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Correction for Atmospheric Refraction in an Airborne, Operational Environment

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31 August 1984

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LEXINGTON, MASSACHUSETTS

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FOR THE COMMANDER

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J.M. SORVARI

Group 38

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ABSTRACT

A scheme for correcting for the effect of atmospheric refraction is described. The scheme minimizes the amount of computation required in real-time applications. Values derived reproduce observed data very well. There is a residual uncertainty in typical airborne applications; using the simplest scheme this amounts to about 33 μr. A few suggested improvements to the scheme reduce the uncertainty below 10 μr.

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I INTRODUCTION

Recent years have seen increasing deployment of complex, airborne systems for which highly accurate pointing calculations are needed. These calculations are used for real-time pointing of various sensors at selected targets and for reduction of observed target locations to a common reference frame. One of the limitations on the accuracy which can be achieved in these calculations is the displacement of the apparent target location by atmospheric refraction. Ground-based observers can use empirical data to correct for this displacement. Airborne observers must predict the magnitude of the effect for their individual locations using some sort of refraction model.

In a recent report (reference 4) K-P. Dunn describes one such model. Dunn's algorithm corrects for refraction for arbitrary observer and target locations. The calculation is geometrically exact and requires a rather great amount of computation for each correction. Such a large computational burden may be difficult to support in real-time applications.

This report discusses refraction from a somewhat different viewpoint and lays out a correction scheme specifically designed to minimize real-time computation. In addition, the sources of uncertainty are evaluated for typical operating conditions. Methods for reducing several of the contributions to the total uncertainty are proposed.

II ASTRONOMICAL REFRACTION

The precise, instantaneous image displacement due to atmospheric refraction is very complicated and depends on detailed atmospheric data which are not usually available, especially in real time. As a result, an exact calculation is not possible. Fortunately, the actual behavior usually differs only slightly from a mean displacement which is simply an increase in the apparent elevation of an object with no change in the apparent azimuth. The magnitude of this change in elevation for an astronomical (i.e., "infinitely" distant) object is called the astronomical refraction. It may be calculated using various models and may be accurately measured at appropriate ground-based observatories. There exist standard tables of astronomical refraction; reference (1) is a frequently used example.

Astronomical observations are typically made at elevations greater than about 15°. In this range the calculated refraction is insensitive to details of the model used and the results are well represented by

$$REF = R_o \text{ cotangent (elevation)} \qquad (1)$$

where R_o is a refraction parameter which depends upon atmospheric conditions at the observer's (surface) location. At lower elevations the model details become important, and the relation in EQN (1) fails. In this case recourse to empirical tables is ordinarily made. There are no empirical tables appropriate for airborne environment so a physical model will be needed. The physics needed for this is straightforward so this approach is likely to be reliable, but there is a caveat. The empirical data, against which the technique of refraction modeling is proved, consists of measurements made at locations specifically chosen for their benign atmospheric characteristics. There remains some chance that anomalous vertical refraction (as in "mirage" phenomena) and horizontal refraction (azimuth changes) may be present.

Figure 1 shows the geometry of the refraction calculation. Following usual convention, a local zenith distance is used instead of its complement, the local elevation. Numerical tables developed in this report will, however, be listed by elevation. An outward going ray* at geocentric distances r is incident at angle ϕ on the interface between media with refractive indices n and $n + \Delta n$. The ray is refracted by $\Delta\Gamma$ and proceeds outwards to the next interface at $r + \Delta r$ where it is incident at angle $\phi + \Delta\phi$. The flight over Δs from r to $r + \Delta r$ subtends angle $\Delta\theta$ at the Earth's center. The physics involved is Snell's Law:

$$n \sin \phi = (n + \Delta n) \sin (\phi + \Delta\Gamma).$$

This can be solved for $\Delta\Gamma$:

$$\sin \Delta\Gamma = \frac{n \sin \phi \left\{\cos \phi - \left[\cos^2\phi + 2\,\Delta n/n + (\Delta n/n)^2\right]^{1/2}\right\}}{n + \Delta n} \tag{2}$$

or in the infinitesimal limit:

$$d\Gamma = -\frac{dn}{n} \tan\phi. \tag{2a}$$

*It is somewhat easier to model the behavior of an outgoing ray instead of the relevant incoming ray. Fortunately Snell's Law contains nothing about the sense of the propagation direction so the results obtained for the outgoing ray will apply as well to an incoming ray.

