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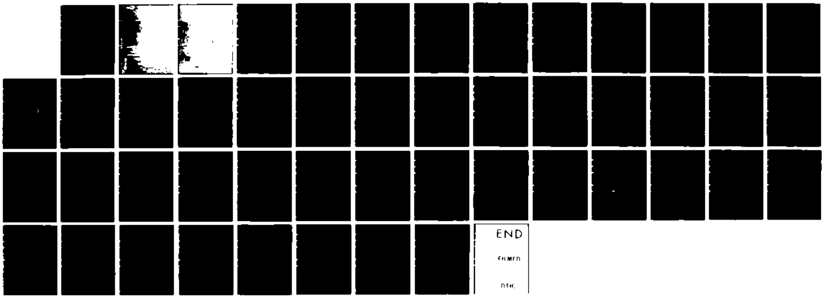

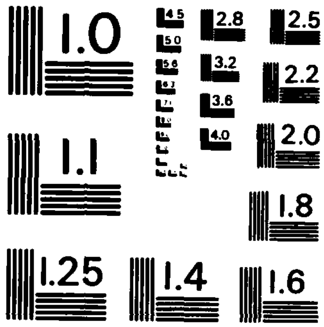

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THE LIMP FLYWHEEL

R.M. LERNER

Group 68

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LEXINGTON MASSACHUSETTS

ABSTRACT

It is proposed to design and construct energy storage flywheel rotors as statically limp tubes containing liquid mass, and to drive and support this rotating system (at least in part) directly, rather than through separately engineered subsystems. If the "liquid" is presumed thixotropic or viscous, nominally stiff structures subject to plastic flow are included. At one extreme of the design range, nearly all the mass is in the liquid and the only significant stresses are those in the wall of the containment; at the other extreme, the statically limp structure is nearly "dry" and is formed into an oblate surface by the centrifugal force of its own mass. The plausibility of the approach is argued by analogy with various physical examples, ranging from the spinning lariat to the design of reinforced concrete.

Results include the conclusion that in a limp structure, bonding between matrix and fibers is not a primary issue, and that a thin rim of liquid restrained by radially looped fibers has the same efficiency in using fiber material as when the fibers are used to support a solid rim as radial spokes (standard thin rim efficiency). It is also argued that both energy exchange and support can be provided for the flywheel rim itself, without the need to supply either through the central axle.

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Chapter 1

INTRODUCTION

The flywheel is a universal, but not always appreciated, form of energy storage. For example, the so-called "spinning reserve" of the power utility grid, which includes both the rotating machinery at the power plant and at user installations, will hold a system together for about 10 minutes after a major interruption.[1] This time scale is only an order of magnitude short of that required for load peaking for the utility, or for energy storage through shadow for a communication satellite. To deliberately design flywheel for either of these purposes is largely a matter of the will and the technology to do so.

Kinetic energy is stored as $mv^2/2$, even if the mass is rotating and the velocity is rotational rather than linear. Mass costs money, especially if it is engineered into a high strength spinning structure.

One figure of merit for a flywheel is thus the energy storage per unit mass, which is clearly proportional to the square of the speed with which that mass moves. An energy storage of 1 Watt-hour per pound implies a peripheral speed of about 400 ft/s for mass concentrated on the rim of a flywheel. At 25 w-hr/lb, typical of many orbital and land-mobile requirements, the peripheral speed is about 2000 ft/s.[2] (The units are bastard, but such is the nature of practical assertions.)

These speeds have two consequences. They imply high rates of rotation and consequent problems in the design and operation of long-lived, low-friction, high-speed bearings. They also imply a major problem with windage loss. At 400 ft/s windage can be controlled by used of hydrogen fill gas; but 2000 ft/s requires a vacuum.

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[1] The northeast power blackout of November of 1965 was caused by cumulative tripping of circuit breakers in the tie lines between Niagara Falls and downstate New York. With Niagara Falls gone, there was a massive power outflow from metropolitan New York. The spinning reserve held up for about 15 minutes while the man at the switches tried to get authorization by telephone to cut the city free. At that point, the system unravelled. It turned out that the rotors of several large New York city generators were lubricated by pumps which did not have backup emergency power. Bearings were lost; and it was several days before the New York City had full power again.

[2] It can be argued that one should add the weight of the flywheel support and energy conditioning systems to that of the wheel, thus raising the speed for the net w-hr/lb. However, it can also be argued that if one is not limited to 60 Hz as an internal working frequency, these extra weights should be minimal.

The other outstanding technology problem with flywheels is strength of material. Flywheel energy storage is directly proportional to the working tensile stress and to the total volume of material so stressed. All other things being equal, the cost of a material rises sharply with usable working stress; while the cost of the support system varies with total system volume.

Conventional wisdom tends to overlook the support cost; the conventional figure of merit for flywheel materials is a strength to density ratio. Certainly if stainless steel and Kevlar have comparable strength and cost per pound, and the latter has one sixth the weight of the former, for a given energy storage a Kevlar wheel is not only lighter but cheaper. However, E–glass has a strength only slightly less, and a density only somewhat greater, than Kevlar; but it is an order of magnitude cheaper. It is the current material of choice for flywheel power peaking, provided cyclic fatigue failure can be controlled.

Thus three factors limit the potential usefulness of flywheels for energy storage:

- Cost.
- Weight.
- Technology.

In terrestrial applications, cost is the most important consideration. In this connection, the cost of coping with the engineering constraints, including getting power into and out of the flywheel, can easily overwhelm the cost of the flywheel itself. In orbit, weight takes the place of money.[3]

Table 1 lists some possible flywheel energy storage applications, together with estimates of the energy storage and weight required. (Applications requiring frictionless bearings[5] are omitted.) Except for the first entry, the specific energy tabulated implies the use of "high strength" materials in the flywheel. These include

- Steel (e.g. piano wire or stainless steel spring wire)
- E–glass or S–glass fiber
- Kevlar
- Graphite fiber

Each of these materials has a tabulated tensile strength as high as 500,000 psi; each has a production strength of about 350,000 psi, of which at most about 200,000 psi is usable as static working stress. Typical vendor data tabulates only the highest strength on this list when making global comparisons[4] between materials. A typical example is

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[3] The monetary equivalent of weight could be obtained by prorating the entire cost of the mission or missions required to assemble an entire system in orbit.

[4] Extensive statistical data related to the strength of newer materials is difficult to acquire. Extensive histograms, based on hundreds or thousands of individual stress tests from commercial melts are available for many alloys in the *Metals Handbook*.

TABLE 1

Energy and Weight Allocations

Application	*Energy Storage*	*Weight*	*Watt-hrs per pound*
2 kW motor generator set	10 kW for 15 seconds to start motors.	25 lbs	1 to 2
Automotive engine size reduction	50 kW for 6 minutes	10% of 2 tons vehicle weight.	12.5
Trailer Truck hill braking and climbing	300m hill	2% of 50 ton vehicle weight	21
Commercial TV transmitter in orbit*	150 kW for 45 min	5000 lb	23

*The storage time through shadow is nearly independent of altitude from low through synchronous orbit. However, the time between shadows varies from 45 minutes just outside the atmosphere to several months at 24 hour orbit.

the comparison of specific tensile strength and tensile stiffness for the materials in the list above, published by Dupont in 1977, reproduced here as Figure (1). If T_{max}/ρ^* is the specific tensile strength in inches (as in the Figure), then one formula for the ideal specific energy storage E/m of a rotor built to operate at that stress level is

$$E/m = T_{max}/64000\rho^* \qquad (1)$$

where E/m is measured in W–hr per pound of material (see Appendix A.3). On the basis of this formula and the Figure one would expect energy storage ranging from 30 W–hr/lb for steel up to 160 w–hr/lb for Kevlar.

However, on the basis of the more realistic 200,000 psi useful working stress, the data translates into flywheel E/m ranging from 12 W–hr/lb for steel to 65 W–hr/lb for Kevlar. In fact, flywheels have been demonstrated in steel at 3 W–hr/lb; but have had problems reaching 20 W–hr/lb with Kevlar composites. This indicates the difficulty of actually using materials (with real-world stress concentrations, stress cycling, and safety factors) within a factor of three of a *repeated* stress of 200,000 psi. If design for 10^7 cycles of loading and unloading is required, it is found in Section 3.3 that 100,000 psi is a more realistic repeated maximum.

Another major problem is the diversity of design technology in the flywheel support subsystems. The tendency has been to regard each of the following as a separate design problem:

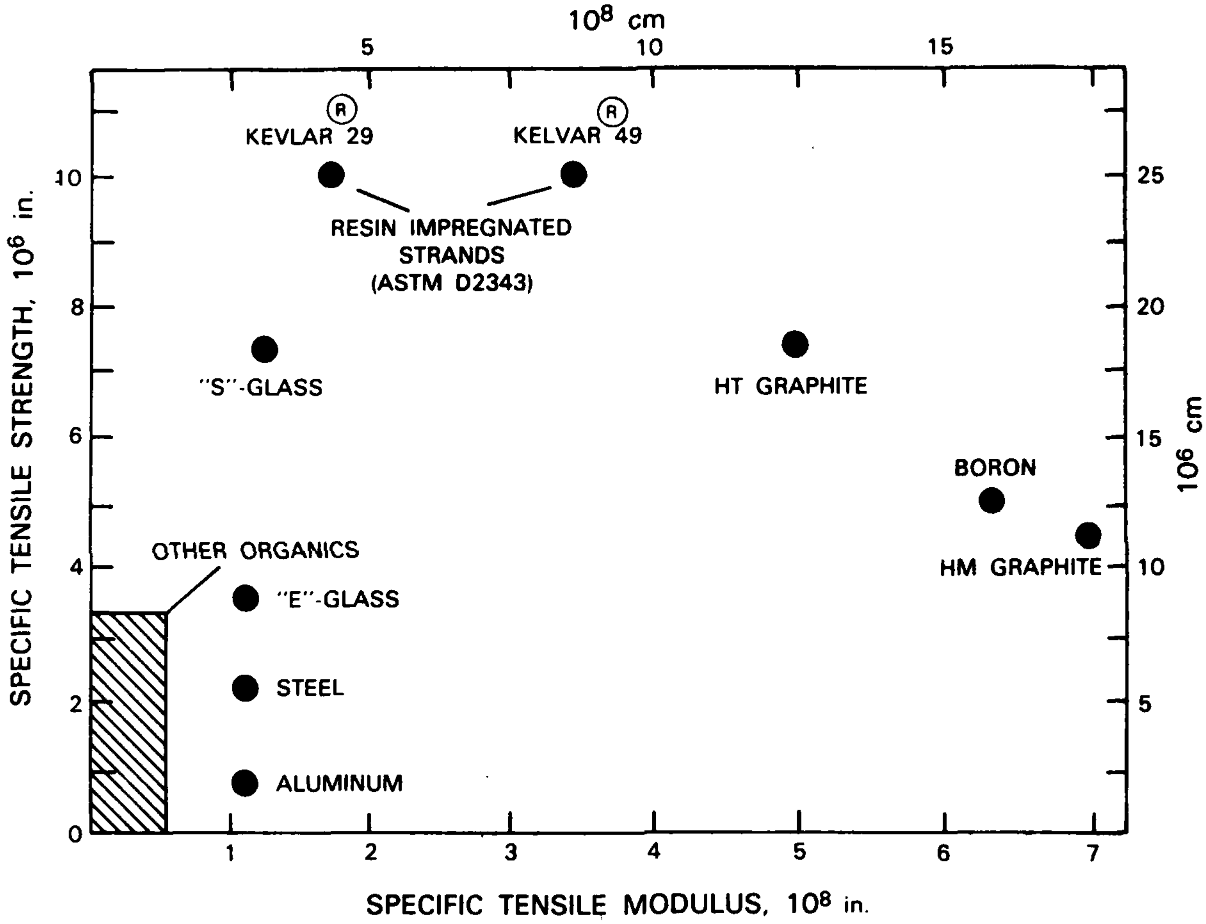

Figure 1: Specific Strength Data Dupont 1977

- Power conditioning.
- Flywheel drive and power takeoff.
- Flywheel suspension.
- Structural vibrational control.
- Flywheel rotor design.

Each of these engineering designs tends to acquire a life of its own. The result is a total cost in one study[5] which is roughly five times the cost of the flywheel rotor! Even in space, where component dollar cost is not the primary issue, the complication of five engineering enterprises to make one component work detracts from the credibility of the technology. To make the flywheel cost–effective on the ground and credible in space it will be necessary to combine several of these problems so that the cost and apparent system complexity will collapse also.

