$$

$$U_{j+1} = (I - K \, H_j) \, U_j$$

$$K_j \quad = U_j \, H_j \, (H_j \, U_j \, H_j^T + \sigma_j^2)^{-1}$$

Here y_j is the state vector detemined from previous measurements, m_j is the scalar measurement, h_j is the associated measurement function, U_j the covariance matrix for y_j, H_j is the Jacobian of h_j, namely

$$H_j \quad = \frac{\partial h_j}{\partial x} \text{ evaluated at } \vec{x} = y_j$$

and σ_j is the standard deviation associated with the measurement m_j. The matrix K_j is the gain matrix. In Fig. 3

$\vec{x}(t_k \mid t_{k-1})$ is $\vec{y}_0$, $P(t_k \mid t_{k-1})$ is U_0

$\vec{x}(t_k \mid t_k)$ is $\vec{y}_5$ and $P(t_k \mid t_k)$ is U_5.

Similiar associations hold for other configurations of the Sequential Update.

4. EXAMPLE OF TRACKING WITH TRISTATIC FILTER

This example is drawn from the mission GT100. The tristatic filter estimates of target position and velocity can be used to calculate the strobe times for the remote sites. Estimates generated by the TRADEX monostatic tracker can also be used for this purpose. The difference between the expected time of arrival of the peak return, as determined by the tracking filter, and the actual time for it are plotted for each site in Figs. 7, 8, and 9. Prior to 1662 seconds after launch the TRADEX monostatic tracker is directing the laying of the strobes at the remote sites. After the 1662 second mark, the tristatic filter is directing the remote sites. The strobes at TRADEX are always directed by the TRADEX monostatic tracker. These plots show a dramatic improvement in the accuracy of the laying of strobes when controlled by the tristatic filter.