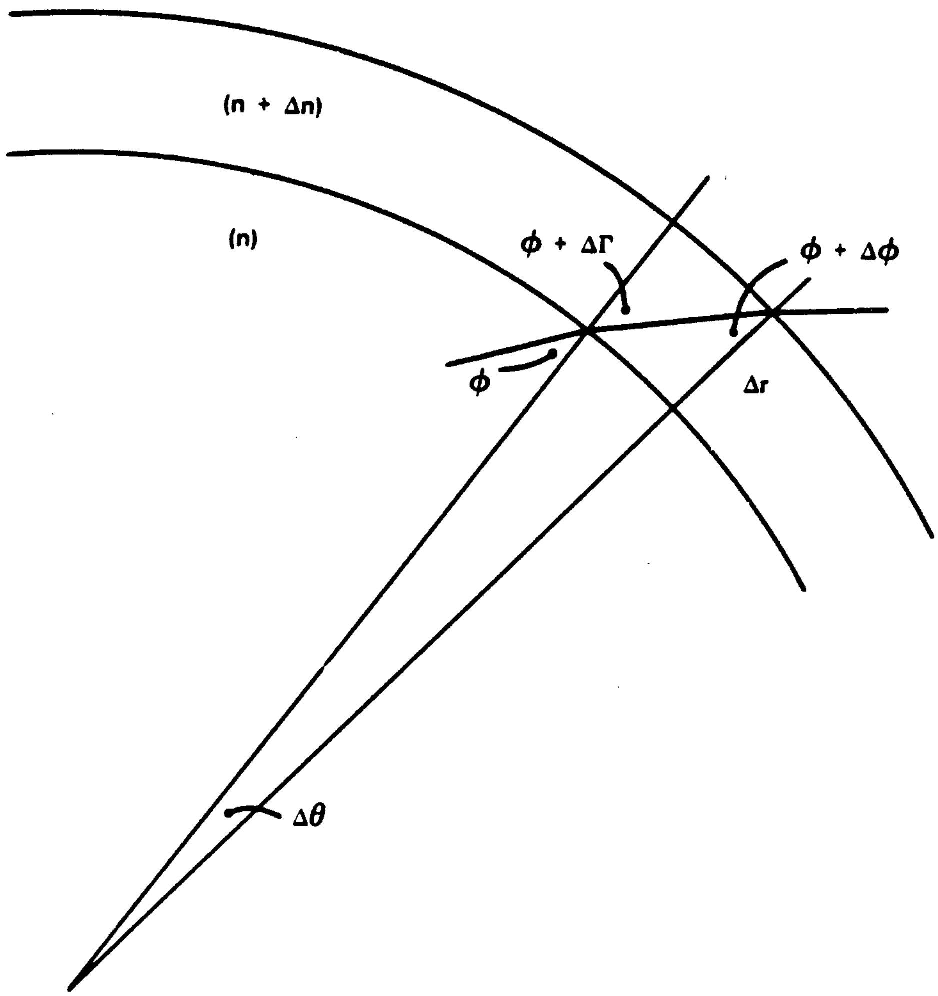

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Figure 1. Geometry for the calculation of atmospheric refraction.

Adding the interior angles of the triangle gives:

$$\Delta\phi + \Delta\theta - \Delta\Gamma = 0, \tag{3}$$

or

$$d\phi + d\theta - d\Gamma = 0 \tag{3a}$$

Finally, applying the cosine law to the triangle, leads, after considerable manipulation, to:

$$\sin \Delta\theta = \frac{\sin\phi'}{r} \left\{r' \cos\phi' - \left[r'^2 \cos^2\phi' - (r'^2 - r^2)\right]^{1/2}\right\} \tag{4}$$

where

$$\phi' = \phi + \Delta\phi, \text{ and}$$

$$r' = r + \Delta r$$

This reduces to:

$$d\theta = \frac{dr}{r} \tan\phi \tag{4a}$$

Inserting (4a) and (2a) into (3a) leads to

$$d(nr \sin \phi) = 0.$$

Evaluating the product at the observer's location gives the useful relationship:

$$nr \sin \phi = n_o r_o \sin \phi_o \tag{5}$$

Note that ϕ_o is the observed (apparent) zenith distance of the ray arriving at the observer's location.

Although equations (2), (3), and (4) are the exact equations for finite steps, and equaticns (2a), (3a), and (4a) give only approximate values, it should be noted that the finite steps are already an approximation of the continuous atmosphere. In addition, equations (2), (3), and (4) comprise a set of coupled transcendental equations -- a nasty business. Integration was therefore carried out using the infinitesimal forms. These were recast to avoid problems with tan ϕ at small elevation angles ($\phi \approx \pi/2$). Defining a refraction parameter

$$\gamma = -\frac{1}{n}\frac{dn}{dr}$$

The equations for the model become:

$$\Delta\Gamma = \gamma \sin\phi \, \Delta s,$$

$$\Delta\phi = (\gamma - 1/r) \sin\phi \, \Delta s, \quad \text{and}$$

$$\Delta r = \cos\phi \, \Delta s.$$

These equations were then used to integrate outward from the observer (at r_o, ϕ_o) to a point at which negligible additional refraction is produced. Initially the step size Δs was set very small and the upper limit on r very large. The step was then increased and the upper limit decreased until an error of about 1 μr was produced. This point was arbitrarily chosen as the optimum point in the trade-off of accuracy vs. speed.

