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RETURN DIFFERENCE FEEDBACK DESIGN FOR ROBUST UNCERTAINTY TOLERANCE IN STOCHASTIC MULTIVARIABLE CONTROL SYSTEMS

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INTRODUCTION

The increased demands for quick and precise control over aircraft and space vehicle response that are anticipated in the coming decades will have to be matched with automatic control systems that can respond instantaneously, without moment-to-moment human guidance, anticipating vehicle response insofar as is possible and, more importantly, continually and automatically monitor vehicle response and re-adjust control signals to correct for unpredicted deviations from the desired response. Such unpredicted response variations can result from external disturbances (e.g., wind gusts) and from the impossibility of employing a sufficiently complex and accurate model of the vehicle's dynamics to account for every vibrational mode, every nonlinearity, ..., every variable affecting system response.

While the state-space-based mathematical theory for controlling systems without substantial uncertainty regarding dynamical response grew relatively sophisticated during the two decades of the 1960's and 1970's, there was almost no significant progress concerning the control of systems having uncertain response since the 1940's and 1950's when great strides were made in the development of the "classical" transfer-function-based theory for the control of simple single-input-single-output (SISO) linear time-invariant LTI) systems with uncertainty. Consequently, when this research project was begun in October 1979 there was no adequate theory to provide engineers with an efficient, systematic procedure for the design of precision controllers for more complex multi-input-multi-output systems such as the highly unstable, fast responding, control configured aerospace vehicles that are expected to be operating in tomorrow's combat environment.
The objective of the present research has been to develop this badly needed theory so that the engineers who must design tomorrow's aerospace vehicles will have more than intuition, trial-and-error simulation and mystical "seat-of-the-pants" insight for guidance in designing uncertainty-tolerant automatic controllers for these vehicles.

**BACKGROUND AND PROGRESS 10/79 - 9/82**

Since work began on this project in October 1979, the research effort has been generally successful in achieving its main objective of relating the return difference matrix to the uncertainty tolerance properties of a system. These properties are also known as robustness properties or feedback properties. Our results, together with some related results useful in the actual synthesis of robustly uncertainty tolerant feedback controllers, were reported in the paper "Feedback Properties of Multivariable Systems: The Role and Use of the Return Difference Matrix" [1]. This paper discusses the central roles in feedback theory of the return difference matrix (denoted I+L(s)) and the inverse-return difference matrix (denoted I+L^{-1}(s)).

Among the new theoretical results in [1] are the following:

(i) A new method for exact evaluation of the sensitivity of multi-variable feedback control systems which overcomes significant practical limitations associated with previously known methods. Sensitivity to large plant and sensor variations is directly related to the nominal system's return and inverse-return difference matrices. (See Theorem 2.1 and Theorem 2.2 in [1]).

(ii) Significant drawbacks of characteristic locus analysis methods
(cf. MacFarlane and others) are described. Return and inverse-return difference singular value plots are found to overcome some of the drawbacks of characteristics loci. The results have been found to be useful in quantifying some fundamental limits on the achievable performance of feedback control systems. (See Section 3 and Section 4 of [1]).

(iii) A technique, based on stochastic linear quadratic Gaussian (LQG) optimal control theory, has been developed to aid the shaping of the return and inverse-return difference singular value plots. Though the technique is to a certain extent a trial-and-error design technique, it continues to be substantially more systematic than any other method that is currently available for synthesizing multivariable control systems to meet specifications requiring a robust tolerance of disturbances, noise and plant/sensor modeling errors (see Section 5 in [1]). To demonstrate the viability of the technique, the theory has been applied to the synthesis of an automatic controller for the longitudinal dynamics of an advanced control configured vehicle (CCV) aircraft, viz., the NASA HIMAT remotely piloted aircraft (see Section 6 of [1]).

More recently, research effort has focused on several issues.

First, substantial effort has been focused on the important practical issue of how to solve the so-called "Inverse Problem of Linear Quadratic Gaussian (LQG) Optimal Control" in the general setting in which the controller is dynamical and the plant is subject to plant and sensor noise. This inverse problem is as follows: Given a realizable closed-loop control return-difference matrix, the plant transfer function matrix \( P(s) \), and the plant and sensor noise power spectra matrices, say \( \Sigma_d(s) \) and \( \Sigma_n(s) \), find the linear quadratic cost matrices \( R(s) \) and \( Q(s) \) such that the closed-loop system is optimal in the sense that the following stochastic cost is
minimized:

\[
J = \frac{1}{2\pi j} \int_{-j\infty}^{+j\infty} \text{Tr} (Q(s)\Sigma_y(s) + R(s)\Sigma_u(s)) \, ds
\]

where \( \Sigma_y(s) \) and \( \Sigma_u(s) \) denote the respective closed-loop power spectra matrices of the sensor output and control input signals. We are pleased to report that the solution to the problem, together with extensive discussion of its ramifications regarding a number of related problems is in hand [5]. This work was supported under AFOSR Grant 80-0013.