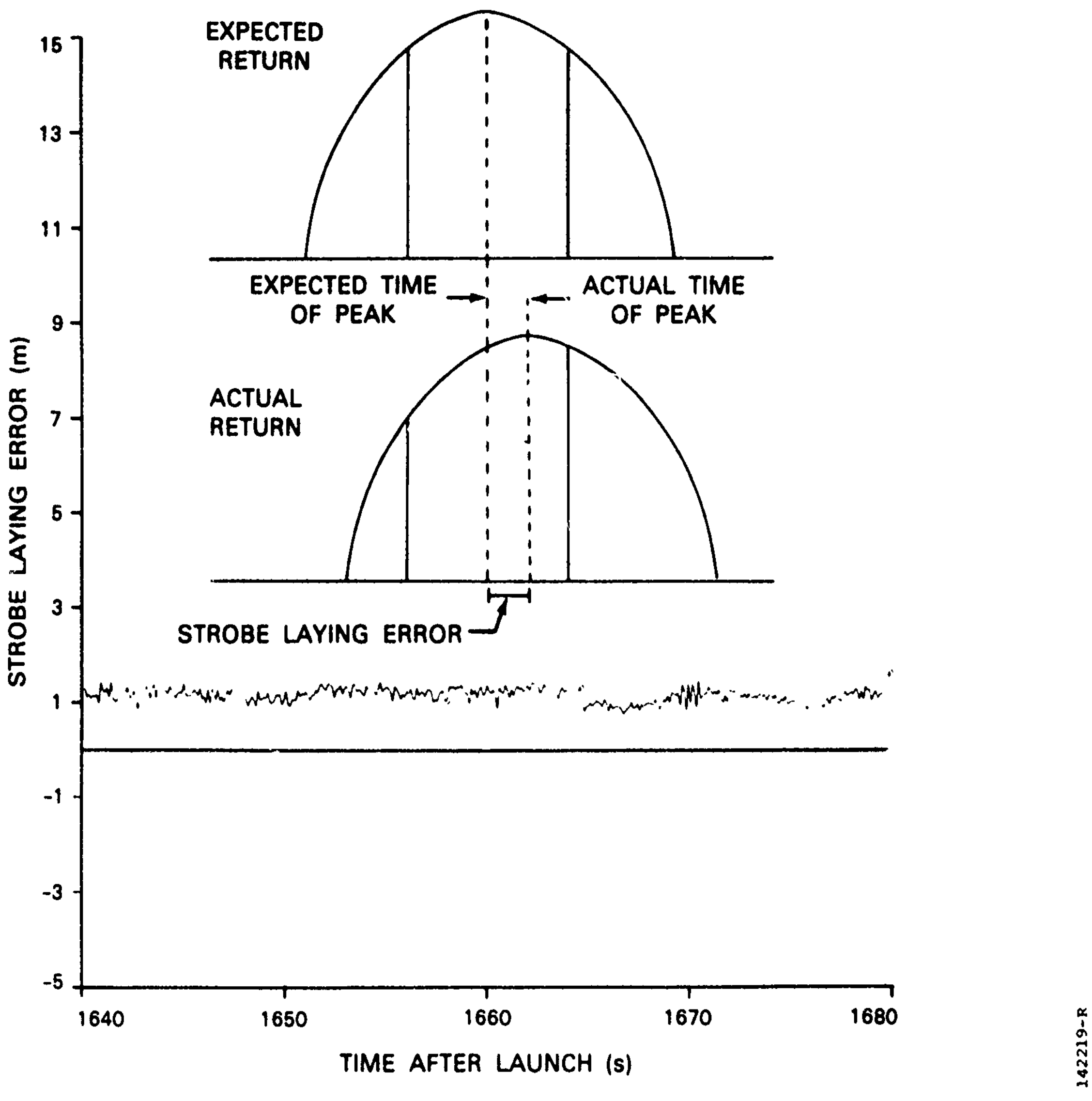

Fig. 7. Strobe laying error for TRADEX range.