The "natural" input parameter for calculation of Γ is the apparent zenith distance ϕ_o. When treated as a function of the apparent zenith distance, the refraction will be denoted Γ_1. By iterating on the input angle it is also possible to produce values of Γ treated as a function of ($\phi_o + \Gamma$) -- i.e., the true zenith distance of an astronomical object. When this is done, the refraction will be denoted Γ_2. Each version has its place. Throughout most of this report Γ will be written without subscript because the choice is irrelevant to the point under discussion. In fact, the numerical examples were obtained with version Γ_1, except for a single Γ_2 value on page 24.

In order to calculate the refraction it is necessary to derive values of the refraction parameter Υ. Several possibilities exist. The simplest would be to invent a model of n as a function of r. A better approach would be to model the atmospheric pressure and temperature profiles and calculate n from these. The approach taken here was to use the temperature and pressure profiles given in the set of standard supplemental atmospheres (reference 2) and calculate n from these. The supplemental atmospheres represent a combination of physical theory and empirical data; they are given for several latitudes and for summer and winter conditions. The refractive index was then calculated using currently adopted relationships (reference 3):

$$(n-1) \times 10^6 = 272.6\, q(\lambda)\, f(p,t)$$

where

$$q(\lambda) = 0.236 + 108.2/(146 - 1/\lambda^2) + 0.937/(41 - 1/\lambda^2)$$

with λ in μm, and

$$f(p,t) = \frac{p\left[1 + p(1.049 - .0157t) \times 10^{-6}\right]}{720.893\ (1. + .00366t)}$$

with p in mm Hg and t in °C. The wavelength factor goes to unity for increasing wavelength; $q(.55) = 1.019$; $q(10) - 1 << .001$. The thermodynamic state factor is unity at 760 mm Hg and 15°C. Seven sets of n have been compiled: one for the year-round tropical atmosphere (TRSW); two for midlatitudes, a summer (MLSU) and a winter (MLWI); and four for subarctica, a summer (SASU) a mean winter (SAWI), a cold winter (SAWC), and a warm winter (SAWW). The index of refraction was calculated for 1 km steps up to a height of 50 km. In the ray tracing calculation a four-point interpolation was used to give n and its derivative as functions of h. Beyond 50 km a constant exponential decrease, which was derived from the 40-50 km points, was assumed.

It would be highly desirable to test the refraction model described above against empirical data obtained under a variety of conditions. Unfortunately the only such data available is the accumulated ground-based experience, obtained under a very narrow range of conditions. Table I shows the results of refraction calculated using the MLSU and MLWI atmospheres with the observer at sea-level. In order to facilitate comparison with published discussion of refraction and measured values of refraction, the astronomers' angle unit, the second of arc, is used; 1" = 4.85 μr. Also listed is the empirical data given in reference (1). The agreement is satisfactory. The last column shows the refraction calculated using Eqn (1). Good agreement is obtained above 40° but serious discrepancies have developed by 10° elevation.

TABLE I

ASTRONOMICAL REFRACTION AT SEA-LEVEL

Apparent Elevation	ΔElevation (arc-sec) Calculated, Mid-Latitude Winter	Summer	Allen AQ	Ro" Cot(e)
45°	61	56	59	59
40	72	67	70	71
35	87	80	84	85
30	105	97	101	103
25	130	120	125	127
20	166	153	159	163
15	224	206	215	222
10	334	307	319	337
8	412	378	394	423
6	533	488	509	565
4	741	674	706	849
3	911	824	865	1133
2	1163	1043	1103	1701
1	1566	1384	1481	3403
0.5	1874	1638	1760	6945
0.0	2279	1963	2123	∞

III UNCERTAINTIES

Linearity and Separability

Apart from the simple geometric functionality, EQN (1) carries two implications that are ordinarily assumed true even if a more complicated elevation dependence is used. These are that the refraction depends linearly on some refraction parameter (R_o, which may be an arbitrary function of temperature, wavelength, etc.) and that the geometrical and refractive effects are separable. That this cannot be strictly true is easily seen by examining the details of refraction in the atmosphere. Consider two rays outward bound at O, with the same zenith distance but having different wavelengths and consequently seeing different refractive index. At the first interface the ray seeing the higher refractive index will experience greater refraction in proportion to its greater refractive index. At the second interface this same ray will experience greater refraction both because of its higher index <u>and also</u> because of its greater zenith distance. The total refraction must thus increase faster than linearly with R_o. Because the geometrical dependence is not linear with elevation, the magnitude of the supralinearity must depend on the elevation, and so the effects couple.

This behavior can be seen by exercising the model. Using MLWI, a ground-based observer, elevations of 5° and 10°, and values of $q(\lambda)$ of 1.0 and 1.2 (physically implausible) both effects are produced. At 5° the total refraction for $q = 1.2$ is 20.3% higher than for $q = 1.0$ instead of the linear 20%. This implies generation of an error of 9.4 μr if the linear assumption is used. At 10° the increase has dropped to 20.1%. In addition to verifying the coupled, nonlinear behavior, these numbers show that the effects are small. We shall therefore adopt the form of EQN (1), using the model to get the geometrical relationship at a nominal value of R_0 and then making a linear adjustment for actual wavelength and environmental conditions.