The introduction of the concept of a limp, fluid–filled flywheel does not by itself solve any of these problems. What it does do is to replace the concept of a rigid wheel restrained by a six axis support system with external drive and sensitivity to a host of vibrational instabilities. The substitute is a floppy structure that seeks its own equilibrium shape at speed and which is so weakly coupled to the outside world that integral design of the wheel, drive, and support systems becomes almost a necessity.

A further advantage is that the wheel can be run (for a given stored energy) at modest speed heavily loaded with liquid, so that the only structures close to ultimate design stress are the reinforcing fibers in the containment structure. Then, by off–loading liquid, the same wheel can be operated at progressively higher speeds at the same energy. This process allows isolation and correction of the (inevitable) design deficiencies one by one in the same model.

In principle, energy can be exchanged with the flywheel by pumping fluid in and out of the spinning structure. Indeed, if excess fluid can be introduced as the wheel slows down, it is possible to run it at constant stress and avoid the problems of fatigue under cyclic loading!

The objective of this paper is to use the concept of the limp flywheel as much to identify problems as to solve them. Indeed, engineering designs, even "on the back of an envelope", are beyond the scope of this effort. In what follows, the concept of the limp flywheel is used to a review a number of issues of support stability, energy exchange, and electromagnetic interactions. Various strength issues are also addressed, including the role of fibers and matrix, fiber and flywheel geometry, stress transfer, and fatigue failure, as well as the idiosyncracies of some fiber materials.

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[5] "A Flywheel Energy Storage and Conversion System for Photovoltaic Applications – Final Report" by Philip Jarvinen, March 1882, DOE/ET/20279–159

Chapter 2

THE FLUID LOADED LIMP FLYWHEEL

What is suggested here is the use of a new kind of "composite" flywheel in which the strength is supplied by filaments of a high strength material, and in which the mass is, at least conceptually, a liquid. The obvious analogy is the automobile tire, in which the strength comes from radial or circumferential belts, while the main mass is in the rubber.[6] The division of the tasks of strength and mass between separate materials reduces the nominal specific energy storage of the material used. But this is traded off for a greater design certainty about what happens, in both the practical and theoretical senses. As a result, the net specific energy of an actual rotor may be higher on the basis of a non–rigid design (with separation of function) than for the same material used for both functions in a rigid rotor.

Our objective is to establish that such a flywheel system is both plausible and potentially advantageous. Such discussions fall naturally into three categories

1. Stability.
2. System implications.
3. Rotor support.
4. Strength of materials.

In each case, primary reliance will be placed on physical analogy. Analysis will either be omitted or used only as an illustration.

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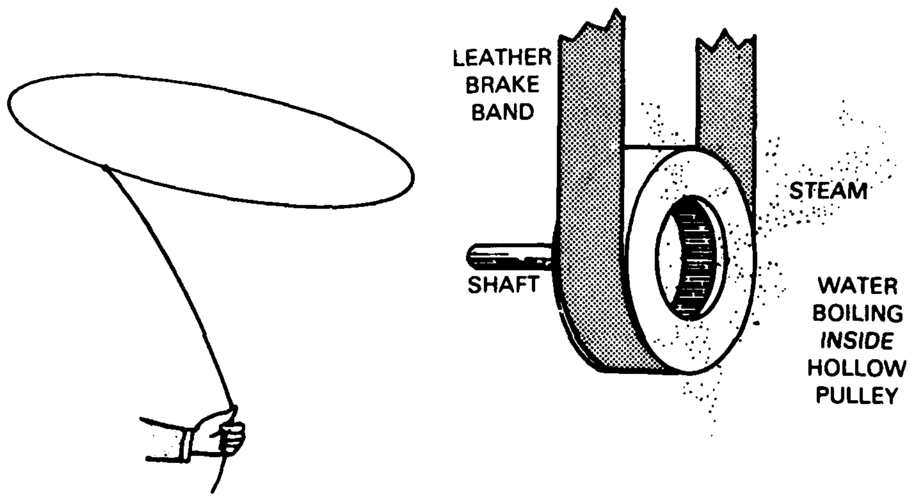
[6] The analogy is closer than one might at first imagine. At low mechanical frequencies, the properties of rubber are those of a high molecular weight liquid (Poisson's ratio is 0.5) which happens to have a small residual d.c. stiffness in addition to viscosity. Moreover, material similar to Kevlar (aramid fibers) is used for reinforcement in the higher grades of tires (as are glass and steel!). The reader should note that truck tires, when they overheat, fracture radially just as composite flywheel rotors do. Here the analogy comes to an end; the liquid mass is postulated within the tube in the position of the tire air, rather than as an external application like the wearing surface of the tire.

2.1 STABILITY

Stability considerations come in two flavors: those related to the gross stability of the mechanical arrangement being considered, and those related to control of incidental mechanical resonances.

2.1.1 Gross Stability

The gross stability question is simple: does a rotating flexible tube (such as a tire) tend to a toroidal shape or does it tend catastrophically to a collapsed or distended shape? A related question is, if a tire is partly filled with a liquid such as water, does the (inside) free surface of the liquid tend to lie smoothly at a uniform radius, or does it tend to form ripples and lump up? The answers from experience are favorable.



(a) Lariat

(b) Prony Brake

Figure 2: Stable Whirling Systems

The loop of a lariat is a whirling system which is statically limp. Dynamically, it arranges itself into a circle. While this fact does not *prove* that a circle will result with some other form of support than the slightly eccentric out-of-plane rope to the hand, the example makes the circular equilibrium shape at least plausible. (See Figure (2).)

The quiescence of the inside surface of a liquid within a whirling tube is established by the behavior of water in the Prony Brake, also shown schematically in Figure (2). This is a device used to load and measure the power output of rotating machinery. The friction of the brake is supplied by a leather strap which slips on a rotating pulley. The pulley is hollow; it is filled with water which boils to dissipate the heat generated by friction. The observation is that the water distributes itself uniformly around the inside periphery when the pulley speed is high enough to cause centrifugal acceleration to exceed that of gravity.[7]

This result can be justified by elementary analysis. The velocity c of a gravity wave is known to be given by the formula[8]

$$c^2 \leq g\lambda/2\pi \tag{2}$$

where equality holds only in the limit in which the depth of the liquid becomes comparable with a wavelength λ of the (assumed sinusoidally time dependent) disturbance, and g is the local acceleration normal to the surface. The normal acceleration of a free surface rotating at radius R and at angular frequency Ω is

$$g = R\,\Omega^2 \tag{3}$$

Now suppose a stable disturbance of the liquid, so that λ is $2\pi R/N$, N an integer Combining the two previous equations then yields

$$c \leq \Omega\, R/\sqrt{N} \tag{4}$$

But ΩR is simply the peripheral speed of the liquid surface. It follows from this that the resonances of the surface waves at the free surface always occur at frequencies *below* the spin frequency of the rotor. If the liquid surface is stable at one speed for which centrifugal accelerations dominate wave motion, it will be stable at all such speeds.

2.1.2 Resonances

All mechanical systems have resonances, often with high Q's. In the vicinity of a high–Q resonance stray parameters can cause dynamic instablities in an otherwise stable configuration. This can happen either as a result of amplification of motion to the point where neglected finite amplitude effects take over, or by providing coupling between parts of the system that were assumed not to be coupled.

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[7] Colleague Richard Sullivan (private communication) also points out that when a toroidal fuel tank is spun up in a 0–g environment, the liquid contents is initially a cloud of droplets; above a certain rotation rate, the liquid transforms from a droplet cloud to a stable belt hugging the outer perimeter of the tank. He also characterizes the resulting spun system as "self–stabilizing" (albeit the spin rates for his tests were much lower than those considered here).

[8] This formula is for a standard infinitesimal amplitude gravity wave on at the boundary of a half space, in which the wave motion is pure potential flow (no rotation). For a finite depth h, the formula for c^2 becomes $kg \coth(kh)$, where k is $2\pi/\lambda$. There is another type of gravity wave, the Goerstner wave, with purely rotational finite amplitude flow, having the same wave–speed formula. See Lamb, *Hydrodynamics*, 6th Edition, Dover, 1945, Sect 221. Evidently, a combination of the two wave types, with the usual modifications in radial dependence from exponential to modified Bessel functions, should be able to satisfy boundary conditions on a rotating free circular cylinder surface.

The cures are threefold:

- Design to avoid unstable structural resonance or coupling.
- Move structural resonances outside the frequency range of the spinning rotor, so that only asymptotic effects of such resonances need be considered.
- Thoroughly damp any resonances remaining within the range.

The most powerful methods of investigating stability of resonance and coupling include a consideration of the non–resonant wavespeeds of the modes being considered:

- If the square of the wavespeed is negative, the mode is unstable.
- If two modes are coupled, energy will flow from the mode with the higher wavespeed to the one with the lower wavespeed.
- Coupled modes never cross wavespeeds; instead each mode changes type, with a maximum or minimum in wavespeed at the nominal crossing point.

An example of negative square wavespeed is provided by the "gravity wave" on the *outer* surface of a rotating fluid cylinder. If the negative spring constant of the acceleration is not overcome by the stiffness of the stretched string constraint (which nominally holds the fluid in place), there can be at least one dynamically unstable mode of the outer surface of the flywheel.

Resonances may be moved outside the spin rate range either by making structure very stiff, so that mechanical resonances *lie above* the spin rate; or by making structure very floppy, so that resonances lie *below* the spin rate. The prototype flywheel reported by Jarvinen[5] rotated in the range between 100 and 200 Hz. Reaching E/m of 25 W–hr/lb will require faster wheels, moving at up to 1000 Hz. It becomes very difficult to prevent structural resonances in massive bodies and support systems as the frequency rises to such values.

Jarvinen[5] and Millner[9] reported steps towards alleviating such problems by moving a major resonance, the primary resonance of the suspension system, *below* the operating frequency range by the use of active magnetic bearings. The present proposal takes this philosophy one step further, by reducing static stiffness to a perturbation on a system whose resonances are almost entirely due to dynamic forces, such as the "gravity" wave just treated or the violin string modes of stretched filaments. Since these forces are determined in turn by the spin rate, the result is a *resonance pattern* which tends to track the spin rate. The design which is satisfactory at one speed should work over a wide range of speeds.

Evidently, a flywheel rotating with peripheral speed ΩR can couple energy (for example, by hysteresis) into any lower speed peripheral mode, which could result in excitation of subharmonic modes; however, such modes are elastically "floppy" and thus are easy to damp. An advantage of the liquid model of the flywheel is that the identification of such modes is simplified by unidirectional principal stiffness of the

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[9] "Flywheel Components for Satellite Applications" by Alan Millner, MIT Lincoln Laboratory Technical Note 1978–4, 16 May, 1978

constraints.

2.2 SYSTEM IMPLICATIONS

Specific advantages of the liquid (or statically floppy) model are

- The containment of high pressure fluids by uniaxial stress members is well understood in the design of pressure vessels.

- Safety against overspin can be obtained by venting fluid at overpressure. As a consequence, the loaded container can be operated much closer to its stress limit than would be the case if the empty container were spinning at a rate that might cause it to come apart.

Some materials, such as lead, are stiffish at room temperature unstressed, but will tend to behave as viscous liquids at the centrifugal accelerations met in the flywheel. If the fluid is more conventional, so as to have low viscosity under all conditions, further simplifications and combinations of function are possible:

a) Exchange of liquid with a reservoir, or recirculation of liquid can be used as the means of delivering and extracting energy.

   1. The flywheel can now exchange energy by both speed change and mass change. In some combination, this means that the speed of the wheel can be steadier for the same energy change, or the depth of energy extraction can be greater for the same speed change.

   2. The pump or pumps which connect with the flywheel can be placed line with the normal power train of the equipment being served, with no need for alignment with the axis of the flywheel.

   3. The use of efficient multistage turbine pumps eliminates the need for the turbine and the power train to turn at compatible shaft speeds.

b) The rotating joint through which the fluid flows can be designed as a thrust bearing to bear part of, or all, of the weight of the rotor.

c) Shock overloads (perhaps due to catastrophe elsewhere in the power train) on the rotating fluid need not result in shock overloads on the containment system, and vice versa. Indeed, through the operation of overload relief valves, the shock may never reach the flywheel.

In the last sense, the liquid system may be superior to an electrical system, in which it is very difficult to prevent a momentary short circuit from affecting the drive motor-generator.[10]

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[10] An important difference between the two systems is that the fluid flow equations are inherently non-linear in the practical flow regime, while the electrical equations are inherently linear. While the relief valve has its analog in the Zener diode, there is no useful electrical equivalent of a metering constriction in a pipe. In con-

For some applications direct electric drive connection to the rotating fluid is necessary or desirable. In this connection, the peripheral speeds of the liquid are high enough (over 500 m/s for a specific energy storage in excess of 10 W–hr/lb) that all liquid metals are good enough conductors for efficient coupling even at modest (3000 gauss) magnetic field densities. Moreover, there is nothing to prevent a part of the "liquid" from being otherwise loose insulated copper wires suitably wound and supported by bands of high strength non–conductor such as Kevlar.

