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AN INTRODUCTION TO FUZZY REGRESSION ANALYSIS WITH A BALLISTIC APPLICATION

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US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
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Regression models are frequently used for predicting, but quite often the value assumed by the independent variable is somewhat vague. That problem is overcome in this paper by utilizing a combination of fuzzy set methodology and conventional statistical techniques; specifically, fuzzy regression analysis. Fuzzy regression analysis is applied to a particular vulnerability problem, and the difference between this approach and one which utilizes only probability theory is briefly examined.

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I. INTRODUCTION

In November 1982, the Vulnerability/Lethality Division of the Ballistic Research Laboratory organized and sponsored the Joint Working Group on Fuzzy Set Methodology, whose purpose has been to explore the possibility of applying this relatively new approach to existing problems in vulnerability analysis. The work presented in this report is a direct consequence of my participation in this group.

Frequently, a regression model is developed based on past data which are precise; that is, measured. However, when this model is used for predicting purposes, the value of the independent variable cannot be accurately determined. For example, consider a model in which the independent variable is federal government budget deficit. The values of past budget deficits are available; however, the exact amount of the 1990 budget deficit, although we know it will be "high," is presently unknown and can only be estimated. If we want to use the regression model for predicting the value of the dependent variable in 1990, then we must quantify what is meant by "high"; this is done subjectively and through the use of fuzzy set methodology. "High" is represented not as a single point, but rather as a range of values characterized by a central value and a measure of spread.

The early concepts of fuzzy sets were developed by L.A. Zadeh [1]. Results presented here utilize some of the subsequent research concerning properties of fuzzy sets and operations on fuzzy numbers; in particular, that published by Zadeh [2], S.M. Baas and H. Kwakernaak [3], D. Dubois and H. Prade [4], M. Mizumoto and K. Tanaka [5] and R.R. Yager [6]. Especially useful was the work in fuzzy regression analysis by Yager [7] and H. Tanaka, S. Uejima, and K. Asai [8].

II. FUZZY SET METHODOLOGY

Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of elements and S be a subset of X. Then we can define the characteristic function μ_S: $X \rightarrow \{0, 1\}$ such that

$$\mu_S(x_i) = \begin{cases} 1 \text{ if } x_i \in S \\ 0 \text{ if } x_i \notin S \end{cases} \cdot \tag{1}$$

If, however, X is the set of men and S is taken to be the set of old men, there may be some vagueness about the membership of certain x_i in S. Is a 50-year-old man a member of S? I used to think so; but now that I'm older, I'm not quite so sure. Suppose we let μ_S take on values other than 0 and 1; in particular, any value between 0 and 1 so that μ_S: $X \rightarrow [0, 1]$.

In this case, S is called a fuzzy subset of X and μ_S is called the membership function (M.F.) of S. Each x_i has associated with it a value $\mu_S(x_i)$ representing a degree of membership in S. The closer this value is to one, the more completely the associated x_i is a member of S. The fuzzy subset S is written

$$S = \{x_1 | \mu_S(x_1), x_2 | \mu_S(x_2), \cdots, x_n | \mu_S(x_n)\}. \tag{2}$$

Numerical data can be represented by equating X with the set of real numbers, in which case S is called a fuzzy number.

Consider the fuzzy number "near 7" with an associated M.F.

$$\mu(x) = e^{-(x-7)^2}. \tag{3}$$

A graph of the membership function is shown in Figure 1. Note that it has a central value of seven ($\mu(7) = 1$) and that the measures of spread to the left and to the right are equal ($\mu(x)$ is symmetric). This membership function is not unique but rather is a subjective effort to represent the fuzzy number. A number that is not fuzzy is called crisp, and a membership function can also be defined for it. For example, crisp 7 has an associated M.F. equal to the characteristic function

$$\mu(x) = \begin{cases} 1 \text{ if } x = 7 \\ 0 \text{ if } x \neq 7 \end{cases}. \tag{4}$$