Use of Tables and Interpolation

Although the refraction calculation was adjusted to run as quickly as possible without producing significant error, it still takes about 10 milliseconds to run on the VAX 11/750. This is a burden in real-time operation and should be avoided if possible. An alternative is use of a table and some more or less complicated interpolation scheme. The simplest case would be regularly spaced sample points with a linear interpolation. Table II shows the results of such a scheme. First the model calculation was used to give refraction values at half-degree intervals. The SASU atmosphere at 11.3 km was used in the calculation. Next values for half-odd-integer values of elevation were obtained by averaging the surrounding integer elevation values. The resulting error is given in the last column.

Because of the nature of the dependence of refraction on elevation, linear interpolation always overestimates refraction. The errors at the mid-points can thus be reduced by subtracting half the error from the values for whole degree elevations. Table III shows such a revised table, interpolated values, model calculations, and interpolation errors. As expected, the error at the half integer elevation is reduced at the expense of adding an error at the integer elevations.

TABLE II

LINEAR INTERPOLATION ERROR

Apparent Elevation (deg)	Calculated Refraction	Interpolated Refraction	Interpolation Error
	(- - - - - - - - - - - - -	μr - - - - - - - - - - - - -	)
9.0	461.7		
8.5	487.2	488.6	+1.4
8.0	515.5		
7.5	547.1	549.1	+2.0
7.0	582.6		
6.5	622.7	625.5	+2.8
6.0	668.4		
5.5	720.7	724.9	+4.2
5.0	781.3		
4.5	852.0	858.4	+6.4
4.0	935.5		
3.5	1035.5	1045.9	+10.6
3.0	1156.2		

TABLE III

INTERPOLATION ERROR WITH ADJUSTED TABLE

Apparent Elevation (deg)	Adjusted Refraction	Interpolated Refraction	Calculated Refraction	Interpolation Error
	(- - - - - - - -	- - - - - - - μr	- - - - - - - -	- - - - - - - -)
4.2		900.9	900.3	+0.6
4.1		916.1	917.6	-1.5
4.0	931.4	931.4	935.5	-4.1
3.9		953.2	954.0	-0.8
3.8		975.0	973.2	+1.8
3.7		996.8	993.1	+3.7
3.6		1018.6	1013.8	+4.8
3.5		1040.4	1035.3	+5.1
3.4		1062.1	1057.6	+4.5
3.3		1083.9	1080.8	+3.1
3.2		1105.7	1104.9	+0.8
3.1		1127.5	1130.0	-2.5
3.0	1149.3	1149.3	1156.2	-6.9
2.9		1182.3	1183.5	-1.2
2.8		1215.3	1211.9	-3.4

Although the interpolation error is strictly deterministic in nature we may get some feeling for its importance by analysing it as a random error. Following the procedure above, the RMS interpolation error in the range 0° to 1° elevation is 30 μr or about 1.2% of the refraction at 0.5° elevation. This is, at worst, not the dominant source of error and is, in any event, negligible above a few degrees of elevation. Thus use of a linear interpolation on a table of values at 1° intervals provides satisfactory accuracy and vastly improved computational speed. Construction of appropriate tables should be carried out in the mission planning phase.

Sensitivity to Observer Height

The height h of the observer is an input parameter for the refraction model. As in the case of the refraction parameter, there is no reason to expect the exact dependency of total refraction upon h to be simple. For small enough departures from nominal height h_o we can, however, expect to be able to use an equation of the form

$$\Gamma(h,e) = f(h,e)\ \Gamma\ (h_o,e)$$

in order to simplify calculation of Γ when a range of h is anticipated. In fact, for typical airborne operational altitudes good results can be obtained with a very simple expression for f:

$$f(h,e) = f_o + f'(h - h_o).$$

Using SASU and h_o = 11.3 km the best fit in the range $10.3 \leq h \leq 12.3$ km is given by

$$f_o = 1.0035455$$

$$f' = -0.1494091$$

For e = 10° the "RMS error" introduced by this method is about 1.3 μr. The typical uncertainty in the observer's height, about 30 m, propagates through either the full calculation or the linearized correction to give an uncertainty of 1.9 μr at the same nominal point.

Variation with Latitude

Reference (2) gives model atmospheres for latitude 15°, 30°, 45°, 60°, and 75°. These differ not only in the mean temperature, but also in subtler ways such as scale height. In order to determine the effect of latitude on the refraction, calculations were carried out for tropic, mid-latitude, and sub-arctic atmospheres. Table IV shows the results for the tropic atmosphere and for the middle of the ranges spanned by the several mid-latitude and sub-arctic atmospheres. The calculations were carried out for q = 1.0 and h = 11.3 km. The differences, while not large, are significant at low elevations. Clearly it is worth the trouble to chose the appropriate atmosphere. For intermediate latitudes there is no choice but to interpolate. This is clearly not physically correct, but seems unlikely to introduce more than a few μr error at e = 10°.