2.3 ROTOR SUPPORT

Most flywheels are either directly supported by conventional bearings (as is the case for the flywheel of the conventional automobile engine) or have a secondary support system that takes over as the rotor comes up to speed. For reasons which are not entirely clear to the writer, the secondary support systems are merely at–speed versions of concepts that could be engineered to be stable at all speeds. They are not what is suggested here, a secondary support system that is dynamically stable even if it is statically unstable. The advantage of such systems is that they can be simple.

The example of the lariat is once again suggestive. Not only does the lariat take the form of a circle as its loop is brought up to speed; it rises to a position over the performer's head. By moving his position (and that of his arm) the performer can actually throw the rotating loop as if it were a solid hoop. Thus, not only does a small centripetal unbalance levitate the loop; there is sufficient weakness in the coupling between the loop and the constraint (the rest of the rope) to allow some motion as independent structures.

The author does not have the background in mechanical or aeronautical engineering to suggest dynamically stable support systems from these arts. However, electromagnetic systems can be dynamically stable. Now, there is a provable theorem in *magnetostatics* (Ernshaw's theorem) that there are no stable equilibria. However, this theorem depends on subtle assumptions about the environment in which it applies. A system with diamagnetism, either real or effective, can exhibit stable static magnetic equilibria.

Everyone is familiar with the unity diamagnetic susceptibility of a superconductor; and the demonstration in which a small magnet will float within a superconducting bowl, keeping to the center. Some years ago, the writer and Bradford Howland mounted an office wall magnet on a cork and floated the combination (magnet down) in a water glass full of a saturated ferric sulphate solution. The latter is significantly paramagnetic; the surroundings were diamagnetic by comparison. The magnet positioned itself stably on the central axis of the glass.

All conductors are dynamically diamagnetic. In a superconductor, the dynamic condition is metastable; in an ordinary conductor it must be constantly re–established by the use of alternating fields. A closer analysis shows that the current induced in a

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sequence, it turns out to be impractical to provide overload fault protection between the generators in a central station power plant and the low voltage bus–bars in the switchyard. A short will either burn free or irreversibly damage the machinery before circuit breakers could act.

conductor by an alternating field exerts a reaction force only to the extent that it is out of phase with the induced electric field. It follows that the alternating magnetic field will tend to repel conductors in its vicinity if

a) the electrical skin depth in the conductor is thin compared with its actual depth, or

b) the induced voltages cause currents to flow around a circuit whose admittance is primarily inductive.

Repulsion (and, ultimately, levitation) by skin effect is not an efficient process. The in-phase and reactive components of the net current are equal; the in-phase current results in loss without first order force. The second alternative, in which the current is constrained to flow around a mainly inductive path, is efficient.

Except when deliberately designed and operated otherwise, the circuits of rotating machinery tend to be inductive. The minimization of such effects, referred to as "controlling armature reaction" is one of the objectives of normal machinery design. Use of the inductive current for levitation requires two changes from ordinary rotating machinery design:

a) A magnetic circuit design change in the armature, to facilitate rather than to discourage flux paths through the pole pieces when there is a flow of armature current, and

b) The elimination of magnetically soft materials. (A series motor would tend to expel its rotor if it were not for the overwhelming attractive forces in the iron.)

The two design changes are mutually consistent; both are made possible by the availability of high-energy product permanent magnet materials to replace "field coils" on either rotor or stator (but not both at once).

If 50 kg of a 200 kg rotor is devoted to copper in a configuration in which the field is 3300 Gauss and the flywheel has a 2m circumference, then a current of about 3000 amps in the copper can support the flywheel for a power loss of about 100 W.[11] and so-called armature reaction from the motor-generator will keep it centered radially.

The practical device which demonstrates these effects bears little relationship to a flywheel. It is the constant current transformer that was used in series street lighting circuits until fairly recently. A sketch of such a transformer is shown in Figure (3). This is a variable reluctance device in which control over the output voltage appears in the equivalent circuit as a rms-current dependent leakage inductance. The constant voltage input winding is at the bottom. An unusually long E-core extends upward, resulting in substantial leakage flux between the central tine of the E and its sides. As a result, the flux in the central tine decreases upwards. A partly counterweighted

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[11] 50 kg is roughly 1/180 m^3 of copper. A 2-m length therefore has a cross- section of 1/360 m^2 and a resistance of $1.2 \times 10^{-5}\ \Omega$. A current of 3000 amps through 2 m at 0.33 mks flux density units produces a force of 2000 newtons, which will l.vitate 200 kg. This same current through the above resistance produces an I^2R loss of 110 Watts.

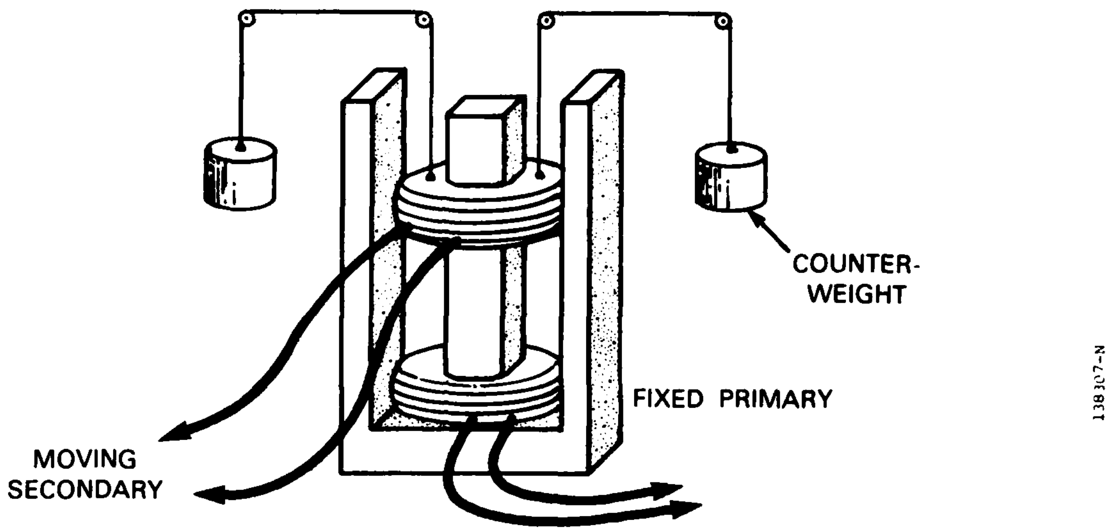

Figure 3: Constant Current Series Lighting Circuit Transformer

output coil floats in the leakage field, levitated by the current through it as a result of the action of the induced voltage on the external resistive load. If the rms current tend to go up (for whatever reason) the coil rises on account of the increased force; in consequence, there is less flux density through its center from the tine and the voltage drops to tend to re-establish an equilibrium current.

Chapter 3

STRENGTH CONSIDERATIONS

It is possible to postulate a limp flywheel constraint system made entirely out of cloth, treated with some kind of waterproofing material to keep the fluid from oozing through the pores in the cloth under high pressure. This is indeed the ideal for which one strives. However, the addition of the "waterproofing" produces a composite material system which may itself cause stress or ooze, or worse, redistribute stresses in such a way as to initiate progressive catastrophic overloading of the .nain load bearing fibers.

The fact that the limp flywheel adjusts its shape under load avoids some of the possible difficulties. Nevertheless there are some combinations of high strength fibers and matrix materials whose properties are ill–suited to formulation of composites for the flywheel application. For conventional designs, these include low shear strength of bonds to the matrix and non–compatibility of thermal expansion coefficients. One resolution is to avoid fiber materials entirely, and to use a solid rotor. This was the "low technology" route taken by Jarvinen et al[5] However, this road appears to be a dead end. The strongest materials are those which have been drawn as fibers in the direction of the maximum working stress, or which have been grown as uniaxial single crystal whiskers. Ultimately, to make flywheels competitive with batteries in cost and weight, these fiber materials must be used. This means identifying, and where necessary synthesizing, combinations of material properties that will work reliably at high stress and strain as fiber composites.

The investigations of this section include several departures from conventional wisdom. The limp flywheel structure is inherently one that lacks shear stresses, at least to the first order in small quantities. The matrix has only two functions, to distribute the compressive load of the fluid onto the fibers, and to hold the fibers in place apart from one another. If the function of lubrication (which is part of holding fibers in place) is omitted, then it can be argued that the matrix isn't necessary at all so long as there is a gasket to prevent the fluid from working through the interstices. Indeed, experiments with glass fiber wraps on pressure bottles support just such a conclusion. However, the matrix can prevent the fibers from cutting one another where they cross; and can hold a loose fiber in place if it snaps prematurely. If the strength of matrix–fiber bonds, and resistance to delamination by longitudinal shear are no longer important in the limp structure, it *is* important that the matrix resist liquefaction under pressure. Ideally, the matrix material should stiffen as the compressive or tensile strain upon it increases.

Just as the matrix is not called upon to resist major tension or shear, the loaded fibers do not need to sustain column compression.[12] In this context, the flexure and

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[12] It can be argued that shrinkage of the matrix during cure will put the fiber reinforcement under compression. To the extent that this is true, the composite must

rotating beam types of stress cycling tests are are too severe, since they subject the fibers to equal compressive and tensile loads. Ideally, the fiber should fail in tension with plastic flow, rather than to snap at a conchoidal fracture in shear. Some plastic flow before breaking permits the wheel to fail gracefully with an initial give before it ruptures.

It is regrettable that for most materials processing for high strength reduces the region of plastic flow, or eliminates it entirely. It is even more regrettable that in most cases the stress that can be sustained on repeated cycling between high an low levels is a modest fraction, 30% is typical for 10^7 cycles, of the loading that the material can take once or twice. For materials such as glass fiber, which is subject to water vapor corrosion and to mechanical damage by rubbing, the loss in strength on cycling can be much higher unless the surface treatments in manufacture and the matrix are properly chosen.

Calculation reveals that a limp flywheel whose fibers are radial loops (beginning and ending at the axle and extending in a plane through the axle) is as efficient in energy to mass ratio as if the fibers were radial spokes restraining the same rim load. This calculation applies to the case when the mass of fluid loading in the rim is large compared with the mass of fiber. Other cases, including toroidal or helical windings, not considered here.

3.1 REINFORCED SOLIDS

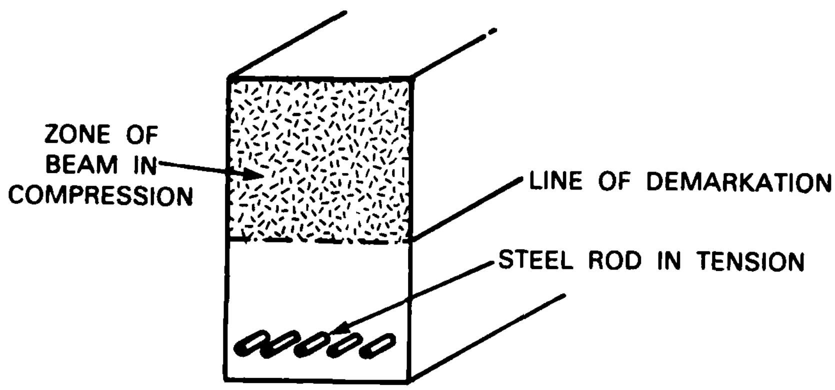

Figure 4: Reinforced Concrete Section

Figure (4) shows the design of a typical reinforced concrete beam, near its center. Concrete has strength in compression and shear, but almost no strength in tension. Since a tensile stress generally appears at 45° to the direction of an applied shear, the

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be able to withstand statically any tendency of the matrix to peel away from the fibers which hold it extended to its pre-cure length.

shear strength (50% of the compressive strength) is mostly unusable. Compressive strength depends on the amount of water used in mixing and ranges from 1000 to 4000 psi, almost in inverse proportion to the amount of water.

Steel bars imbedded in the concrete are used to shoulder the tensile loads. The scheme works because

a) The steel is much stiffer than the concrete.

b) The stiffness of the concrete and its compressive strength are proportional to one another.

c) Steel and stone have comparable thermal expansion coefficients.

In the design of reinforced concrete it is conventional to assume that the entire compressive load is borne by the concrete; the entire tensile load by the steel. The existence of concrete beyond the outermost layer of steel is largerly a matter of environmental protection; building codes typically require a thickness of concrete cladding determined entirely by fire resistance considerations.