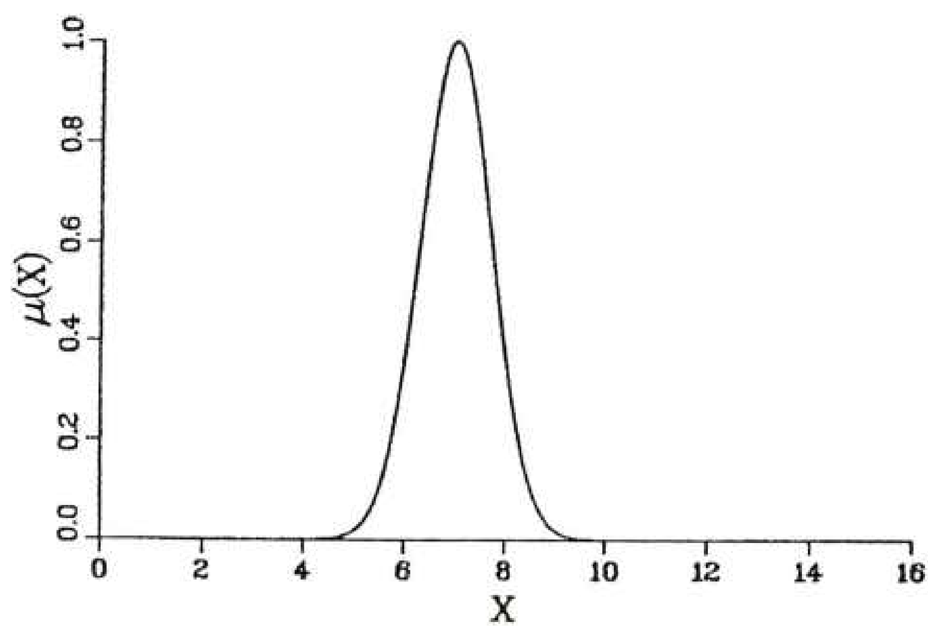

FIGURE 1: MEMBERSHIP FUNCTION ASSOCIATED WITH "NEAR 7"

The Extension Principle [2] provides a method by which functions can be extended to act on fuzzy sets. Let $F: X \times Y \rightarrow Z$. Then if A is a fuzzy subset of X with associated M.F. $\mu_A(x)$ and B is a fuzzy subset of Y with associated M.F. $\mu_B(y)$, then C is a fuzzy subset of Z where its M.F. is defined as

$$\mu_C(z) = \underset{\substack{x,y \\ F(x,y)=z}}{\text{MAX}} \left[\text{MIN}\left(\mu_A(x), \mu_B(y)\right)\right]. \tag{5}$$

We can apply this principle to fuzzy numbers. Using an example from Yager [7]:

$$\begin{aligned} &\text{Let A} = \text{"near 7"} \quad \text{with } \mu_A(x) = e^{-(x-7)^2} \\ &\text{and B} = \text{"near 2"} \quad \text{with } \mu_B(y) = e^{-(y-2)^2}, \end{aligned} \tag{6}$$

and we can evaluate the sum of these two fuzzy numbers. In this case, our function is addition where $F: R \times R \rightarrow R$. Then, by the Extension Principle, C is a fuzzy subset of R;

and its M.F. is defined as

$$\mu_C(z) = \underset{\substack{x,y \\ x+y=z}}{\mathrm{MAX}} [\mathrm{MIN}(e^{-(x-7)^2}, e^{-(y-2)^2})] \qquad (7)$$

$$= \underset{x}{\mathrm{MAX}} \;[\mathrm{MIN}(e^{-(x-7)^2}, e^{-(z-x-2)^2})]$$

since y is equal to z - x. Results, explained in the succeeding paragraph, due to Dubois and Prade [4] concerning operations on fuzzy numbers show that the above expression simplifies to

$$\mu_c(z) = e^{-\left(\frac{z-9}{2}\right)^2}. \qquad (8)$$

Therefore, C is a fuzzy number that might be interpreted as "close to 9."