TABLE IV

LATITUDE EFFECT

Apparent Elevation	Refraction (μr) TR	ML	SA
20°	220	211	197
18	246	235	220
16	278	266	248
14	318	305	285
12	371	355	333
10	443	424	398
8	547	523	491
6	708	679	638
4	988	950	893
2	1556	1510	1426
0	2966	2997	2924

Variation with Season

Reference (2) gives four sub-arctic atmospheres, one for summer, and three for winter. The accompanying text points out that the winter-warm and winter-cold atmospheres do not represent extremes of a continuous population but rather are typical members of an essentially bimodal distribution. Table V shows the refraction for the four atmospheres; again $q = 1.0$ and $h = 11.3$km. The seasonal effect can be seen to be comparable to the latitude effect, so again it is worthwhile to use the appropriate atmosphere in the calculations. In the case of intermediate seasons, the bimodal distribution in winter makes hash of the idea of interpolation. A plausible course might be to use SASU in summer, choose somehow (weather data, perhaps) between SAWW and SAWC in winter, and use a table midway between SASU and SAWI the rest of the year. This will undoubtedly introduce some error. Judging from the table, this error should be negligible above 20°, be about 4-5 μr at 10°, and grow to about 30 μr by 0°.

Effect of Weather

An airborne measurement of an atmospheric parameter, say temperature, will, in general, differ from the corresponding value in the atmospheric profile used in the refraction calculation. For example, consider a winter flight at 11.3 km with the SAWC atmosphere chosen as appropriate. At that height the model profiles predict about 225K, 222K, 217K, and 217K for SU, WW, WI, and WC respectively. A measurement of 219K would have contributions from measurement error, a local perturbation from the nominal profile, and a global

TABLE V

SEASONAL VARIATION OF REFRACTION

Apparent Elevation	Refraction (μr) SASU	SAWI	SAWW	SAWC
20°	206	192	188	192
18°	230	214	210	214
16°	260	242	237	242
14°	298	278	272	278
12°	348	324	318	324
10°	416	387	380	387
8°	514	479	469	479
6°	666	622	609	622
4°	933	872	853	872
2°	1489	1396	1363	1394
0°	3034	2876	2813	2860

shift of the entire temperature profile. These are completely different physical situations; how should the +2K temperature shift be apportioned? Even if the first two contributions could be ruled out, the situation is no better. Should the refraction be <u>increased</u> by interpolation between WC and SU (217 and 225) or <u>decreased</u> by interpolation between WC and WW (217 and 222)? It appears that it will not be useful to attempt to correct for weather. This might introduce an uncertainty of about the same magnitude as the uncorrected seasonal variation. More likely the effect will be considerably smaller because airborne operations take place near or above the tropopause.

IV THE POSITION CORRECTION

The astronomical refraction Γ and the apparent change in elevation ε are equal only for stars. For nearby objects the ray which will reach the observer after refraction does not start out parallel to the radius vector from object to observer. The geometry is shown in Figure 2. A ray (dashed line) starts from the target point T towards the apparent observer O', with local zenith distance ϕ. The ray is refracted through angle Γ and arrives at the observer O with apparent zenith distance ϕ_o. The true (geometric) zenith distance of the target is ζ. If enough of the geometry is known, the elevation correction ε may be calculated. Two cases occur: (i) an object is observed at ϕ_o and it is desired to find the true zenith distance ζ (or elevation, $\pi/2 - \zeta$); in this case the appropriate argument for Γ is clearly the apparent zenith distance, which is known. (ii) the true zenith distance of an object is calculated, and it is necessary to determine the apparent position at which to point the telescope. In the second case the argument for Γ is a problem. The value of ϕ_o is the unknown, and $\phi_o + \Gamma = \zeta$ only for very large distances. As a result it is necessary either to iterate or to accept an approximation.

Case (i) is appropriate for the reduction of observed locations to a standard frame. Angle COT can be seen to be $\pi - \phi_o - \varepsilon$, angle OTC is $\phi - \Gamma + \varepsilon$, and consequently angle OCT is $\phi_o - \phi + \Gamma$. The observed zenith distance ϕ_o is known and Γ can be found as a function of ϕ_o. The trigonometric