The "balanced" approach to the design of reinforced concrete is to select the amount of steel so that roughly the same cross section is under compression as in the case of a strong homogenous beam. If the concrete cures stiffer than planned, the line of demarcation between tension and compression will move away from the steel so as to reduce the area of concrete available to resist the compressive load; but the remaining concrete is also stronger than planned and will so be able to accept the increased loading. Similarly, if the concrete is somewhat weaker than planned, the demarcation will shift to put more of the concrete under compression, and so to spread out the load.

In this connection, concrete exhibits a remarkable property: its stiffness (Young's modulus) Y and its strength T_c are proportional.

$$Y = 1000\, T_c \tag{5}$$

Now, the bond between reinforcement and matrix is mechanical, not chemical. All except the thinnest (1/4") re-bar is made with a dimpled surface to improve the bond. If there is doubt about the bond, e.g. if the bar is short, the ends of the reinforcement are bent to form hooks. The integrity of the bond is aided by the compatibility of the thermal expansion of concrete and steel: the bond does not tend to work free with temperature cycling.[13]

All elements of a fiber reinforced composite must be capable of the same *strain* along the fiber direction; the composite can be no stronger than the most fragile component in strain. Formula (5) above indicates that the concrete can sustain a

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[13] However, if moisture gets into the contact between bar and matrix, alternate freezing an thawing can destroy the bond. The high thermal conductivity of the steel is a disadvantage in this process, in that it provides a high conductance path to an infinite thermal reservoir for the freeze-cycling of small pockets. This is a particular problem with reinforced steel highway bridges, where occasional stress overloading of the concrete can produce cracks which are pathways for the water.

maximum strain of roughly 0.1%. In "balanced" construction, this limits the tensile stress in the steel to 0.1% of its elastic modulus, or 30,000 psi. (Construction codes generally reduce this to 15,000 psi or even 10,000 psi for safety). There is no point in attempting to use a high strength steel in such an application.

All high strength materials (tensile strength over 300,000 psi) attain that strength by virtue of being

- stiff, and

- able to withstand repeated cycling to several percent strain.

The reinforced concrete analogy makes it clear that such properties are wasted in a composite if the matrix cannot by itself withstand even greater strain. The need to withstand greater strain is especially critical at the ends of the fibers, or at points where one of the fibers fails. The unloading of the fiber at its end results in strain concentrations in the adjacent matrix, since such a loose end cannot be bent to form a hook as in concrete. The exact increase is difficult to calculate; for an isolated fiber in an otherwise homogenous matrix, it is typically a factor of four.

There are thus two possibilities for the matrix material for a truly high strength composite:

1. The matrix can be non–crystalline and rubbery. It stretches but does not fail, by fatigue or otherwise, at several percent strain.

2. The matrix can be a ductile crystalline material with a recrystallization temperature below operating (room) temperature. It experiences plastic flow every time there is a major change in the state of the wheel, but spontaneously anneals each time.

In either case the matrix material must have a finite strain threshold (greater than a few percent) below which the material exhibits a minimum long term shear strength of at least a few thousand psi (the typical bond strength of interfaces between matrix and fiber). A closely woven fabric impregnated with such a material can withstand roughly as much pressure (in units of such shear strength) as there are layers of material.

A non–trivial example of the first class of materials is "rubber" itself, the kind used in bonding abrasive cutting wheels or automobile tires. Here, the matrix is itself a composite of an organic and colloidal carbon (lampblack), the latter being as much as 30% by weight of the matrix. There is a broadly based industrial technology for handling such materials and bonding fibers with them. There is even a non–destructive process for estimating the pressure that a bonded welt will withstand: the pressure required to force the material into the welt in manufacture at a temperature well above that at which it will operate. A practical difficulty is that many rubbers have relatively low extrusion pressures (2000 psi).

Rubberlike materials have another important property. At large strain the sample tends to stiffen reversibly, rather than to yield and weaken (as does a ductile material). Thus it eventually snaps in brittle fracture rather than to pull like taffy as does a metal. Other materials which behave like rubber are wool and, to some extent, rayon and silk. The increasing stiffness is especially important to the problem of confining the strain effect of a fiber break.

The second class of materials includes zinc, tin, indium, and (unalloyed) gold. The recrystallization temperature of zinc is too close to room temperature (70° F) to result in a useful matrix. (Zinc is stiff, 1.5×10^7 psi, and strong, 3×10^4 psi, short term but flabby, $< 10^6$ psi, and weak, <6000 psi, long term). Tin and indium both give off an audible "cry" when bent, the noise being caused by dimensional rearrangements that accompany recrystallization. Tin wets iron alloys (as in the "tin can"); it has a Young's modulus of 10^7 psi and a yield strength of 5000 psi. If used as a matrix, every change in flywheel speed will be accompanied by plastic flow of the tin; a liner may be needed between the welt and the flywheel liquid to prevent radial extrusion of the tin during these changes. Although, in principle tin could be soaked up by a dry welt while the former is in a molten state, the better method is probably impregnation under pressure and sintering, as with rubber, just below its melting point. In this connection, the admixture of colloidal alumina (or silica) with the tin by powder metallurgy prior to impregnation will strengthen the matrix and raise the pressure for impregnation.

A final strength consideration is the strain-induced temperature change in the high strength material. Part of the change is a rise due to hysteresis. This is assumed small, lest the material itself fall prey to fatigue failure.

There is also an elastic effect, due to the difference between adiabatic and isothermal coefficients of response to changes, elastic or thermal. For materials with a high volume expansion coefficient with temperature, such as zinc, tin, and Kevlar (the latter, 105 ppm°C) and high bulk modulus, the difference between the adiabatic and isothermal elastic constants is as as much as 10% of either. By analogy with pulling the piston of a cylinder full of gas, the stretching of such a material tends to cause a temperature drop.[14] However, the situation is complicated by the fact the many fibers have unusually low, even negative, thermal expansion coefficients along their fiber axis. (Such fibers include graphite and Kevlar).

Materials like tin would be used only in a matrix; the elastic energy stored or dissipated as heat is a small fraction of that stored (at the same strain!) in the fibers. At 200,000 psi, the adiabatic temperature change in Kevlar is about 5°C; this is not quite inconsequential, but it is not necessarily a major effect. Other high strength materials have bulk expansion modulus β substantially smaller than that of Kevlar (the effect is proportional to β^2); the temperature change is an order of magnitude less in these materials.

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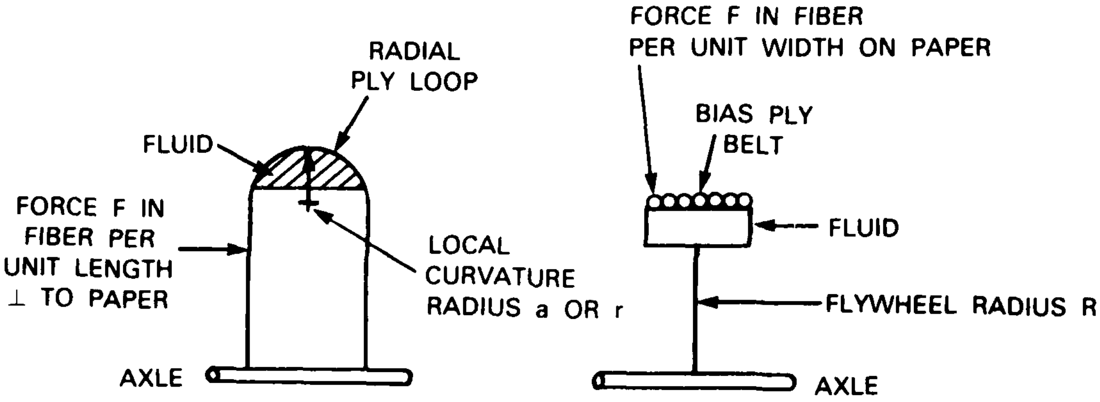
[14] See, for example the *AIP Handbook, 3rd Ed.* Section 3f-3, American Institute of Physics, New York, 1972

3.2 FIBER FORCES TO CONTAIN FLUID PRESSURE



(a)Radial Model **(b)Belt Model**

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Figure 5: Stress and Pressure

There are two basic geometries for restraining the mass of a centrifuge, called here the "*radial restraint*" and the "*belt restraint*". See *Figure* (1). *In either case,* the fluid mass exerts a pressure P against the restraint, to be offset by a surface force F (per unit length of surface perpendicular to its direction) along the surface. F is numerically proportional to P and to the total local curvature $1/r^*$:

$$P = F/r^* \tag{6}$$

Referring to Figure (5.a), there are two principal components to the curvature. The radius of rotation R about the central axis (in a plane perpendicular to that axis), and the local radius of curvature r of the flywheel cross section in a plane which contains the central axis

$$1/r^* = 1/R + 1/r \tag{7}$$

For the radial constraint, the term $1/r$ is dominant; for the belt, $1/R$.

Suppose that the average thickness of the fiber layer is ϵR for the belt; and ϵa for the radial, where a is the smallest value of r. Suppose further that we consider only the dominant constraint in each case. Then the fiber stress T is $F/\epsilon a$ for the radial case, and $F/\epsilon R$ for the belt. Since T is a given design constraint it follows that P for either case is given by

$$P = \epsilon T \tag{8}$$

Thus, by keeping ϵ small, the stress in the fluid may be kept small compared with the fiber stress. This is important, because the fiber stress model will work only if the stresses elsewhere are well within the elastic limits of secondary structural materials elsewhere. At 200,000 psi for T, a modest 0.1 for ϵ nevertheless implies fluid stress at a substantial 20,000 psi, well beyond the plastic limits of many materials.

Of course, the nominal configurations shown in Figure (5) are idealized. If the belt is not also curved (or some similar constraint supplied) in plane which contains the axis of rotation, the fluid will simply slip out the open ends of the cylinder which "contains" it. If the radial ply is not supplemented by some kind of belt, the fluid will simply push adjoining loops aside and spew out between them. Thus the actual "winding" of a reinforcement takes the form of a helical twinding around a closed circular axis; if the direction of the turns is mainly radial, we call this radial reinforcement; if mainly, circumferential, belt reinforcement.

3.2.1 Flywheel Volume

Appendix (A.2) shows that to hold a mass m by a belt or spokes so as to store energy E at fiber stress T requires a volume V of belt or spoke. Since the pressure is related to the allowable fiber stress by a geometric factor, the pressure indirectly determines the volume of the flywheel. For some applications, including spacecraft, this volume is important because of limits on physical space as well as limits on weight or cost.

In the case of the belt constraint, all of the fiber is contained within a cylindric shell whose thickness is ϵR. By scaling, it follows that the total volume of the hollow cylinder enclosed by the belt is V_w

$$V_w = E/\epsilon T = E/P \tag{9}$$

In the case of the radial constraint, the initial thickness of the top (or bottom) of the hollow pancake is ϵa, where a is the radius of curvature of the tip of the pancake. The thickness of the covers of the pancake must grow as one moves in towards the axis, in order to maintain the total cross section carrying stress. The volume of two such covers is $4\pi\ \epsilon a R^2$.

In Section 3.2.2.2 below, formulas are given from which a (r_0 in the notation of that section) can be calculated in terms of the separation $2x_0$ of the covers. For an otherwise "optimum" design, the covers are separated by roughly $2.45a$. Assuming thin covers, the hollow volume is thus $2.45\pi\ R^2 a$. Scaling then gives the nominal flywheel volume in terms of E and P as

$$V_w = 1.22E/\epsilon T = 1.22E/P \tag{10}$$

The radially reinforced wheel is nominally larger, but not much larger, than the belt reinforced one.

3.2.2 Equilibrium Shape of Radial Ply

In this section we describe the equilibrium shape of a radial reinforcement fiber. There are two cases: the constant cross-section fiber non-uniformly stressed along its length by the centrifugal accelerations acting on its own mass, and the fiber under constant stress which restrains a liquid under pressure.