Dubois and Prade [4] demonstrate that fuzzy numbers whose membership functions satisfy certain properties can be represented as an ordered triple (m, α, β) where m is the central value (the point at which the membership function achieves the value one) and α and β are measures of spread in the left and right directions respectively. When we deal with a membership function of the form $e^{-\left(\frac{x-m}{\alpha}\right)^2}$, the ordered triple would be (m, α, α), since it is symmetric about the central value. Dubois and Prade provide definitions of addition and subtraction and approximations for multiplication and division, all consistent with the Extension Principle. Crisp numbers are a special case in which a number n is represented as the ordered triple (n, 0, 0). Assuming both m and n are positive, the operations are defined as follows:

$$\begin{aligned}
(m, \alpha, \beta) \oplus (n, \gamma, \delta) &= (m+n, \alpha+\gamma, \beta+\delta) \\
(m, \alpha, \beta) \ominus (n, \gamma, \delta) &= (m-n, \alpha+\gamma, \beta+\delta) \\
(m, \alpha, \beta) \otimes (n, \gamma, \delta) &\approx (mn, m\gamma+n\alpha, m\delta+n\beta) \\
(m, \alpha, \beta) \oslash (n, \gamma, \delta) &\approx (m/n, (m\delta+n\alpha)/n^2, (m\gamma+n\beta)/n^2).
\end{aligned} \qquad (9)$$

If either m or n is negative, the approximations are slightly different. In any case, the approximations are most nearly accurate when α and β are small compared with m, and γ and δ are small compared with n; in other words, when the numbers are less fuzzy. Notice that in many cases, the product and quotient are more fuzzy than the individual factors, implying that repeated use of these approximations may lead to inaccuracies.

Therefore, our original example of the addition of "near 7" and "near 2" can be represented as

$$(7, 1, 1) \oplus (2, 1, 1) = (9, 2, 2) \qquad (10)$$

which means that $\mu_c(z) = e^{-\left(\frac{z-9}{2}\right)^2}$ as shown before. A graph of the membership functions of all three fuzzy numbers ("near 7," "near 2," "close to 9") is shown in Figure 2. Notice the larger spread for the membership function associated with "close to 9." This is reflected in the larger denominator in the fractional exponent of μ_c.