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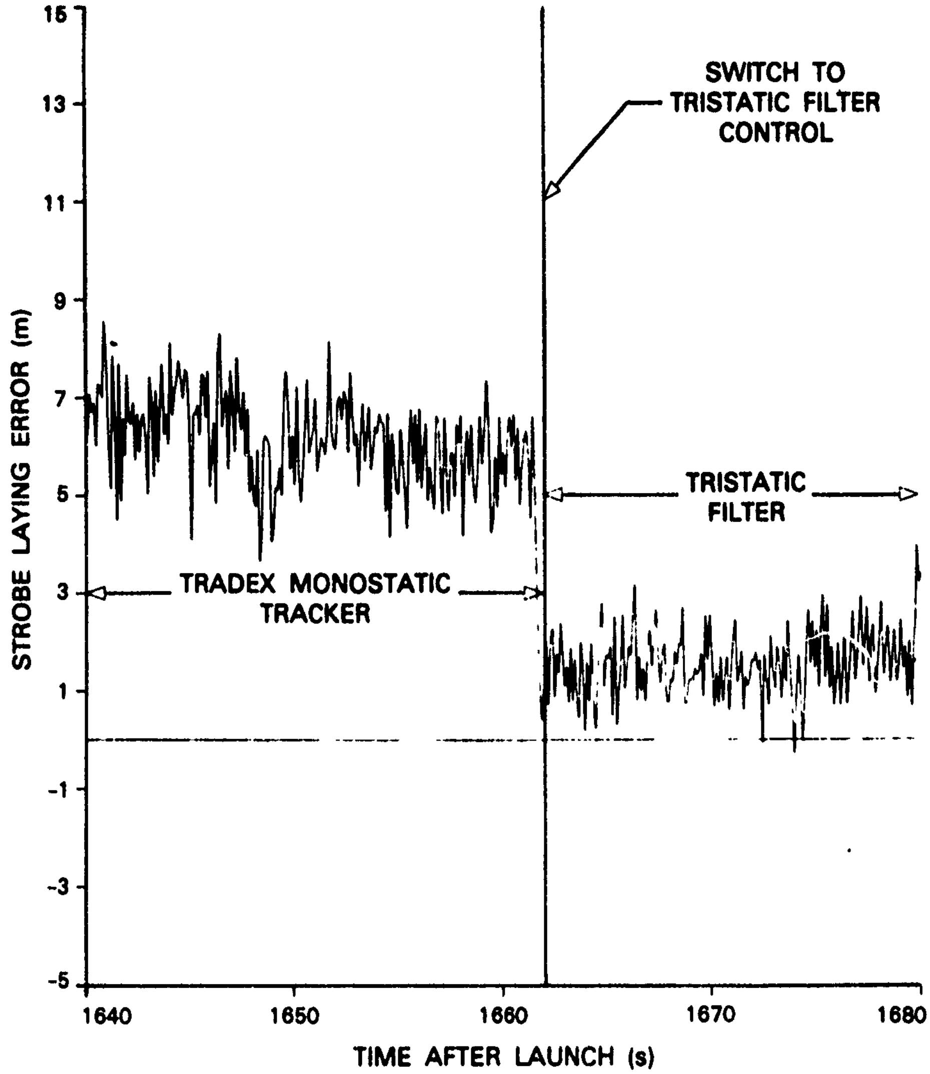