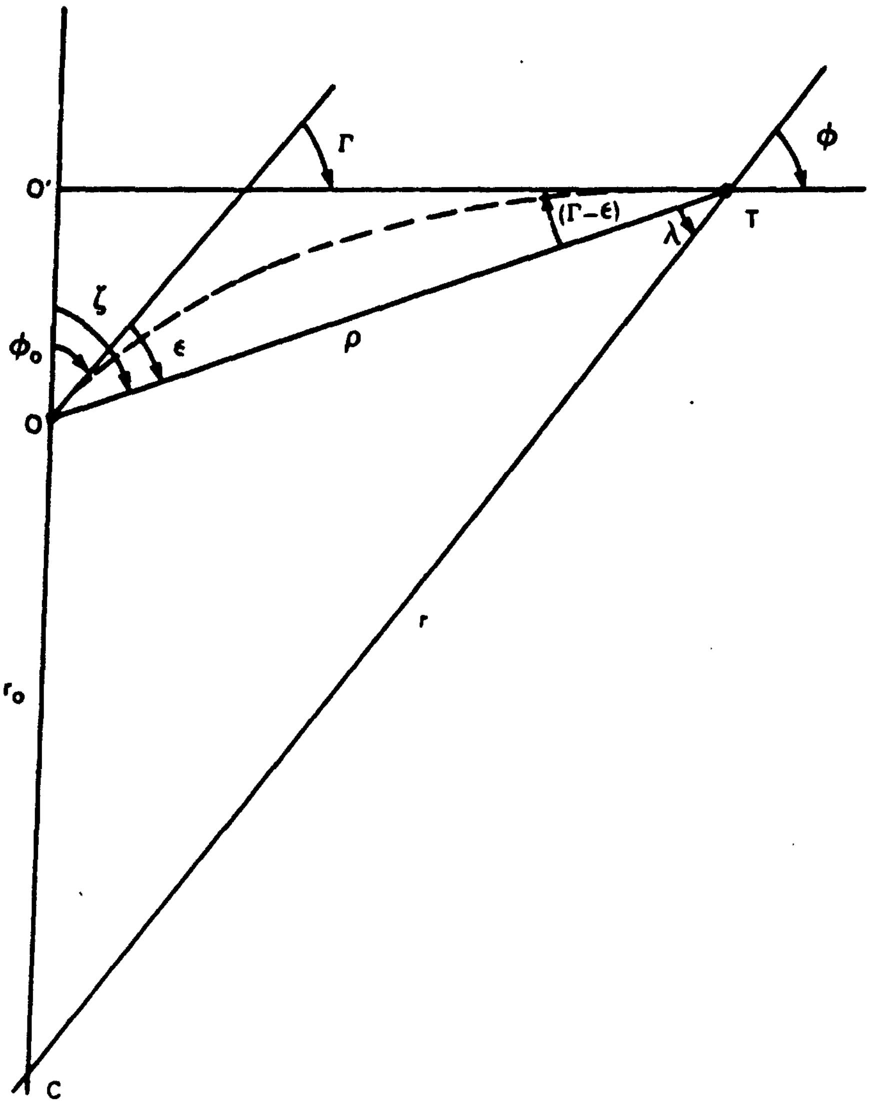

Figure 2. Geometry for the calculation of the elevation correction.

equations plus equation (5) form a set which can be solved for ε. After significant manipulation:

$$\tan \varepsilon = \frac{n_o \cos \Gamma - n_o \cot \phi \sin \Gamma - n}{n \cot \phi_o - n_o \cot \phi \cos \Gamma - n_o \sin \Gamma}$$

This can be recast in a somewhat more illuminating form by eliminating ϕ in favor of ρ, the geometric range to the target, and solving for the difference between the refraction and the elevation correction. Assuming the refractive index at the target is unity:

$$\sin(\Gamma - \varepsilon) = \frac{r_o}{\rho} \left[n_o \sin\phi_o - \sin(\phi_o + \Gamma)\right]$$

Note that this agrees with the astronomical limit: for $\rho \to \infty$, $\varepsilon \to \Gamma$.

Case (ii) is appropriate for calculation of an apparent position from physical data. In this case the triangle OCT is completely known. Combining the trigonometric equations with equation (5) leads to:

$$\tan \varepsilon = \frac{nr \sin (\lambda + \Gamma) - n_o r_o \sin \zeta}{nr \cos (\lambda + \Gamma) - n_o r_o \cos \zeta}$$

Again, a handier relationship may be obtained for $\Gamma - \varepsilon$. For $n \to 1$,

$$\tan(\Gamma - \varepsilon) = \frac{n_o \sin (\zeta - \Gamma) - \sin \zeta}{-n_o \cos (\zeta - \Gamma) + \cos \zeta + \rho/r_o}, \text{ or} \tag{6}$$

$$(\Gamma - \varepsilon) \approx \frac{r_o}{\rho} \left[n_o \sin (\zeta - \Gamma) - \sin \zeta\right] \tag{6a}$$

As was noted above, neither of the possible arguments for Γ is known. A numerical example shows how to deal with this and also puts the entire problem in proper perspective. For input, use the following:

Observer Height: $h = 11.3$ km

Atmosphere: SAWI

True Zenith Distance: $\zeta = 80°$

Range to Target: $\rho = 400$ km

Wavelength Factor: $q = 1.0$

Observer Geocentric Distance: $r_o = 6375.15$ km

The first approximation for the refraction can be obtained using ζ as the argument either in $\Gamma_1(\phi_o)$ or $\Gamma_2(\phi_o + \Gamma)$. These give:

$\Gamma_1(\zeta) = 389.2\ \mu r$ and

$\Gamma_2(\zeta) = 388.4\ \mu r$

The remainder of the example will continue with the Γ_1 version. Next, $\Gamma - \varepsilon$ can be calculated either from EQN (6) or EQN (6a):

$\Gamma - \varepsilon = 26.8\ \mu r$ (EQN 6)

$\Gamma - \varepsilon = 26.6\ \mu r$ (EQN 6a)

From this a new approximation to $\phi_o = \zeta - \epsilon$ may be obtained:

$$\phi_o = 79.979236°,$$

leading to a new Γ

$$\Gamma_1 = 388.4 \ \mu r,$$

and a new $(\Gamma - \epsilon)$

$$\Gamma - \epsilon = 26.7 \ \mu r,$$

a new ϕ_o

$$\phi_o = 79.979276°,$$

a new Γ

$$\Gamma_1 = 388.4 \ \mu r,$$

and so on. Clearly this is far enough. The following conclusions are now apparent:

- The first approximation using Γ_2 or the second approximation using Γ_1 are both adequate.
- Equation (6a) is sufficiently accurate (unless ρ is very small)
- The elevation correction ϵ is approximately equal to the refraction Γ.