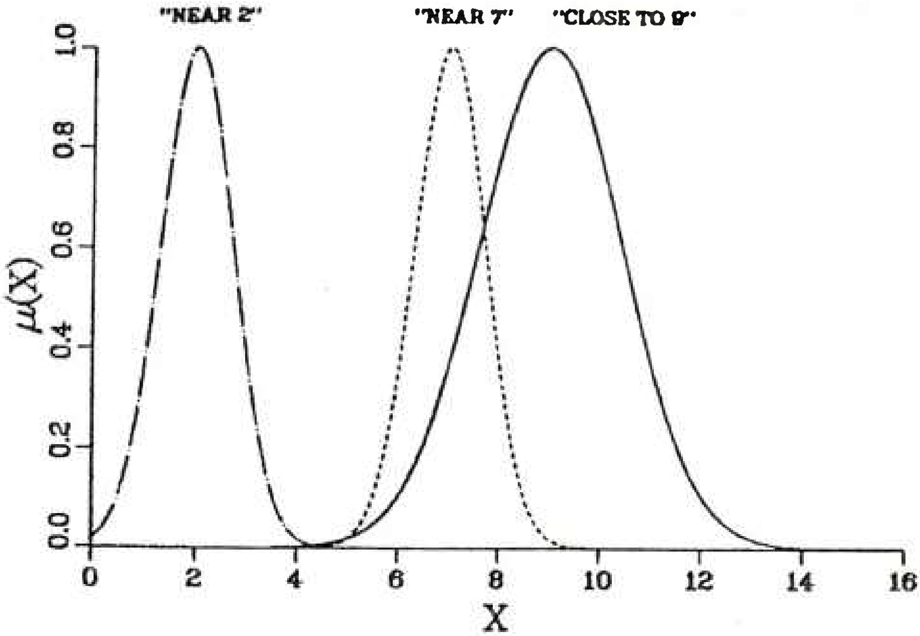

FIGURE 2: MEMBERSHIP FUNCTIONS ASSOCIATED WITH TWO FUZZY NUMBERS AND THEIR SUM

III. ARMY APPLICATION

In a Ballistic Research Laboratory report by R. E. Shear, W.O. Ewing, F. S. Brundick, and J.J. Smyth [9], a simple linear regression model ($Y_i = \beta_0 + \beta_1 x_i$) was developed based on the firing of right-circular-cylinder penetrators of various weights against several types and thicknesses of armor plate. The independent variable, X, was striking velocity of the penetrator; and the dependent variable, Y, was the hole diameter in the armor plate divided by the diameter of the penetrator, which was denoted as scaled hole diameter. A graph of the data and the resulting least-squares fit to the model is shown in Figure 3. The important parameters are:

$$\begin{aligned} &\text{number of data points} \ldots\ldots\ldots\ldots && n = 319 \\ &\text{estimate of } \beta_0 \ldots\ldots\ldots\ldots && b_0 = 0.7064 \\ &\text{estimate of } \beta_1 \ldots\ldots\ldots\ldots && b_1 = 0.000733 \\ &\text{estimate of error variance} \ldots\ldots\ldots\ldots && s^2 = 0.004 \\ &\text{mean of X} \ldots\ldots\ldots\ldots && \bar{x} = 1154.57 \\ &\text{sum of squared deviations of X} \ldots\ldots\ldots\ldots && \sum (x_i - \bar{x})^2 = 23860335.56 \end{aligned} \tag{11}$$