Fig. 8. Strobe laying error for Gellinam bistatic range.

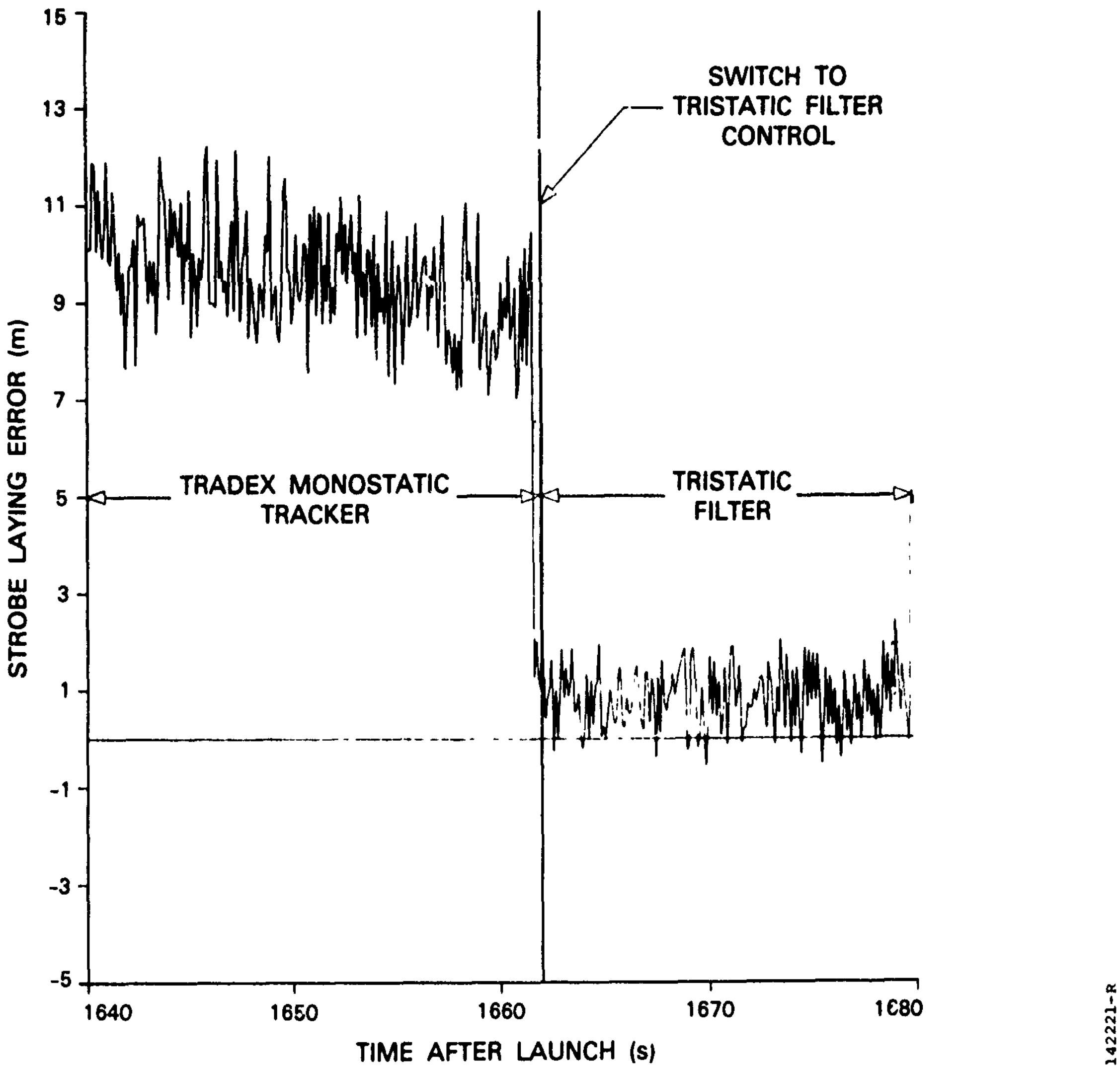

Fig. 9. Strobe laying error for Illeginni bistatic range.

APPENDIX A DETAILS OF THE EQUATIONS OF MOTION

In Section 2 the equations of motion of the target are discussed. This appendix provides the mathematical details. The acceleration due to gravity acting on the target at (x, y, z) is $(g_x, g_y, g_z)^T$ where

$$g_x = x\, g_c$$

$$g_y = [y - R_s \sin(\beta - \alpha)]\, g_c + \cos\beta\, g_d$$

$$g_z = [z + H + R_s \cos(\beta - \alpha)]\, g_c + \sin\beta\, g_d$$

$$g_c = \frac{\mu}{R^3}\left[1 - J_2 \left(\frac{R_E}{R}\right)^2 P_3'(\sin\gamma)\right]$$

$$g_d = \frac{\mu}{R^2}\, J_2 \left(\frac{R_E}{R}\right)^2 P_2'(\sin\gamma)$$

R = distance of target from center of earth

$$R^2 = x^2 + [y - R_s \sin(\beta - \alpha)]^2 + [z + H + R_s \cos(\beta - \alpha)]^2$$

R_E = Equatorial radius of earth

R_s = Radius of earth at TRADEX

H = height of TRADEX above earth

μ = Gravitational constant for earth $(3.986 \times 10^{20}\ \text{cm}^3/\text{sec}^2)$

γ = Geocentric latitude of target

$$\sin\gamma = [R_s \sin\gamma + H \sin\beta + y\cos\beta + z \sin\beta] / R$$

J_2 = Second harmonic of earth's gravity potential

γ = Geocentric latitude of TRADEX

β = Geodetic latitude of TRADEX

P_2 and P_3 are the second and third order Legendre polynomials. Their derivatives are:

$$P_2'(X) = 3X, \qquad P_3'(X) = 1/2\ (15\ X^2 - 3)$$

The target's motion and the earth's rotation lead to the Coriolis acceleration components,

$$A_{cx} = -\ 2\ \omega\ z\ \cos\ \beta + 2\ \omega\ y\ \sin\ \beta + \omega^2 x$$

$$A_{cy} = -\ 2\ \omega\ x\ \sin\ \beta + \omega^2 y\ \sin^2\ \beta$$
$$-\ \omega^2\ \sin\ \beta\ \left[R\ \cos\ \alpha + (H + z)\ \cos\ \beta\right]$$

$$A_{cz} = 2\ \omega x\ \cos\ \beta - \omega^2 y\ \sin\ \beta\ \cos\ \beta$$
$$+\ \omega^2\ \cos\ \beta\ \left[R\ \cos\ \alpha + (H + z)\ \cos\ \beta\right]$$

where

ω = angular velocity of the earth.