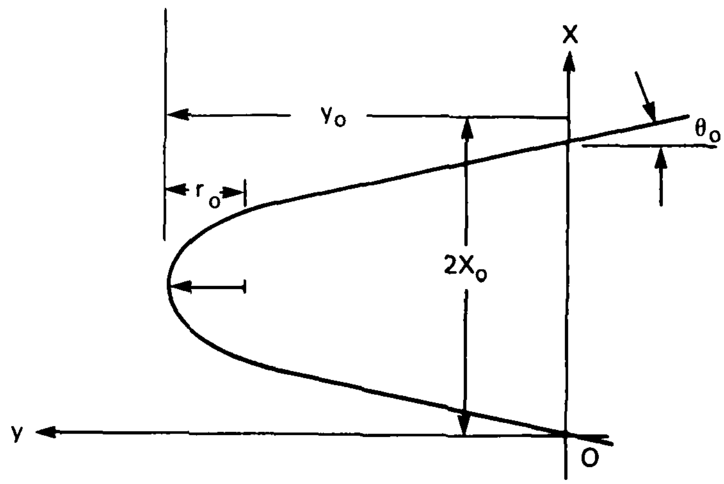

Figure 6: Geometry of Radial Reinforcement Fiber

The general geometry of Figure (6) is applicable to both cases. In the former case, the x-axis is the radius of rotation; the fiber is attached so as to make an angle θ_0 with it. In the latter case, the x-axis coincides with the free surface of the liquid; the filament continues past it in a straight line with fiber stress T per unit length normal to the plane of the diagram. (Note that in our notation dy/dx is $\cot\theta$, rather than $\tan\theta$.)

In both cases, r_0 is the radius of curvature at the outer tip edge of the rotating containment. For the free fiber, the distance y_0 is identical to the radius R of the flywheel; for the case of liquid containment, y_0 is the radial depth of the liquid.

Both cases lead to second order differential equations whose ultimate solutions are part of the theory of elliptic functions. Indeed, one (admitedly flip) writer introduces the first problem with the remark, "The following problem I am working, not because of any practical importance, but because it affords a mechanical means of generating the curve y = Sn(x)."[15]

[15] "The Elliptic Functions As They Should Be" by Albert Eagle, Galloway and Porter, Cambridge England, 1958 (§11.17–18).

3.2.2.1 Self Loaded Uniform Radial Loop

The differential equation for the first case is derived from the fact that the accelerations (gravity and other structure neglected) are entirely radial; hence the x–component of the force directed along the fiber must be constant. The resulting differential equation relates the angle θ at any point to the local radius of curvature *r*:

$$d \cot\theta/dx = -y\, r/r_0 \tag{11}$$

by various substitutions, this can be integrated once. The next stage of integration leads to elliptic integrals and to the conclusion that

$$y/x_0 = (\pi/K^2(1-k^2))\, \mathrm{Sn}(k,\pi x/2x_0) \tag{12}$$

The integration (discussed in Appendix (A5.2.1)) leads to a number of results, including formulas for the length L of the fiber in terms of complete elliptic integrals, and a variety of relationships between the elliptic parameter k and the initial angle θ_0. For k=0, the Sn curve is a sine curve. As k increase to unity, the curve becomes progressively flatter, so that the particular value of k must be deduced from a knowledge of x_0 and either y_0 or the length L. As Eagle[15] remarks, this deduction requires the numerical solution of a transcendental equation, for example

$$y_0/x_0 = 2k/(1-k^2)K(k) \tag{13}$$

Once k is found, then the geometry of the Sn curve yields ratios which do not depend on the scaling used in the definition of the elliptic function:

$$\cot \theta_0 = 2k/(1-k^2)$$

$$y_0/r_0 = 4k^2/(1-k^2) \tag{14}$$

3.2.2.2 Fluid Loaded Radial Loop

The problem of describing the equilibrium shape of the radial containment fiber when the dominant stress is that due to fluid pressure is much easier, because the differential equation is formally equivalent to that of a thin stiff wire or bar bent to large excursion by forces acting along its unbent axis. The resulting shapes are studied in elementary strength of materials courses, and have been described by every classic textbook on elasticity as "elastica".[16]

The differential equation is the result of stating, on the one hand, that the fluid pressure is in equilibrium with the (invariant) force in the fiber, and related to it by the local radius of curvature, as above; and observing, on the other hand, that in a layer thin compared to the overall flywheel radius R, the fluid pressure varies linearly from zero at its free surface to a maximum at $y=y_0$. This leads to the second order differential equation

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[16] Eagle, *loc cit* §11.19*ff*. See also A.E.H. Love , "The Mathematical Theory of Elasticity", 4th Edition of 1927, Dover Publications, New York

$$y''/(1+y'^2)^{1.5} = \Gamma\,(1 - y/y_0) \tag{15}$$

where Γ is the ratio y_0/r_0. Integrating once (See Appendix A.5.2.2) gives an equation for $\sin\theta_0$:

$$\sin\theta_0 = 1 - \Gamma/2 \tag{16}$$

Evidently, we want the upper and lower surfaces of the pancake to be more or less flat, so that the fibers can be continued (eventually) past the axle without attaching them there (so as to form by progressive winding a thin layer like the red wax coating on an edam cheese). This requires Γ to be close to 2, so that θ_0 will be close to zero.

This choice (Γ=2) also leads to the minimum value of fiber stress for a given mass of fluid (see Appendix (A.5)). With Γ=2 the stress in the fiber is the same as it would be if the rim were being held by radial spokes of the same cross section. Thus, the radial looped fiber is in fact a practical means of executing the massive–rim flywheel with radial spoke support.

Since the parametric equations of elastica are well described in the literature, there is no need to go over that ground here. The one necessary tie–in is the observation that Γ and the elliptic parameter *k* are related simply by

$$k^2 = \Gamma/4 \tag{17}$$

The parameteric equations themselves are

$$y = 2k\cos\phi$$

$$x = 2E(k,\phi) - F(k,\phi) \tag{18}$$

where E and F are elliptic integrals, and ϕ is the parameter. For the choice $k^2=1/2$, $E(k,90°)$ is 0.85 and $F(k,90°)$ is 1.35. From this, one calculates that $2x_0/r_0$ is 2.45.

3.3 FIBER AND MATRIX RHEOLOGY

Most glassy or polycrystalline solids have an isotropic bulk compression modulus λ ranging from roughly 2×10^6 psi to 80×10^6 psi. The shear modulus and Young's modulus for such materials are up to a factor of two lower. When such materials fail, they either snap while still in the linear region of the stress–strain curve (as do glass fibers); or they give and stretch as does steel. See the curve labelled "steel" in Figure (7)[17]

Other materials, such as rubber and wool exhibit exactly opposite behavior; they are easily extendible at low strain and stiffen at high strain. (On the scale of the Figure, cotton appears to be like glass; in fact, the behavior is similar to rubber and wool, but on a different scale.) It is possible for a material to exhibit both kinds of behavior simultaneously, the rubber type along a fiber axis, and the metal type at right

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[17] Taken from Sect. 41 of *Elasticity, Plasticity, and Structure of Matter* by R. Houwink, 2nd edition, Harren Press, Washington D.C., 1953, p275

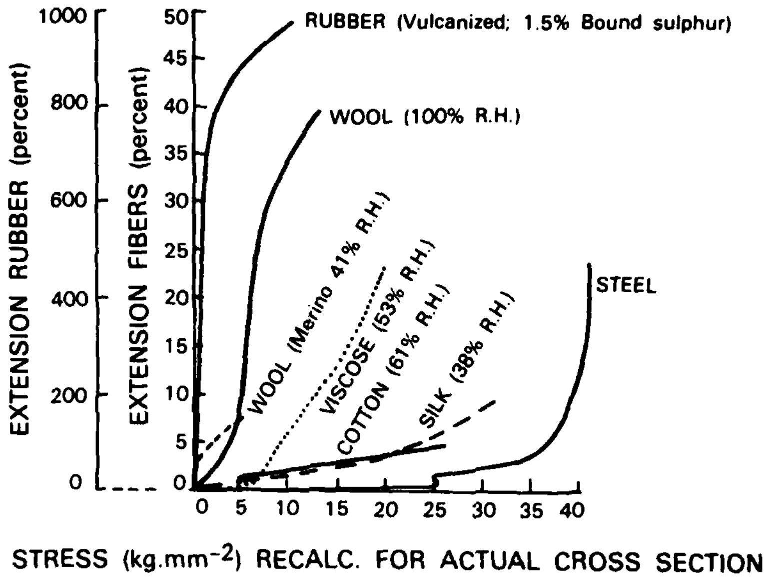

Figure 7: Stress Strain Curves for Various Materials

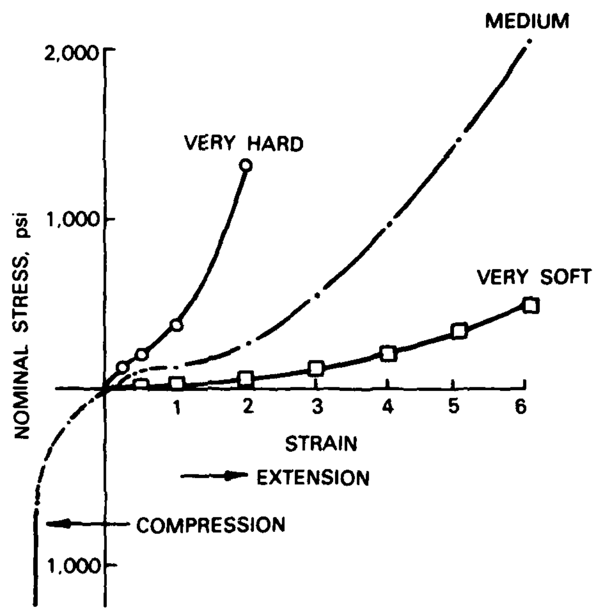

Figure 8: Typical Stress–Strain for Rubber.

angles to it.

Rubbery behavior is the consequence of an unusually flaccid low frequency component to the shear stiffness. This high compliance component can be frozen out by lowering the temperature, or by using a measurement frequency above a few hundred Hertz, or by introducing large strain. (The material must stiffen a high compressive strain, which, after all, cannot exceed unity.) Figure (8) shows stress–strain for a typi–

cal rubber.[18]

The elastic stretching of both fibers and matrix under repeated loading and unloading involves both long term complications and short term effects. One of the long term complications is elastic relaxation, mentioned in connection with the elastic properties of zinc above.

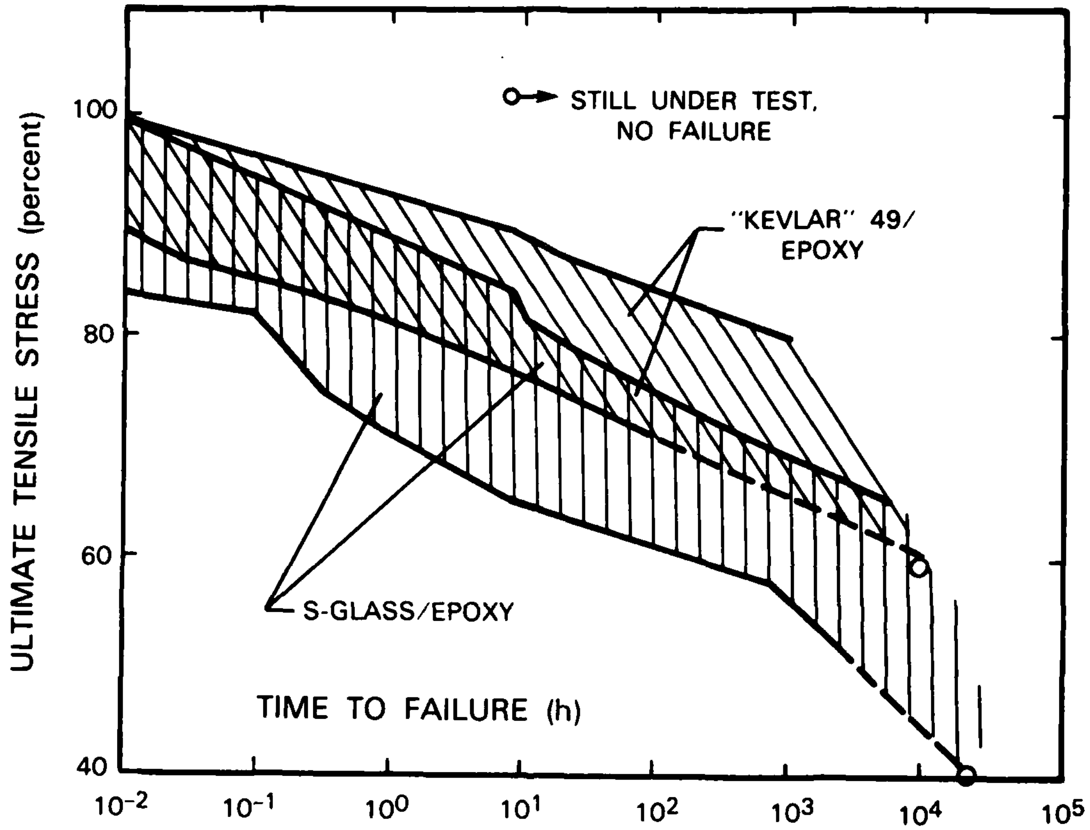

Figure 9: Relaxation Failure

All fibrous materials are subject to a difference between the short and long term loads that they can safely withstand. For wood, the strength ratio between a few hours of loading (say, roof loads in a storm) and a few years of loading may be a factor of two. Figure (9) shows a similar phenomenon with Kevlar and glass epoxy composites. (This figure was taken from the Dupont Kevlar 49 Handbook.)