The importance of the LQG inverse problem to the proper understanding of robust multivariable feedback control system design cannot be understated. LQG multivariable feedback designs are preferred for a variety of reasons: Good computer software is readily available for solving the LQG design equations (e.g., [10]), LQG designs optimize inherent trade-offs between robustness properties such as sensitivity versus stability margin [1], and many engineers in the aerospace industry are familiar with the basic LQG concepts. The solution to the inverse problem plays a vital role in our understanding of how one can use the LQG theory to place the poles and zeros of \( I+L(s) \) in order to achieve acceptable robustness and sensitivity singular-value Bode plots [1] and acceptable transient response and asymptotic tracking properties [5]. Also, as it is common practice to iteratively adjust the LQG cost matrices when "fine-tuning" a control design to meet various robustness specifications, the solution to the inverse problem provides a starting point for fine-tuning non-LQG multivariable feedback designs.

A second focus of the research effort is one of rather fundamental significance in linear feedback theory: realizability. A closed-loop
feedback system's return-difference \( I+L(s) \) is said to be realizable for a given plant \( P(s) \) if for some controller \( C(s) \)

(i) \( L(s) = P(s)C(s) \), and

(ii) \( (I + L(s))^{-1} \) is stable, and

(iii) \( P(s) \) and \( C(s) \) are "coprime" in the sense that there are no unstable pole-zero cancellations in the product \( P(s)C(s) \). A new simplified realizability result (Lemma 1, [4] and [55]) has been developed which characterizes realizability directly in terms of the poles and associated residues of the return-difference and inverse-return difference. This is an improvement over previous multivariable results ([3], Lemma 3) which require a solution of the so-called "Bezout" equation and give only limited insight into the constraints imposed by realizability on the set of achievable return difference and inverse-return difference matrices.

The realizability question arose in connection with the previously mentioned LQG inverse problem, and our new realizability result in [4] plays a crucial role in the solution of the LQG inverse problem in [5], in addition to laying the groundwork for the decoupled multivariable \( L^\infty \) optimization problem solved in [4].

The \( L^\infty \) optimization problem that naturally arises in sensitivity and stability margin optimization has been another sign of our research effort. With the aid of results in [1] which show that stability margin is inversely proportional to the \( L^\infty \) norm of the inverse of the inverse-return difference matrix \( I + L^{-1}(s) \) and the aid of our new realizability result (Lemma 1 of [4]), it was shown in [4] and [55] that the problem of designing a feedback controller to maximize stability margin (subject to decoupling and, perhaps, asymptotic tracking constraints) is mathematically equivalent to the minimization
\[
\min_{x_j(s) \in H^\infty} \| x_j(s) \|_{\infty}, \quad (j = 1, \ldots, m)
\]

subject to complex interpolation constraints of the form

\[
x_j(s_{iu}) = w_{ij}, \quad (i = 1, \ldots, n), \quad \text{Re}(s_i) > 0,
\]

where \( s_{ij} \) and \( w_{ij} \) are complex constants and \( H^\infty \) is the Hardy space of stable transfer functions with the \( L^\infty \) norm,

\[
\| x(s) \|_{\infty} \triangleq \sup_{\omega} | x(j\omega) |
\]

A simple solution to this interpolation problem requiring only the calculation of certain eigenvectors and eigenvalues is available in the mathematics literature [39]. This leads in turn to the multivariable feedback controller having maximal stability robustness. It also points the way to improving and extending to multivariable systems certain recent results of Zames and Francis [40] concerning single-loop feedback sensitivity minimization with respect to the \( L^\infty \) norm. The results recently have been generalized to the decoupled multivariable case by the principle investigator and Ph.D student, B. S. Chen [4, 5].

**PROGRESS 10/82 TO 9/83 AND CURRENT RESEARCH**

During the past year our research has lead to progress in several directions as reported in the thesis and seventeen new publications and reports appended to this report.

One focus of the research effort has been on the robustness of control systems for flexible mechanical structures, e.g., space-borne
antennas and telescopes. References [48-51] and [53] address this problem. Results reported therein relate the singular values of Moore's balanced state-space realization [8] to the poles and associated damping ratios for lightly damped large-scale-structures [49]. It has been found that for such systems "balanced" and "modal" state-space coordinates coincide asymptotically as the damping goes to zero [50]. Also, improved bounds on the sensitivity of the Lyapunov equation associated with such systems have been obtained [51, 53]. The Lyapunov equation plays a central role in robustness and stability analysis.