This last conclusion is strengthened when greater ranges are considered. For $\rho = 600$ km, $(\Gamma - \epsilon) = 17.8 \ \mu r$, and for $\rho = 800$ km, $(\Gamma - \epsilon) = 13.4 \ \mu r$.

Airborne systems are not ordinarily controlled in azimuth a and zenith distance z, but in an inertial coordinate system. For this memo the coordinate system will be taken to be the astronomical right ascension (α), declination (δ) system, although there will probably be a flight-constant offset from true α. Thus it will be necessary to calculate corrections to α and δ which are the result of the correction ε to the zenith distance. The relation between the two coordinate systems involves both a constant tilt which depends on the astronomical latitude B and longitude Λ, and a slip which involves the Greenwich sidereal time θ_g. An intermediate coordinate system - hour angle h, declination δ is usually defined. This system has the same pole as the (α,δ) - system but uses the observer's meridian as the zero of the cyclic coordinate h, which is a left-handed angle. The relationships are:

$$h = \theta_g + \Lambda - \alpha, \quad \text{and}$$
$$\delta = \delta$$

Figure 3 shows the relationship between (h,δ), and (a,z). In this figure Z is the observer's zenith, P is the north celestial pole, and T is the two-dimensional location of the target. The systems are related by a fixed rotation:

$$\begin{pmatrix} \sin z \cos a \\ \sin z \sin a \\ \cos z \end{pmatrix} = \begin{pmatrix} -\sin B & 0 & \cos B \\ 0 & -1 & 0 \\ \cos B & 0 & \sin B \end{pmatrix} \begin{pmatrix} \cos\delta \cos h \\ \cos\delta \sin h \\ \sin\delta \end{pmatrix}$$

which can be solved to give (a,z) as a function of (h,δ) and B or (h,δ) as a function of (a,z) and B. Alternatively, the spherical triangle PZT may be solved to provide these equations.

The straightforward way to calculate the corrections to h and δ is to solve for (a,z) corresponding to (h,δ), correct for refraction by $a' = a$ and $z' = z - \varepsilon$, and finally solve for (h',δ') corresponding to (a',z'). This is clearly an inappropriately large amount of computation. It is possible to reduce the amount of computation somewhat by taking advantage of the small