Unless there is a bad unbalance or vibration, the stresses developed in a flywheel are static stresses so long as the speed remains constant. ±10% variations in the stress generally do not induce fatigue failure; but changes much larger than this may necessitate design for a much lower static stress in the fibers in order to avoid failure under

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[18] Taken from the *Engineering Materials Handbook*, edited by Mantell, Figure 32–4.

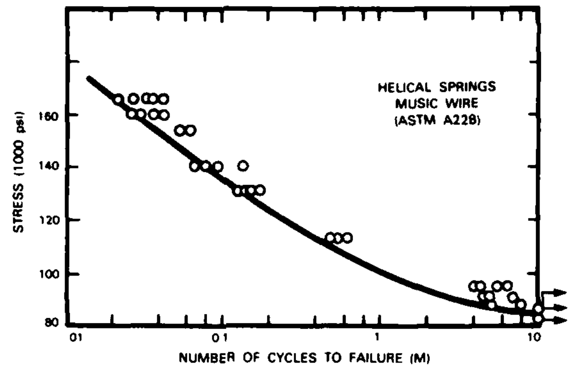

Figure 10: Fatigue failure in steel springs

repeated cycling. Fatigue failure in steel springs is a thoroughly investigated problem. Figure (10) shows a decrease of a factor of at least two between the one time loading strength and the 10^7 cycle strength. The long term and fatigue effects are not additive; to a certain extent repeated cycling merely hastens the onset of what would have happened eventually under sustained load. However, additional abrasion damage to fibers as they flex is known to be an important degradation mechanism to glass fibers; it may be important for other brittle materials also.

For some purposes, for example a once–per–day loading and unloading, design for 10,000 stress cycles might be adequate. This would be called a 25 year design. For a once an hour cycling, as might be met in low earth orbit, 25 year design calls for 3×10^5 cycles. For the likely land mobile applications, trucks, buses, and constantly used passenger cars such as taxis, conservative design would call for stress derating to 10^7 cycles. Note that the flywheel is *not* designed to fail after this number of cycles; the stresses are merely to be derated on cyclic use basis.

A basic view of this memorandum is that the fibers do *not reinforce the matrix; the matrix merely holds the fibers in place.* Where a continuous filament can be layer wound to provide the strength, there is experimental evidence to support this view. In one set of experiments glass fibers (sized with "HTS", an adherent for epoxies) were layer wound to reinforce a thin walled cylindric bottle. The bursting stress in these fibers (>300,000 psi) "laid up dry" was consistently as high as when they were impregnated with a matrix material.[19]

Nevertheless, the conventional experimental composite R&D has been directed towards the reinforced matrix. Accordingly, supplementary sources of information about potential matrix materials are important. One such source is the use of these materials in filamentary form as textiles. Some forms of cellulose acetate and nylon thread are capable of 100% elastic recovery after being stretched to 10% strain; these also have static tensile strength over 10,000 psi. Of course, the context is an extruded fiber which has undergone some initial alignment of molecules along the fiber axis.

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[19] *"Fiber Reinforced Plastics" by L.J. Broutman, Chapter 13 in* "Modern Composite Materials", Broutman & Krock, Editors, Addison Wesley, Reading Mass, 1967

This class of materials (high strain recovery, high static strength) merits careful examination for potential use as the matrix in a limp fly wheel design.

Inasmuch as the flywheel necessarily operates with at least a few percent strain, the properties of its materials should be selected with large strain and failure of individual stands or pockets of material in mind. It is argued below that the high strength fibers should ideally exhibit some plastic flow at failure (see subsection on steel wire below); while the matrix in which they are imbedded should stiffen as tensile strain increases (see subsection on stress transfer below).

3.3.1 Glass Fibers

There is an extensive literature and practice in the use of glass fibers for reinforcement. Individual filaments, rapidly drawn and coated to prevent surface corrosion, exhibit breaking strengths between 500,000 and 700,000 psi for so–called glass types E and S; fused silica fibers have reached 1,200,000 psi. These are the numbers quoted in the textbooks.

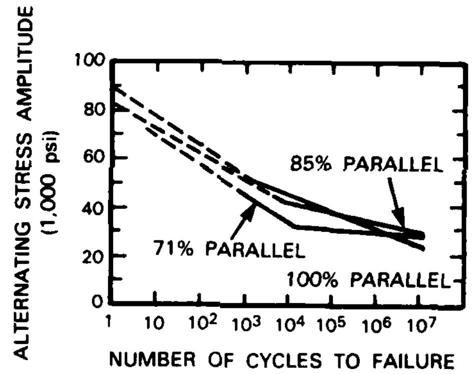

Figure 11: Flexural stress failure for E–glass in epoxy
Glass strands nearly parallel to stress

Unfortunately, the commercial reality is a far cry the promise. The strongest commercial glass fibers are drawn continuously in bundles of several hundred strands and sized with a interface adhesive and lubricant as the multi–strand is formed. Whether because the lubrication is not wholly adequate, or because grit also enters the thread, the resultant "fiber" is subject to tendonitis. Laid up dry in the form of windings, such fibers exhibit a bursting strength of 300,000 psi, a number which does not change by much as epoxy is added. However, on repeated flexure, the rupture strength drops substantially. Figure (11) shows a drop of 3–to–1 in the strength of E–glass at 10^7 flexure cycles.

It seems established that the problem with glass fibers is to protect them from mutual abrasion and from stress corrosion by water vapor, as well as to eliminate imbedded sharp cornered micro particles. Evidently, these are problems in manufacture that cannot be corrected after the fact. Interaction with manufacturer's applications engineers is particularly important to be sure that the product that one would like to use is

- available
- appropriate
- amenable to the handling that it will receive.

Failing such care in specification (including specification of the coupling or sizing material), comparisons become meaningless. Nevertheless, one of the clear advantages of the Kevlar class of materials is that they are tough rather than brittle; and the manufacturing process results in a well-lubricated thread ('roving') rather than an abrasive one.

3.3.2 Steel Wire

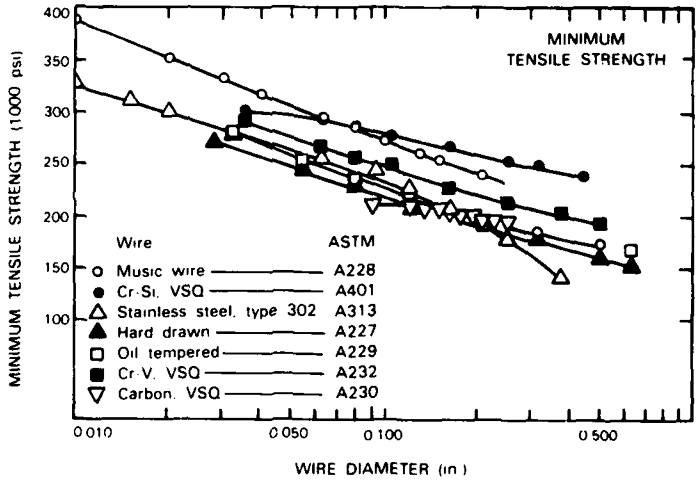

Figure 12: Variation of Strength with Wire Diameter

The temptation is to dismiss steel as too heavy and too expensive a material for use in high energy storage flywheels. Nevertheless steel for use in springs is one of the best understood and reliable of engineering materials. This fact is illustrated by a handbook plot of the strength of steel wire *vs* diameter, shown in Figure (12). What is plotted in ***minimum*** strength from samples from a variety of melts, rather than an "average" strength or mere research sample result.

In addition, steel is tough and not subject to brittle fracture. The working stress is a factor of three or four (for 10^7 cycle design) above that available from glass fiber. Moreover, steel is not necessarily expensive (see below).

Two main types of high strength steel are used in springs, a high carbon steel called "music wire" and type 302 stainless. A variant of music wire, called "piano wire" has about 30% greater strength (literally, over the long pull). Figure (12) shows the dependence of the strength of drawn wires on the diameter: the finer the wire, the stronger. Unfortunately, is is also true: the finer the wire, the more expensive. In wire sizes under a few mils, stainless steel wire is less expensive than high carbon steel. Even so, in 100 pound lots the current (Jan 1984) price of 302 stainless is $6.50 per lb for 10 mil wire, and $9.50 per lb for 5 mil wire. This must be compared with prices for music wire in 100 lb lots, which at the same time was $2.70 per lb for 10 mil wire and only 91¢ per lb for 31 mil wire. In terms of price, the higher strength of the finer wire may not be justified. These prices also must be compared with $12 per lb for Kevlar 49 and under a dollar per pound for continuous E glass fiber. (S–glass cost about $4 per lb.)

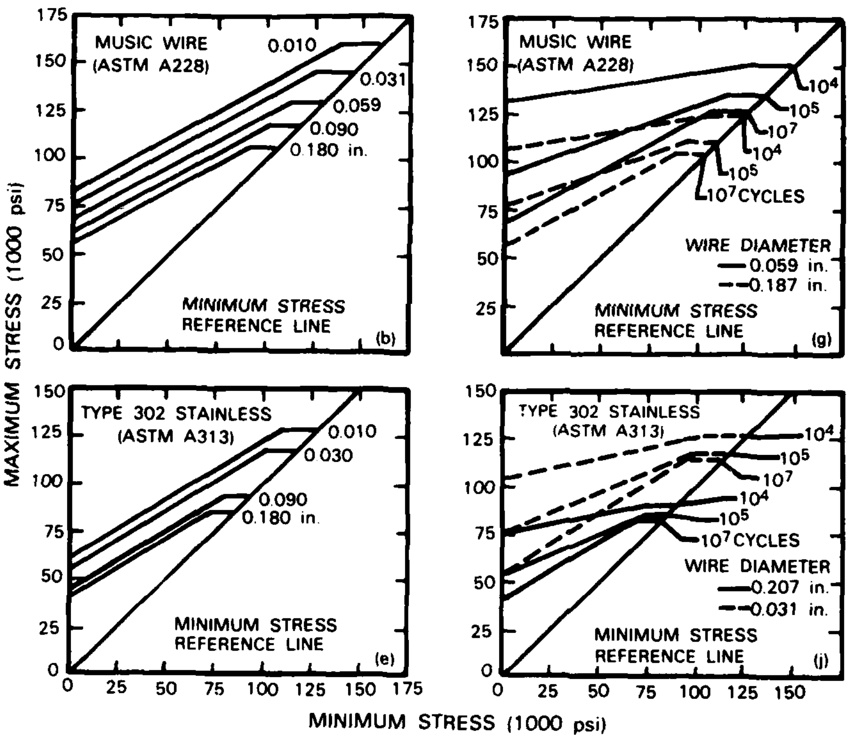

Figure 13: Fatigue Design Curves for Steel Springs

Figure (13) shows a typical set of design curves for the use of two types of steel for multi–cycle stress, taken from the *Metals Handbook.* On the right are curves which give maximum and minimum stress limits for fixed wire diameter and a range of cycles to failure. On the right are design limits for a range of wire diameters and fixed 10^7 minimum cycles to failure. Evidently, a 10^7 cycle design for stress cycling between 25,000 psi and 100,000 psi requires music wire with 10 mils diameter (at $2.70

per pound) However, the sacrifice of 10% in strength or a reduction to 10^5 cycle of loading, would permit the use of 31 mil wire ($0.91 per lb).

In addition to the extent to which the properties of spring steel are documented in dispassionate literature, the use of steel has another outstanding advantage for the experimental construction of "floppy" flywheels. When a strand gives, in neither snaps nor unloads; it merely refuses to take any more load as it is stretched further. In a floppy design, this stretching produces graceful degradation

1. because the stress to be redistributed over neighboring fibers is merely the excess over the yield stress for the weak fiber, not the entire stress. Even if yield results in a slight decrease in stress in the affected fiber, the redistribution is modest. As a result, delamination of the matrix surrounding the weak fiber (or other major elastic event) is not an immediate consequence of "give" in the weak fiber.

2. because the total local strain merely increases slightly when fiber failure occurs, the floppy structure can respond by simply taking on a slightly larger radius. Although this in turn increases the average forces, and can lead thereby to catastrophic failure, the very increase in diameter is an early warning of the impending catastrophe.

Any other material that exhibits plastic flow when it fails in tension would have similar advantages for use in a flywheel.