The existence of solutions to the LQG (Linear Quadratic Gaussian) optimal control problem has been re-examined from the operator theoretic point of view [52]. Results have been obtained relating the spectra of the Wiener-Hopf operator to the existence of stabilizing and antistabilizing optimal feedbacks and to the fundamental role played by the "positivity" of certain related operators. The new approach assumes a great deal about the "fine" structure of the linear-quadratic problem. The results of Safonov and Sideris [63] provide a unified view of the state-space and Wiener-Hopf approaches to the LQG problem, showing that the LQG state-feedback and Kalman-Bucy filter return-difference matrices generate the plant matrix fraction description and the spectral factors of the Wiener-Hopf solution.

Several of our papers deal with various aspects of stability margin or "robustness" analysis for multiloop feedback systems. Reference [55] describes a simple method based on the Perron-Frobenius Theory of non-negative matrices for suboptimally pre-scaling matrices before computing robustness singular values. References [58] and [65] show that with additional computational effort, optimal prescaling can be
accomplished by gradient descent methods and that $\sigma_{\text{max}}^2(DMD^{-1})$ is convex in the diagonal scaling matrix $D$.

Solutions to the problem of synthesizing an output feedback control law to achieve $L^\infty$-norm optimal robustness singular values (and/or sensitivity singular values) are obtained in [4],[54],[57] and [66]. We treated the case involving a closed-loop decoupling constraint in [4] and [54]. The unconstrained case is treated in [57] and [66] by establishing an equivalence with the so-called "zeroth order" optimal Hankel-norm model reduction problem which can be solved in the state-space using the techniques developed by Bettayeb, Silverman, and Safonov [12,13] and improved by Glover et al. [68]. The very recent work of Doyle [69] interfaces with the results of [57,66,68] to provide a complete state-space oriented framework for solving the $L^\infty$ singular value optimization problem. The paper by Safonov [59] gives a brief overview of the state-of-the-art in $L^\infty$ optimization with a comprehensive discussion of the role, use and limitations of the theory for robust control system design. The paper [65] by Safonov clarifies some common misunderstandings about the role of singular values. We believe that the $L^\infty$ singular value approach to feedback control law synthesis has much to commend it in that it provides a direct method for optimizing and shaping singular-value Bode plots. However, singular values have been found to be very conservative measures of performance in many situations. Our current research effort is directed in part toward the problem of expanding the class of problems that the $L^\infty$ theory can handle to include less conservative performance measures than singular values. The design methodology we describe in [59] is a step in this direction.

We have obtained significant new results concerning the stability of nonlinear feedback systems having hysteresis nonlinearities. The results of Safonov and Karimlou [60,62] and the results in the Ph. D. thesis of
Karimlou [67] enable one to treat a broad class of hysteresis nonlinearities as if they were conic-sector bounded, even when the hysteresis includes the origin.

A fundamental role in robustness theory is played by the "conic sector." Indeed, the singular-value may be viewed as simply a method for verifying whether a given transfer-function matrix satisfies a conic sector condition. Of course, conic sectors are much more general in that they may be used for nonlinear and time-varying systems in addition to transfer functions. The paper [56] by Safonov on "Propagation of Conic Model Uncertainty" is of fundamental significance. It shows that simple operations such as feedback, input projection, output projection, and feedforward map conic sectors into and onto new conic sectors whose cone-parameters can be precisely calculated. The result enables hierarchical decomposition of robustness analysis and robust control law synthesis problems involving uncertain large scale systems.
REFERENCES


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B.D.O. Anderson, "Algebraic Properties of Minimal Degree

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# Report Title

**RETURN DIFFERENCE FEEDBACK DESIGN FOR ROBUST UNCERTAINTY TOLERANCE IN STOCHASTIC MULTIVARIABLE CONTROL SYSTEMS**

## Abstract

The objective of the research has been to develop engineering methodologies applicable, but not limited, to aerospace automatic control design problems in which there are performance specifications requiring precise control of system behavior in the presence of stochastic disturbances (e.g., wind gusts) and large-but-bounded uncertainties in the dynamical response of the system (e.g., parameter uncertainty, unmodeled nonlinearities, and so forth). During the past three years of research, a cohesive body of theory has been developed that enables engineers to relate the ability of feedback control systems to meet such specifications directly and quantitatively to the "return difference matrix" associated with the system's feedback loops. Now results enabling the UC optimization of returned difference singular value Bode plots promise to be of great value in robust multivariable feedback controller synthesis. Continuing research is currently being aimed at further tightening the links between this theory and the most recent developments of modern stochastic linear optimal control synthesis theory, and extending the (CONTINUED)
ITEM #19, ABSTRACT, CONTINUED: results to admit more practical problems, so that this theory may be used more effectively by engineers to efficiently and systematically design the feedback gains that determine a feedback system's return difference matrix. Such results substantially reduce the dependence of control engineers on intuition, simulation, and luck and provide the know-how to successfully and efficiently solve the increasingly complex and demanding aerospace control problems of the coming decades.