The drag model is explained in Section 2.

APPENDIX B JACOBIAN OF EQUATIONS OF MOTION

The Jacobian of the right hand side of the system of equations of motion is $\frac{\partial \vec{f}}{\partial \vec{x}}(\vec{x})$. This 6 by 6 matrix is

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial \dot{x}} & \frac{\partial f_4}{\partial \dot{y}} & \frac{\partial f_4}{\partial \dot{z}} \\ \frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial \dot{x}} & \frac{\partial f_5}{\partial \dot{y}} & \frac{\partial f_5}{\partial \dot{z}} \\ \frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial \dot{x}} & \frac{\partial f_6}{\partial \dot{y}} & \frac{\partial f_6}{\partial \dot{z}} \end{bmatrix}$$

where

$$f_4 = - g_x - A_{cx} - Dv\dot{x}$$

$$f_5 = - g_y - A_{cy} - Dv\dot{y}$$

$$f_6 = - g_z - A_{cz} - Dv\dot{z}$$

$$\frac{\partial f_4}{\partial x} = \frac{\partial g_x}{\partial y} - \omega^2 - vx \frac{\partial D}{\partial x}$$

$$\frac{\partial f_4}{\partial y} = \frac{\partial g_x}{\partial y} - v\dot{x} \frac{\partial D}{\partial y}$$

$$\frac{\partial f_4}{\partial z} = \frac{\partial g_x}{\partial z} - v\dot{x} \frac{\partial D}{\partial z}$$

$$\frac{\partial f_4}{\partial \dot{x}} = - D \left(v + \frac{\dot{x}^2}{v}\right)$$

$$\frac{\partial f_4}{\partial \dot{y}} = - 2 \omega \sin \beta - \frac{D\dot{x}\dot{y}}{v}$$

$$\frac{\partial f_4}{\partial \dot{z}} = 2\ \omega \cos \beta - D \frac{\dot{x}\dot{z}}{v}$$

$$\frac{\partial f_5}{\partial x} = - \frac{\partial g_y}{\partial x} - v\dot{y} \frac{\partial D}{\partial x}$$

$$\frac{\partial f_5}{\partial y} = - \frac{\partial g_y}{\partial y} - \omega^2 \sin^2 \beta - v\dot{y} \frac{\partial D}{\partial y}$$

$$\frac{\partial f_5}{\partial z} = - \frac{\partial g_y}{\partial z} + \omega^2 \sin \beta - \cos \beta - v\dot{y} \frac{D}{z}$$

$$\frac{\partial f_5}{\partial \dot{x}} = 2\ \omega \sin \beta - \frac{D\dot{x}\dot{y}}{v}$$

$$\frac{\partial f_5}{\partial \dot{y}} = - D\ (v + \frac{\dot{y}^2}{v})$$

$$\frac{\partial f_5}{\partial \dot{z}} = - \frac{D\dot{y}\dot{z}}{v}$$

$$\frac{\partial f_6}{\partial x} = - \frac{\partial g_z}{\partial x} - v\dot{z} \frac{\partial D}{\partial x}$$

$$\frac{\partial f_6}{\partial y} = - \frac{\partial g_z}{\partial y} + \omega^2 \sin \beta \cos \beta - v\dot{z} \frac{\partial D}{\partial y}$$

$$\frac{\partial f_6}{\partial z} = - \frac{\partial g_z}{\partial z} - \omega^2 \cos^2 \beta - v\dot{z} \frac{\partial D}{\partial z}$$

$$\frac{\partial f_6}{\partial \dot{x}} = - 2\omega \cos \beta - D \frac{\dot{x}\dot{z}}{v}$$