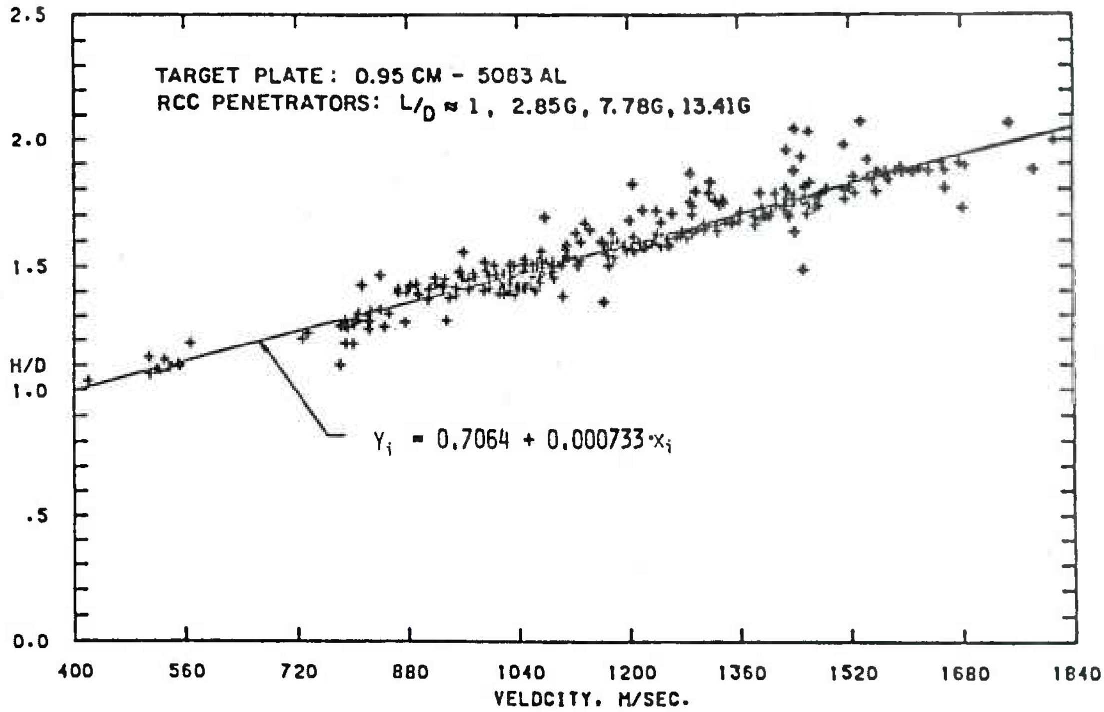

FIGURE 3: SCALED HOLE DIAMETER VS. VELOCITY

Once the model has been established, we can predict a scaled hole diameter, y_*, given a penetrator striking velocity, x_*, by using conventional statistical techniques. Our estimate of y_* would then be

$$\hat{y}_* = b_0 + b_1 x_*, \tag{12}$$

and the variance of this estimate would be

$$s_{\hat{y}_*}^2 = s^2 \left[1 + \frac{1}{n} + \frac{(x_* - \bar{x})^2}{\sum (x_i - \bar{x})^2} \right]. \tag{13}$$

However, suppose the vulnerability analyst is attempting to evaluate a future weapon system, in which case he is unable to obtain an exact value of penetrator striking velocity. In fact, he may not even be able to arrive at a bound for the value. In this case, he might choose a fuzzy number for x_* and subjectively decide on its membership function. This is a difficult problem, for the membership function established for x_* will affect the results throughout the analysis. Therefore, this step should not be treated in a trivial manner.

Call x_* = "close to 1000" and define $\mu_{x_*}(x) = e^{-\left[\frac{x-1000}{100}\right]^2}$. Then we can represent μ_{x_*} as (1000, 100, 100) and evaluate

$$\mu_{\hat{y}_*} = (0.7064, 0, 0) \oplus [\, (0.000733, 0, 0) \otimes (1000, 100, 100) \,] \tag{14}$$

to obtain

$$\mu_{\hat{y}_*} = (1.439, 0.073, 0.073). \tag{15}$$

Then $\hat{y}_*$ has an associated M.F.

$$\mu_{\hat{y}_*}(y) = e^{-\left[\frac{y-1.439}{0.073}\right]^2}. \tag{16}$$

Note the denominator is less than one, which implies that $\hat{y}_*$ is much less fuzzy than x_*; we will call $\hat{y}_* =$ "very close to 1.439." This means that for a penetrator striking velocity of "close to 1000," the estimate of scaled hole diameter is "very close to 1.439."

In a similar manner

$$\mu_{s_{\hat{y}_*}}(s) = e^{-\left[\frac{s-0.065}{10^{-8}}\right]^2} \tag{17}$$

which means that $s_{\hat{y}_*}$ might be called "extremely close to 0.065." Actually, the denominator is so small that this can be considered a crisp number and will be for the remainder of this application.

In some vulnerability models, killing a critical component is a function of hole size, particularly for radiators, fuel lines, and so forth. The vulnerability analyst becomes interested in whether or not a particular hole size exceeds a certain threshold, which itself may be a fuzzy number. In many cases, $\hat{y}_*$ is a normal random variate; and, as such, we can make probabilistic statements concerning it. In particular, we can determine the probability that y_* exceeds this threshold, call it "a." Then conventional statistics provide

$$\Pr\{y_* > a\} = \Pr\left\{z > \frac{a-\hat{y}_*}{s_{\hat{y}_*}}\right\} \tag{18}$$

where z is the standard normal random variate.