Unfortunately, in most material systems, high strength is obtained as the expense of the capacity for plastic flow, so that ultimate failure (breakage) occurs at a comparable value of energy delivered, but at much lower strain and much higher stress. Evidently, if one were able to specify the properties of the material at failure, one would choose so that the strain diagram would have roughly equal regions of non-yield stress and plastic flow before rupture. Such a material would have ample latitude to permit plastic flow in weak fibers, and to allow significant expansion in flywheel diameter as a warning of impending rupture.

3.3.3 Graphite and Aramid Fibers

Glass fiber differs from bulk glass largely on account of the speed with which it is pulled; its physical properties remain not far from isotropic. In steel wire, pulling results in preferred crystal orientation with respect to the direction of pull, and some elongation of the crystal structure; but it remains a strong and tough material in all directions. However, many other kinds of materials exhibit pronounced variations in strength with respect to the fiber axis.

Aramid fibers (Kevlar) and viscose rayon (cellulose acetate) are examples of non-crystalline materials in which pulling the fiber lines up long-chain molecules to give high strength along the fiber axis, and a corresponding shear strength at right angle to it. Such materials can be split apart relatively easily along the fiber axis; although the resulting problem with "split ends" can be less than that of the TV ads for hair dressing treatments, the longitudinal shear strength of such fibers can easily be less than the strength of the glue bond to the surrounding matrix.

Graphite fibers also suffer from relatively low shear strength, in this instance because graphite has a layered molecular structure. In the plane of a layer (which includes the fiber axis), the material has high strength; but the layers themselves separate easily. A bundle of such fibers has high transverse shear strength in part because random twist ensures that some stiff, strong fibers are available to take the transverse loads at any angle. However, the longitudinal shear strength is parallel to the slip planes for all fibers; it is accordingly low.

Notwithstanding the problems of Kevlar and graphite, longitudinal shear weakness (and other transverse strength problems) are not a necessary feature of oriented materials. A modest amount of cross-linking would prevent the plastic fiber from splitting; almost every other single crystal fiber belongs to a system which does not have the laminar structure of the graphite crystal. It is in a sense unfortunate that two high strength materials which are so relatively easy to make are so weak longitudinally.

Graphite fiber and Kevlar fiber both have a physical property which is a bellweather of problems with elastic homogeneity. The temperature coefficient of both materials is *negative* along the fiber axis, while the volume coefficient in both cases is positive and normal for the melting point of the material. This indicates a taut structure along the fiber axis, but a relatively loose structure at right angles. Indeed the linear coefficient of expansion of Kevlar is sufficiently high (50 ppm°C) across the fiber as to raise compatibility problems in some composite systems.

3.4 STRESS TRANSFER

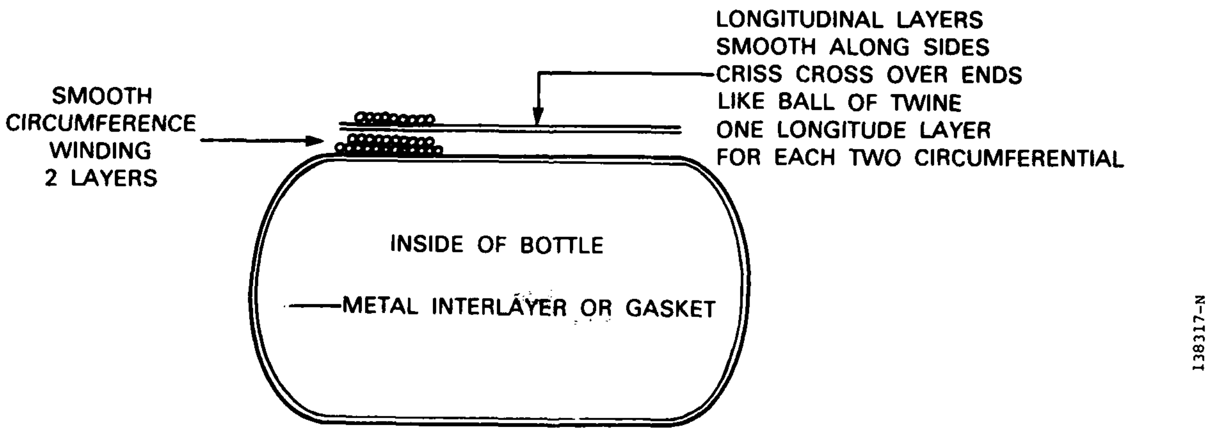

Figure 14: Winding and Metallic Interlayer in Pressure Bottle

If a fiber is loaded mainly by its own spinning mass there is no need to discuss stress transfer. However, if internal pressure is being contained, then some stress transfer mechanism is required. It need not be a sophisticated one. The fact that the inner layers of a winding cannot stretch unless the outer ones do so also suffices. (Note that this doesn't work the other way around. Outer layers can be stretched without stretching the inner ones.)

Figure (14) exhibits the typical manner in which fiber reinforced pressure bottles are made. A metal internal bottle is used as a form on which to wind the fiber, and as a gasket. Continuous filament, precoated with bonding agent is then wound circumferentially around the bottle; and also axially. Two circumferential layers are wound for each axial layer, this being the ratio of forces involved. The wound structure is then impregnated with matrix material and cured.

Bottles of this sort have also been tested laid up without matrix, and impregnated without being cured.[19] Fiber loading up to over 300,000 psi was inferred when bottles were burst by internal pressure. Clearly, no mechanisms for stress transfer are required other than inner fibers pushing on outer ones with the aid of fiber to fiber friction.

Now suppose that the winding is impregnated with an encapsulant which mechanically surrounds the fibers but does not actually bond to them. If a force is applied to this matrix at right angles to the fibers, the fibers will resist on that part of the fiber surface that faces the direction of the force. The land or bridge of material that pushes on each fiber experiences shear stress roughly equal to pressure it exerts on the fiber. If the fluid pressure is P and there are M layers, the pressure on each layer is P/M. Evidently, a matrix that can withstand a pressure P and remain solid can also withstand a shear stress of P/M.

Conventional wisdom holds that longitudinal shear strength is needed at the fiber matrix interface to contain broken fibers. The situation is not amenable to exact analysis. However, if there is a "bond" between the fiber surface and the matrix, then at a break among densely packed fibers there is a strain concentration extending roughly $\sqrt{(Y/G)}$ diameters along the fiber, where G is the shear modulus of the matrix and Y is Young's modulus in the fiber. To sustain such a strain requires a shearing stress along the fiber of the order of magnitude $\sqrt{(G/Y)}$ times the unbroken fiber stress. For most choices of materials that are otherwise compatible with a limp flywheel, this shearing stress is too much. The fiber–matrix bond simply unzips.

However, non–bonding of matrix to fiber is not the end of the story. The matrix is under compression. As a result there is a *static friction* force on the fiber to hold it in place, even if there is no bond. Moreover, the magnitude of this force (determined by a coefficient of friction of at least 0.1, more likely 0.3) tracks with the stress in the surrounding fibers, since both are the immediate result of the pressure P! Evidently, such forces will be most effective in holding load on a broken fiber in the inner layers, where the pressure in the matrix is highest. There will be little benefit in the outermost layer. There, the most that can be expected is that the matrix will hold the broken fiber on the wheel.

Once again, conventional wisdom appears to be confounded. A high strength shear bond is not required, except perhaps in the outermost fiber layers of the wheel.

All materials have a tendency to plastic flow under sufficiently great pressure. If such flow takes place, the fibers at the region of flow will unload, since the differential pressure which loads them has relaxed. The entire pressure load must be taken up by fibers outboard of the flow region. The result is cumulatively catastrophic. As the flow zone moves outward, eventually the remaining fibers can no longer contain the pressure and burst.

Since all materials are non–linear at high strain, one should choose the non–linearity of the matrix so that non–linearity unequally loads the *inner* fibers. As noted above, the presence of the outer fibers will act to distribute the load again uniformly. However, if the matrix weakens so that the inner fibers are understretched, as also noted above, the outer fibers will simply see excess load. Thus the preferred non linearity for the matrix is for it to get stiffer at high stress, not weaker.

Appendix A

FLYWHEEL FORMULAS

Here we catalog some of the formulas applicable to flywheel performance.

A.1 RELATIONSHIP BETWEEN PERIPHERAL SPEED AND E/M

The kinetic energy which can be stored by a mass m moving at a speed v is simply $mv^2/2$. The maximum speed within a flywheel rotor occurs on the periphery, where v is ΩR. The weight of the mass m is mg. Hence the most energy E that can be stored *is related to the weight* w *by*

$$E/w = g\, v^2/2 \qquad \text{(A-1)}$$

When E/m is expressed in W–hr/lb and v in feet/s, this equation becomes

$$E/m = v^2/171000 \qquad \text{(A-2)}$$

This is a maximum performance, unrelated to the materials of which the rotor is built. It is significant in that establishing an E/m goal necessarily establishes a target peripheral speed for the wheel. Even for E/m at 1 W–hr/lb, the wheel must move at over 400 ft/s. This is at least a subsonic speed in air, but one at which the rim can pump air efficiently.

If the goal rises to 25 w–hr/lb, the wheel speed rises to at least 2000 ft/s. This is supersonic with respect to air. If supersonic effects are to be avoided, the housing must either be evacuated to hard vacuum, or backfilled with H_2 (speed of sound 4400 ft/s) or He (speed of sound 3000 ft/s).[20] The windage loss associated with such wheel speeds is discussed in Appendix (A.6) below.

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[20] Large electrical generators are backfilled with H_2 in this country, in order to cut down on windage losses and to improve the cooling available from the circulating gas.

A.2 VOLUME OF HIGH STRENGTH MATERIAL

Suppose that high strength material is used to restrain mass *m*, either by

1. a radial spoke of cross section A and length R, or

2. by a circumferential belt of perimeter $2\pi R$ and cross section A.

The centrifugal acceleration at the position of *m* at radius R is mv^2/R. In either case, the fiber stress in the high strength material is T (along the spoke or along the circumference, respectively)

$$T = mv^2/RA \tag{A-3}$$

The term in the denominator on the rhs of this equation is the volume V of the fibers; the numerator is simply twice the stored kinetic energy E. Thus

$$TV = 2E \tag{A-4}$$

This relationship states that for mass concentrated on the perimeter of the flywheel, and a working stress T in the high strength constraint a volume V of the high strength material will be required, given by 2E/T.

To the extent that all high strength materials are to be loaded to approximately the same working stress, approximately the same volume of each material is required for a given energy storage (remember, most of the total mass is being constrained by the material). It follows that the costs of the fibers have to be adjusted by relative density to obtain relative cost in use in the wheel. In this sense, an aramid fiber at $12 per pound is only a third as expensive as specialty steel wire at $6 per lb, since the latter is six times as dense. However, against music wire at $0.91 per pound, the aramid fiber would have to cost less than $5 per lb to be cost competitive.[21]

A.3 SPECIFIC STRENGTH

By judicious multiplication and division by density in the last formula, and by combination with previous formulas one obtains

$$T/\rho g = 2\ E/m \tag{A-5}$$

We have used a mass density here in order to exhibit the role of *g* in the formulas. Ordinarily, the result is stated using the weight density ρ^*. If T is measured as force per square inch, and ρ^* in the same force units per cubic inch, the specific stress has the units of length (inches). With T/ρ^* in inches and E/m in W–hr/lb, the last equation becomes

$$T/\rho^* = 64000\ E/m \tag{A-6}$$

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[21] Indeed, the price of tire cord grade aramid fiber was roughly $5 per pound. Kevlar 29 (lower stiffness but comparable tensile strength to the '49' grade) was about $6 per lb in January 1984.