$$\frac{\partial f_6}{\partial \dot{y}} = - D \frac{\dot{y}\dot{z}}{v}$$

$$\frac{\partial f_6}{\partial \dot{z}} = - D\ (v + \frac{\dot{z}^2}{v})$$

APPENDIX C MEASUREMENTS AS FUNCTIONS OF TARGET POSITION

The formulae for the measurements in the notation in Fig. 1 are the following

TRADEX range is

$$R_T = \sqrt{x^2 + y^2 + z^2}$$

TRADEX azimuth is

$$A = \arctan(x/y)$$

TRADEX elevation is

$$E = \arctan \frac{z}{\sqrt{x^2 + y^2}}$$

The Gellinam bistatic range sum is

$$S_G = R_T + R_G$$

where

$$R_G = \sqrt{(x-x_G)^2 + (y-y_G)^2 + (z-z_G)^2}$$

The Illeginni bistatic range sum is

$$S_I = R_T + R_I$$

where

$$R_I = \sqrt{(x-x_I)^2 + (y-y_I)^2 + (z-z_I)^2}$$

The Sequential Update in the tristatic filter uses the partial derivatives of these five measurement functions. These derivatives are:

$\frac{\partial R_T}{\partial x} = \frac{x}{R_T}$	$\frac{\partial R_T}{\partial y} = \frac{y}{R_T}$	$\frac{\partial R_T}{\partial z} = \frac{z}{R_T}$
$\frac{\partial A}{\partial x} = \frac{y}{x^2 + y^2}$	$\frac{\partial A}{\partial y} = - \frac{x}{x^2 + y^2}$	$\frac{\partial A}{\partial z} = 0$
$\frac{\partial E}{\partial x} = \frac{xz}{R_T{}^2 \sqrt{x^2 + y^2}}$	$\frac{\partial E}{\partial y} = \frac{yz}{R_T{}^2 \sqrt{x^2 + y^2}}$	$\frac{\partial E}{\partial z} = \frac{x^2 + y^2}{R_T{}^3}$
$\frac{\partial S_G}{\partial x} = \frac{x}{R_T} + \frac{x - x_G}{R_G}$	$\frac{\partial S_G}{\partial y} = \frac{y}{R_T} + \frac{y - y_G}{R_G}$	$\frac{\partial S_G}{\partial z} = \frac{z}{R_T} + \frac{z - z_G}{R_G}$
$\frac{\partial S_I}{\partial x} = \frac{x}{R_T} + \frac{x - x_I}{R_I}$	$\frac{\partial S_I}{\partial y} = \frac{y}{R_T} + \frac{y - y_I}{R_I}$	$\frac{\partial S_I}{\partial z} = \frac{z}{R_T} + \frac{z - z_I}{R_I}$

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER ESD-TR-84-035	2. GOVT ACCESSION NO. AD-A146885	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* Tristatic Tracking Filter Used by the Multistatic Measurement System		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER Technical Report 699
7. AUTHOR(s) Michael L. Smith		8. CONTRACT OR GRANT NUMBER(s) F19628-80-C-0002
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lincoln Laboratory, M.I.T. P.O. Box 73 Lexington, MA 02173-0073		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element No. 65301A
11. CONTROLLING OFFICE NAME AND ADDRESS KMRD BMDSC-R Department of the Army P.O. Box 1500 Huntsville, AL 35807		12. REPORT DATE 12 September 1984 13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)* Electronic Systems Division Hanscom AFB, MA 01731		15. SECURITY CLASS. *(of this report)* Unclassified 15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

None

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

mathematical models	KREMS	tracking filter
algorithms	TRADEX	ballistic trajectory
tristatic tracking filter	Kalman filter	ballistic Kalman filter
multistatic measurement system		

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report describes the mathematical models and algorithms that comprise the ballistic tristatic tracking filter used by the Multistatic Measurement System at the Kiernan Reentry Measurement Site.

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