Let

$$T = \frac{a-\hat{y}_*}{s_{\hat{y}_*}}. \tag{19}$$

As was shown by Yager [7], if a or $\hat{y}_*$ or $s_{\hat{y}_*}$ is a fuzzy number, then T is a fuzzy number with an associated M.F. μ_T. Here, F: $R \rightarrow [0, 1]$. We can apply the Extension Principle to F; this states that $Q = \Pr\{z > T\}$ is a fuzzy subset of [0, 1] with the associated M.F.

$$\mu_Q(q) = \underset{\substack{t \\ q=\Pr\{z>t\}}}{\mathrm{MAX}} [\mu_T(t)]. \tag{20}$$

Note that since F is a function of just one variable, the MIN operator is suppressed. Then, to determine μ_Q, we first need to evaluate μ_T.

For an example, assume we're interested in a threshold of 1.5 for the scaled hole diameter. Then, using the operations defined previously,

$$\mu_T = [(1.5, 0, 0) \ominus (1.439, 0.073, 0.073)] \oplus (0.065, 0, 0). \qquad (21)$$

Thus, T has an associated M.F.

$$\mu_T(t) = e^{-\left(\frac{t-0.938}{1.123}\right)^2} \qquad (22)$$

so that

$$\mu_Q(q) = \underset{q = \Pr\{z > t\}}{\overset{}{\underset{t}{\text{MAX}}}} \left[e^{-\left(\frac{t-0.938}{1.123}\right)^2} \right]. \qquad (23)$$

Since, in this case, for any given q there is only one t such that $q = \Pr\{z > t\}$, we can ignore the MAX operator. We can determine values for the membership function of Q by performing the following three-step process:

1) Choose a value for q,
2) determine the value of t for which $q = \Pr\{z > t\}$,
3) substitute that value of t into μ_T and evaluate the function.

The graph of μ_Q is shown in Figure 4; it represents a fuzzy probability. Note that the central value (the probability at which the membership function achieves the value one) is 0.175. This means that given a penetrator striking velocity of "close to 1000," the probability that the scaled hole diameter will exceed 1.5 is "around 0.175."

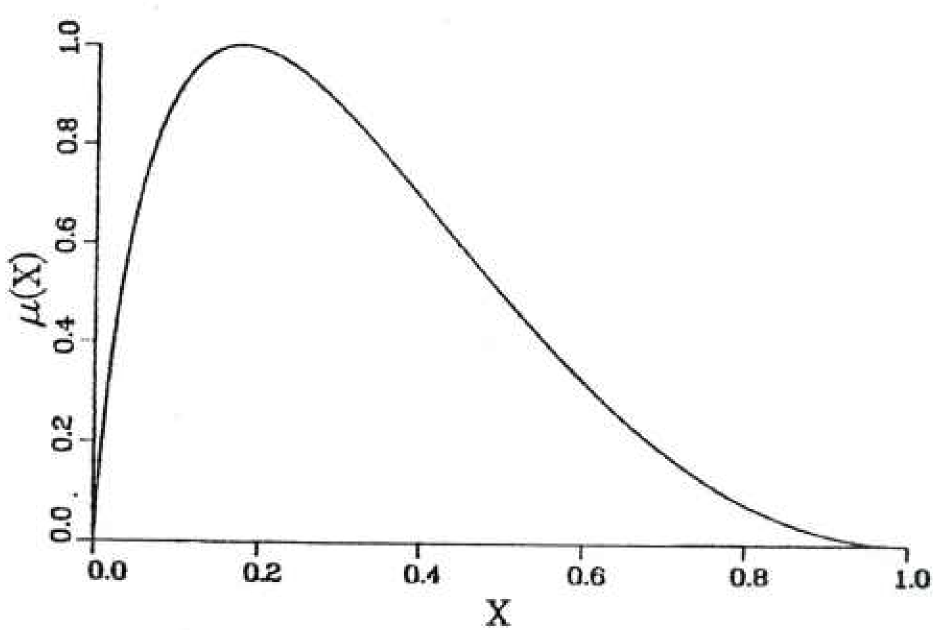

FIGURE 4: MEMBERSHIP FUNCTION ASSOCIATED WITH THE PROBABILITY "AROUND 0.175"

However, notice from Figure 4 that the spread is fairly wide; this is a consequence of the denominator in μ_T, which directly affects μ_Q and can be considered large since q is measured on the unit interval. The figure shows that the value taken on by the membership function remains fairly high (0.498) even at a probability of 0.5. Hence, we have a built-in sensitivity analysis showing the possibility that other values may occur for the probability in which we are interested.