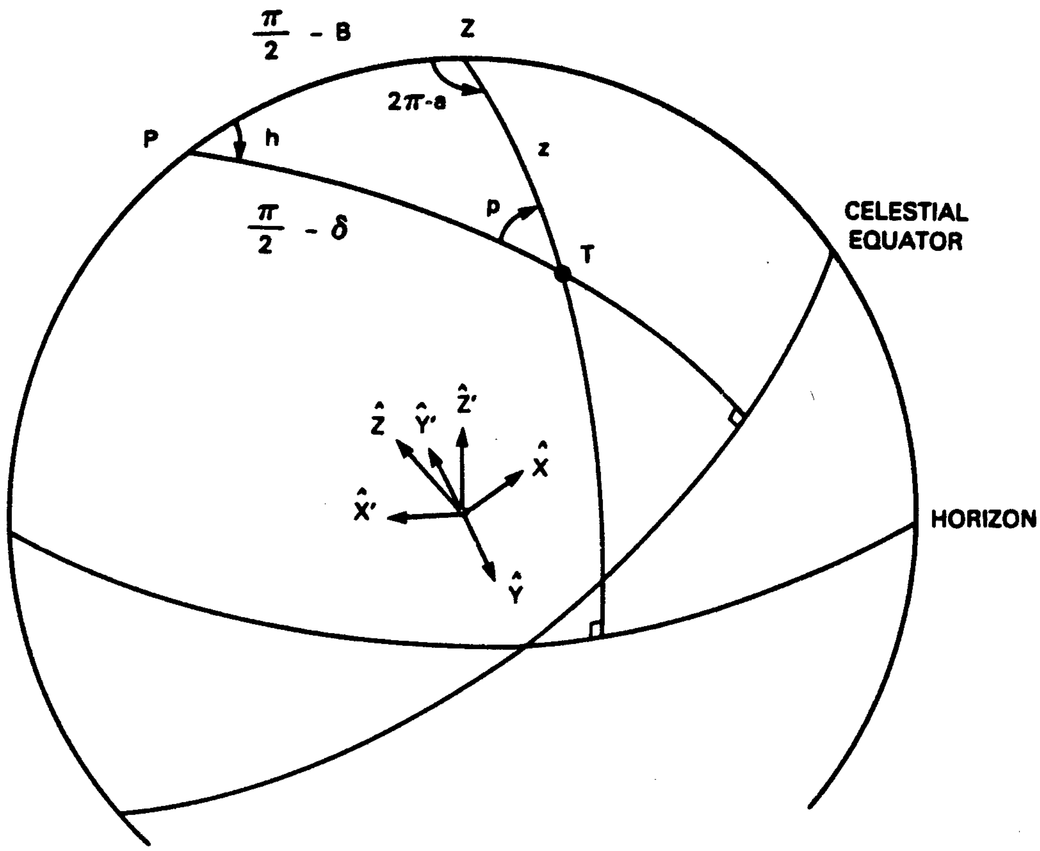

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Figure 3. Relationship between the celestial and horizon coordinate systems.

angle approximations for functions of ε. This is further facilitated by use of an auxiliary quantity, the parallactic angle p in Figure 3. The parallactic angle may be found using the following sequence of calculations:

$$\cos z = \sin B \sin \delta + \cos B \cos \delta \cos h$$

$$\sin z = +[1 - \cos^2 z]^{1/2}$$

$$\cos p = (\sin B - \cos z \sin \delta)/\sin z \cos \delta$$

$$\sin p = \cos B \sin h/\sin z$$

then

$$\delta' - \delta \approx \varepsilon \cos p, \quad \text{and}$$

$$\alpha' - \alpha = h - h' \approx \varepsilon \sin p/\cos \delta' \qquad (7)$$

At $z = 80°$, $a = 300°$, and $\varepsilon = 400\ \mu r$, use of these approximations introduces and error of about a hundreth of a microradian. Further simplifications may be appropriate in specific applications. These include use of encoder values for z instead of their calculation, use of mean or constant values for B and p, etc. Evaluation of the error introduced by such measures is beyond the scope of this study.

V SUMMARY

The following scheme is suggested for correcting calculated pointing angles for refraction in an operational environment.

• During mission planning, calculate tables of astronomical refraction as a function of true elevation (i.e., Γ_2). The input parameters should span the anticipated ranges of observer height, latitude, and season. The tables should be adjusted to correct for the effect of linear interpolation (see p. 12). Values of n_o should also be calculated.

• During mission planning, produce a single refraction table and a single value of n_o from the results of the previous step. These should be evaluated for the nominal observer height, and "interpolated" for specific latitude and time of year. Values of dn_o/dh, f_o, and f' (see p. 16) should also be obtained from the variations over the range of observer heights.

• In near-real time, obtain the appropriate values of Γ_2 for the relevant elevation by linear interpolation from the table. Correct Γ_2 and n_o for the actual observer height.

• In near-real time, calculate the elevation correction ε using EQN (6a) and the pointing corrections $\Delta\alpha$ & $\Delta\delta$ using EQN (7).

Correction for atmospheric refraction on the basis of this scheme involves several assumptions and approximations. Table VI summarizes these; magnitudes of uncertainties are evaluated for 10° elevation, and observer height 11.3 km. Uncertainties in the refraction correction all have very low frequency spectra. The first entry is a reminder that there is no evidence which allows us to rule out the possibility of anomalous refraction. The most problematical of the remaining uncertainties is the weather effect. The value is in fact little better than a guess at the upper limit. If the uncertainties are combined (RSS) it is seen that even a crude treatment is satisfactory for many airborne system missions, giving a total uncertainty of about 33μr. In this case the weather effect is just one of several contributors, so reduction of this figure is of little consequence. It is probably desirable to improve this, or equivalently to produce this acceptable error at lower elevations. This is particularly true in light of the relatively low burden imposed by the improved calculations. In that case, the size of the weather induced uncertainty is crucial. If, as was suggested in Section III, the figure listed is a great overestimate of the actual effect of weather, then the total error can be reduced to less than 10 μr.

TABLE VI

SUMMARY OF UNCERTAINTIES AND ERRORS
IN THE REFRACTION CORRECTION

Source		Magnitude
Assumption that refractive index is monotonic function of height only		Unknown
Assumption of linearity and separability		< 1 μr
Use of table and interpolation		< 1 μr
Sensitivity to observer height		≈ 2.5 μr
Latitude Effect	Uncorrected	≈ 20 μr
	Interpolated	≈ 2 μr
Seasonal Variation	Uncorrected	≈ 15 μr
	Interpolated	≈ 5 μr
Weather		< 15 μr?
Calculation of ε	Set equal to Γ	≈ 15 μr
	Equation (6a)	< 1 μr
Calculation of $\Delta\alpha$, $\Delta\delta$		<< 1 μr

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A scheme for correcting for the effect of atmospheric refraction is described. The scheme minimizes the amount of computation required in real-time applications. Values derived reproduce observed data very well. There is a residual uncertainty in typical airborne applications; using the simplest scheme this amounts to about 33 μr. A few suggested improvements to the scheme reduce the uncertainty below 10 μr.