A.4 PRESSURE AND FIBER STRESS AT CURVED SURFACE

Suppose a curved surface with homogenous surface tension γ per unit length along an imaginary cut through the surface, and let $1/r^*$ be the total local curvature. Then static equilibrium with a fluid pressure P just under the surface is obtained when

$$\gamma = P\, r^* \tag{A-7}$$

where $1/r^*$ is the sum of the two principal curvatures $1/r_1$ and $1/r_2$:

$$1/r^* = 1/r_1 + 1/r_2 \tag{A-8}$$

Suppose next that the surface tension is not homogenous, but instead has a magnitude F in a specific local direction determined by physical lay of fibers. Let r be the local radius of curvature of the fibers. Then the surface fiber force F will be in equilibrium with subsurface pressure P if

$$F = P\, r \tag{A-9}$$

It follows that if the average thickness of the fiber layer is s the fiber stress T is related to the pressure P by

$$P = T\, s/r \tag{A-10}$$

A.5 PRESSURE – STRESS – VOLUME RELATIONSHIPS

The pressure P of the flywheel fluid mass conforms to both local and global constraints. Evidently, if the mass M of the flywheel is distributed roughly uniformly along the surface of a cylinder of height h and radius R, the centrifugal pressure is

$$P = M\, R\Omega^2/2\pi Rh \tag{A-11}$$

Evidently, also if the fluid has a density ρ, the radial dependence of P is locally

$$dP = \rho R\Omega^2\, dR \tag{A-12}$$

A.5.1 Global Relationships

The global pressure relationship, Eq (A-11), has two interpretations:

1. One can cancel the factor of R and combine with results of the previous sub-section to obtain a design formula for the fiber stress T_f

   $$T_f = M\Omega^2\, r/2\pi sh \tag{A-13}$$

2. One can multiply numerator and denominator in Eq (A-11) by R to obtain an energy – volume relationship

$$P = E/V_{cyl} \tag{A-14}$$

where V_{cyl} is the cylinder volume. The conclusion reached is that low pressure implies a large flywheel

$$V_{flywheel}/V_{fiber} = 2T/P \tag{A-15}$$

A.5.2 Radial Loop Fiber Stress

Note that the angle θ in Figure (6) is measured to the y-axis. Thus, in terms of the Figure, dy/dx is $\cot\theta$ (rather than $\tan\theta$ if θ were measured to the x-axis). Note also that we follow here Eagle's[15] notation in ascribing to the sine-type Jacobian elliptic function a real period of 2π and unit magnitude residues at its poles. The resulting "Sn" function (scaled from the conventional "sn" function) has real zeros at x=0 and $x=\pi$ as does the sine function. The maximum at $x=\pi/2$ has a numerical value of $2kK(k)/\pi$, where K( ) is the conventional complete elliptic integral of the first kind. The slope at x=0 is $(2/\pi)^2kK^2$. (Eagle uses "h" for $2K/\pi$, resulting in the formula kh for the maximum and kh^2 for the slope at the origin).

A.5.2.1 Rotating Loop

The *U-shaped loop is assumed attached to the x-axis as an axle* at an angle θ_0. The force per unit length normal to the plane of the paper is assumed to be T(x). Since the filament (neglecting gravity) is not subject to forces in the x-direction, the x-component of T, viz. $T\sin\theta$, is constant at some value F. It follows that the y-component of the linear stress in the fiber is $F\cot\theta$. Now let *ds* be an element of length along the fiber. The force acting on the element *ds* is $\cot\theta\, dF$, in equilibrium with acceleration force $\sigma y\,\Omega^2 ds$. Inasmuch as F is constant, this yields

$$F \csc^2\theta\, d\theta/ds = y\,\sigma\Omega^2 \tag{A-16}$$

Now $ds/d\theta$ is simply the local radius of curvature r. At the extreme end of the loop, where $y=y_0$, $\csc\theta$ is unity and $r=r_0$. Eq (16) can thus be reduced to

$$1+\cot^2\theta = yr/r_0r_0 \tag{A-17}$$

Introduction of the geometric identities

$$\cot\theta = dy/dx = y'$$

$$r = (1+y'^2)^{1.5} \tag{A-18}$$

produces after some algebraic manipulation

$$y''/(1+y'^2)^{0.5} = -y/y_0r_0 \tag{A-19}$$

Multiplying both sides by y' produces an equation that can be integrated by inspection on both sides. Choosing the constant of integration so that the angle at the axle is θ_0 then gives

$$\sqrt{(1+y'^2)} = \csc\theta_0 - y^2/2y_0r_0 \tag{A-20}$$

Now, the integration of this equation to obtain y as a function of x requires the sine-type of Jacobian elliptic function. If we use the Eagle notation and set the ends of the loop at $x=0$ and $x=\pi$, then Eagle finds the equation of the loop to be

$$y = (2/h^2(1-k^2))\, Sn(x,k) \tag{A-21}$$

However, there is an intermediate conclusion that can be reached from Eq (20). In particular, at $y=y_0$, y' vanishes, so that

$$\csc\theta_0 = 1 + y_0/2r_0 \tag{A-22}$$

Now, if this were the case dealt with in the next section, where a pressure P proportional to y is resisted by T/r locally, then y_0/r_0 would be the physical ratio P/T. Here such is not the case. However, the ratio y_0/r_0 is clearly a geometric property of the Sn curve which does not depend on the particular scaling assumptions used in describing the curve. In fact, for a Sn (or sn) curve

$$y_0/r_0 = 4k^2/(1-k^2) \tag{A-23}$$

Another geometric property is the slope of the curve at the origin:

$$\cot\theta_0 = 2k/(1-k^2) \tag{A-24}$$

The problem is to determine k for a particular physical situation. Suppose we have available a length L of the fiber, and attach it to the axle. We don't yet know what θ_0 will be. Eagle gives L in terms of the preceding Sn equation as

$$L = \pi(2E/K(1-k^2) - 1) \tag{A-25}$$

where E and K are conventional elliptic integrals. This makes L calculable if k is given; the inversion to obtain k from L is numerical cut and try. If we suppose that we have available a spool of fiber and reel out enough of it to make the connections at the axle and a loop of the right shape reaching to y_0, we still have the problem that θ_0 remains a function of the (as yet undetermined) parameter k.

In Eagle's notation, the relative height of the Sn function is hk. The corresponding distance between tie points is π. But Eagle finds that to satisfy the dynamic conditions

$$y_0 = 2k/(1-k^2)\,(1/kh^2)\, Sn(k,\pi/2) \tag{A-26}$$

But the first factor, in brackets on the rhs of this equation in simply $\cot\theta_0$; and x_0 for these last few equations is $\pi/2$. Moreover, Eagle's h is the usual elliptic integral K(k) divided by $\pi/2$. It follows that

$$y_0/x_0 = \cot\theta_0/K(k) = 2k/K(k)(1-k^2) \tag{A-27}$$

As noted above, this equation can only be inverted numerically to obtain k for a given y_0/x_0.

A.5.2.2 Pressure Loaded Radial Loop

Here we assume that y=0 is the free surface of the liquid being restrained, and that the liquid layer is thin compared with the flywheel radius R, so that the pressure P is proportional to y:

$$P = \rho y R \Omega^2 \qquad \text{(A-28)}$$

Now, the fiber is presumed massless and limp, so that the force per unit circumferential length, F, must be constant. The pressure must then be in equilibrium with F/r. It follows that inside the free surface, where the pressure is zero, r must be infinite (the filament must be straight).

This being so, the angle θ_0 is determined by the geometry of the attachment at the axle. Even though the ultimate solution for the shape of the curve at the tip of the pancake is an elliptic function, in this case θ_0, and consequently the elliptic parameter k, is predetermined.

From the equilibrium condition,

$$F/r = \rho y R \Omega^2 \qquad \text{(A-29)}$$

This equation, of course, applies at the tip, where $y=y_0$ and $r=r_0$. Normalizing Eq (29) by its values at the tip, and making use of the geometric identity that

$$r = (1+y'^2)^{1.5}/y'' \qquad \text{(A-30)}$$

produces the differential equation

$$y''/(1+y'^2)^{1.5} = -y/y_0 r_0 \qquad \text{(A-31)}$$

After multiplying both sides by y', the equation can be integrated as it stands, making use of the substitution of $\cot\theta$ for y' on the lhs. The result is

$$\sin\theta = \sin\theta_0 + y^2/2y_0 r_0 \qquad \text{(A-32)}$$

Evidently, at $y=y_0$ $\sin\theta$ is unity, so that (writing Γ for y_0/r_0)

$$\sin\theta_0 = 1 - \Gamma/2 \qquad \text{(A-33)}$$

The next step in the integration yields elliptic integrals. Let ϕ be an angle parameter. Then the parametric equations for x and y have the form

$$x = 2E(k,\phi) - F(k,\phi)$$

$$y = 2k \cos\phi \qquad \text{(A-34)}$$

where E and F are the conventional incomplete elliptic integrals of the first and second kind, respectively. With respect to the parametric equations, moreover, the total length of the fiber between x-axis crossings is L(k) and the angle at the axis is $\theta_0(k)$

$$L(k) = 2K(k)$$

$$\sin\theta_0 = 1 - 2k^2 \qquad \text{(A-35)}$$

It follows that k^2 is $\Gamma/4$.

Finally, one can integrate 2xdy to obtain the area of the cross section between the fiber loop and the x-axis for the purpose of finding the relationship between the pressure P at the tip and the mass of fluid causing it. In terms of the Eqs (34), this area A is[22]

$$A = 4k \int_{u=0}^{2\pi} (2E - F) \sin(u)\, du$$

$$= 4k\,(1-k^2)^{0.5} \qquad \text{(A-36)}$$

To use this result, note that the pressure at the tip is F/r_0 and that r_0 in turn is y_0/Γ or $y_0/4k^2$. But in terms of Eqs (34), y_0 is 2k. Thus, r_0 is 1/2k and the pressure at the tip is 2kF. But, in terms of the rotational dynamics, the pressure is $\rho y_0 R\Omega^2$. Equating the two yields

$$F = \rho R\Omega^2 \qquad \text{(A-37)}$$

Note that this result for F is independent of the choice of *k*! All other parameters being equal, the maximum stored energy is thus obtained by maximizing the cross-sectional area A of the fluid mass accommodated by a given stress. By inspection of Eq (36), this maximum occurs for $k^2=1/2$, so that A=2.

Since there are two fibers, the force in each supports a mass ρ per unit circumference at centrifugal acceleration $R\Omega^2$. This is exactly same force-mass relationship as would exist in a weightless radial spoke supporting this mass. (A non-optimum *k* leaves the force in the spoke unchanged while the mass supported drops). The conclusion is that for thin fluid rims, the radial loop support system is as efficient in use of high strength fiber as is a spoke support system.

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[22] For the integrals, see Gradshten & Ryzhik, *Tables of Integrals, Series, and Products*, 4th Edition, Alan Jeffry editor of the English version, Academic Press, New York, 1980

A.6 WINDAGE LOSS

The calculation of the windage loss for a rotating disk or cylinder is contained in an out–of–print government publication.[23]

All wheels operating at high peripheral speed are potential pumps. If possible pumping action is not anticipated, the losses can be very high. However, with a close–fitting casing the gas between the wheel and the casing will circulate at one–half the angular velocity of the wheel, and the pumping will be confined to two thin boundary layers, one adjacent to the wheel and the other adjacent to the casing.

Under such circumstances is is possible to define a coefficient C_M that related friction force to speed and area, where C_m depends on the Reynolds number Re. For both sides of a disk which has radius *r*, spinning with angular velocity Ω, the formulas for the total torque τ are

$$\tau = C_M \rho \Omega^2 r^5/2$$

$$\tau = C_M \rho v^2 r^3/2 \qquad \text{(A–38)}$$

where the second equation is obtained from the first by substituting the peripheral speed v for Ωr.

Reynolds number in such cases is computed from v and r. It is generally in the range 10^7, where C_M is nearly independent of Re at 0.005, and for which the boundary layer is roughly one thousandth of a radius thick. Similar formulas hold for the out–side surface of a spinning cylinder, except that the length of the cylinder replaces one power of r.

Evidently, for a given angular speed only the largest radius portions of a spinning assembly produce a significant contribution to the windage loss. The actual power loss V is found by multiplying τ by the angular velocity:

$$V = C_M \rho v^3 r^2/2 \qquad \text{(A–39)}$$

for the sides of the disk. For air at atmospheric pressure, with ρ equal to 1.3 kg/m^3, and r=0.5m, a peripheral speed of 150 m/s would result in a 3 kW windage loss. Replacing the air with hydrogen would reduce the loss to 200 W.

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[23] "Experiments on Drag of Revolving Disks, Cylinders, and Streamline Rods", by Theodore Theodorsen and Arthur Regier, NACA Report No 793, Superintendent of Documents, Washington DC, 1945

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It is proposed to design and construct energy storage flywheel rotors as statically limp tubes containing liquid mass, and to drive and support this rotating system (at least in part) directly, rather than through separately engineered subsystems. If the "liquid" is presumed thixotropic or viscous, nominally stiff structures subject to plastic flow are included. At one extreme of the design range, nearly all the mass is in the liquid and the only significant stresses are those in the wall of the containment; at the other extreme, the statically limp structure is nearly "dry" and is formed into an oblate surface by the centrifugal force of its own mass. The plausibility of the approach is argued by analogy with various physical examples, ranging from the spinning lariat to the design of reinforced concrete.

Results include the conclusion that in a limp structure, bonding between matrix and fibers is not a primary issue, and that a thin rim of liquid restrained by radially looped fibers has the same efficiency in using fiber material as when the fibers are used to support a solid rim as radial spokes (standard thin rim efficiency). It is also argued that both energy exchange and support can be provided for the flywheel rim itself, without the need to supply either through the central axle.

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