In this instance, it may not be sufficient for the vulnerability analyst to claim little probability of damage to the critical component. With fuzzy probabilities he must be aware of not only the critical value, but also the spread reflected in the membership function. The confidence (spread) he has in this fuzzy probability is, of course, directly related to the confidence (spread) he input to his penetrator striking velocity. The point is that the uncertainty present in the penetrator striking velocity should be carried through the analysis, and fuzzy set methodology is a tool with which this can be accomplished.

IV. MEMBERSHIP FUNCTION VERSUS PROBABILITY DENSITY FUNCTION

In the previous section, the independent variable in the simple linear regression model assumed a fuzzy value, with an assigned membership function that was exponential. Therefore, the membership function resembled a normal probability density function and motivated a question concerning the ability to perform this type of regression analysis using probability theory rather than fuzzy set methodology. The concept of fuzziness has been introduced as something completely separate from randomness, a concept which might provide results in areas, such as this, where probabilistic methods alone remain insufficient except in special cases.

The application demonstrated in this report used a regression model which had been derived using crisp data, and only when that model was being used for predictive purposes did fuzzy numbers enter the analysis. In this case, the independent variable was assumed to be a fuzzy number. Had this fuzzy number been considered a random variable with a given distribution, an estimate of the dependent variable (as well as the standard deviation of this estimate) could have been obtained. Given a willingness to accept some normality assumptions, the probabilistic statement could also have been evaluated, resulting in a crisp probability rather than a fuzzy probability. However, the approximation used when evaluating the expected value of the quotient of two random variables is not analogous to the approximation used when dividing two fuzzy numbers; also, when performing arithmetic operations, the measures of spread are combined somewhat differently. All of this, of course, results from the formation of the Extension Principle which was assumed in the fuzzy set methodology. In any event, this is a rather special case; and it is important to consider more general procedures in regression analysis.

Several papers have been written concerning the fitting of straight lines when both variables are subject to error. Assume a simple linear regression model which has the form $Y_i = \beta_0 + \beta_1 X_i$ where $x_i = X_i + u_i$, $y_i = Y_i + v_i$, and u_i and v_i represent uncorrelated errors. A. Madansky [10] notes that ordinary least squares techniques cannot be used in this case to obtain estimates of the coefficients, since this minimizes the error in only the "vertical" direction. In fact, P.A.P Moran [11] shows that it may not be possible to estimate them at all, only identify bounds for the slope and the intercept of the regression line. D.A. Lindley [12] cautions that under some conditions, the regression line ceases to be linear when the independent variable is subject to error. This, of course, makes prediction of the dependent variable more difficult. He provided a method for such prediction when the value assumed by the independent variable comes from a distribution identical to the one from which the regression line was fitted. R.A. Ganse, Y. Amemiya, and W.A. Fuller [13] extended this work to the situation where the prediction distribution is different from the regression distribution.

If this same data (both variables subject to error) are treated as fuzzy numbers, the Extension Principle provides a method by which conventional regression techniques can be applied to the specified model to obtain fuzzy estimates of the coefficients and fuzzy predictions of the dependent variable. Furthermore, probabilistic statements, confidence intervals, and so forth, all fuzzy, can be derived in a straightforward manner. Finally, fuzzy regression analysis can be extended to non-linear models and can utilize fuzzy numbers whose membership functions have no analogous probability density functions.

V. SUMMARY

Fuzzy sets (in particular, fuzzy numbers) are intuitively appealing in the field of vulnerability analysis, where many of the data available to the analyst are somewhat vague. Some of the more-pleasing qualities of fuzzy set methodology have been demonstrated in the preceding application of fuzzy regression analysis. They are briefly restated below.

1) Fuzzy sets allow us to quantify our vagueness. In the example, the penetrator striking velocity, x_* , was given as "close to 1000." This value, of course, means different things to different analysts. However, once the membership function was defined, the likelihood of any other number being a member of that set was precisely determined. Assuming a crisp number for x_* would have affected the results and, possibly, any subsequent decisions.

2) Operations on fuzzy numbers generalize those on crisp numbers. In this example, had x_* been exactly equal to 1000, then the resulting crisp values for $\hat{y}_*$, $s_{\hat{y}_*}$, and $\Pr\{ y_* > a \}$ would have been just the central values which were achieved for the respective fuzzy numbers.

3) Fuzzy set methodology provides opportunities not always available using conventional methods. In regression, we typically assume a model; and then, for a particular value of the independent variable, we measure the dependent variable with some associated error. The example shown here allowed for some uncertainty to be introduced into the independent variable. Although some research in conventional regression techniques has addressed this problem, the results are limited to special cases.

4) For many problems, fuzzy numbers are relatively easy to use. Given fairly well-behaved membership functions, they can be represented by an ordered triple comprised of a central value, a measure of spread to the left, and a measure of spread to the right. Operations on these triples are well-defined, providing some basic conditions are met; therefore, introduction of fuzzy numbers, even into existing computer software, is not difficult.

5) Fuzzy set methodology appears to be very versatile when applied to regression analysis. The problem of independent variables being subject to error has proven very difficult to handle when using conventional statistical techniques; however, the treatment of these variables as fuzzy numbers has provided a straightforward solution to the problem with results that can be reasonably interpreted.

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REFERENCES

1. Zadeh, L.A., "Fuzzy Sets." Information and Control, Vol. 8, 1965.

2. Zadeh, L.A., "The Concept of a Linguistic Variable and its Application to Approximate Reasoning I, II, III," Information Sciences, Vols. 8, 9, 1975.

3. Baas, S.M. and Kwakernaak, H., "Rating and Ranking of Multiple Aspect Alternatives Using Fuzzy Sets," Automatica, Vol. 13, 1977.

4. Dubois, D. and Prade, H., "Operations on Fuzzy Numbers," International Journal Systems Science, Vol.9, No. 6, 1978.

5. Mizumoto, M. and Tanaka, K., "Some Properties of Fuzzy Numbers," *Advances in Fuzzy Set Theory and Applications,* North-Holland Publishing Company, 1979.

6. Yager, R. R., "On the Lack of Inverses in Fuzzy Arithmetic," Fuzzy Sets and Systems, Vol. 4, 1980.

7. Yager, R. R., "Fuzzy Prediction Based on Regression Models," Information Sciences, Vol. 26, 1982.

8. Tanaka, H. Uejima, S., and Asai, K., "Linear Regression Analysis with Fuzzy Model," IEEE Transactions on Sys., Man, and Cyber., Vol. SMC-12, No. 6, 1982.

9. Shear, R.E., Ewing, W.O., Brundick, F.S., and Smyth, J.J., "The Response of Selected Composite Armors to Simulated Fragment Threats," ARBRL-TR-02322, Ballistic Research Laboratory, May 1981, (UNCLASSIFIED).

10. Madansky, A., "The Fitting of Straight Lines when Both Variables are Subject to Error," Journal of the American Statistical Association, Vol. 54, 1959.

11. Moran, P.A.P., "A Test of Significance for an Unidentifiable Relation," Journal of the Royal Statistical Society, Series B, Vol. 18, 1956.

12. Lindley, D.A., "Regression Lines and the Linear Functional Relationship," Journal of the Royal Statistical Society, Supplement, Vol. 9, 1947.

13. Ganse, R.A., Amemiya, Y., and Fuller, W.A., "Prediction when Both Variables are Subject to Error, with Application to Earthquake Magnitudes," Journal of the American Statistical Association, Vol. 78, 1983.

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