CERENKOV RADIATION IN THE NEIGHBORHOOD OF THE EMISSION THRESHOLD

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Cerenkov radiation for constant velocity electrons in an infinite uniform dielectric has a sharp threshold for \( v \) (electron) larger than the speed of light in the dielectric. A medium of finite length produces diffraction which smears the Cerenkov emission angle and lowers the threshold velocity for emission.
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Introduction - Microwave Cerenkov

Cerenkov radiation in the simplest form occurs when a charged particle in uniform motion exceeds the velocity of light in an infinite medium. The radiation is emitted in a cone, with the rays occurring at a sharp angle given by \( \cos \theta_c = \frac{c}{v} \), and for an infinite medium and uniform velocity, the radiation would disappear for \( v < c \). The result above is the consequence of requiring the phase of the radiation, emitted at an angle \( \theta_c \), to remain in phase with the charge as it moves in time. We have studied microwave Cerenkov radiation experimentally and theoretically\(^1,2,3\). These studies describe the microwave or other R.F. emission which is significant for bunches of electrons with dimensions shorter than the wavelength of emitted radiation so that all electrons in the bunch radiate coherently. These effects will be explored elsewhere.

Diffraction Effects

The point of this paper is as follows: If the ideal conditions (constant electron velocity or infinite medium) are changed, the radiation changes, possibly dramatically. In \(^1\) it was noted that for a finite length of medium, diffraction occurred and the Cerenkov angle is smeared. In \(^2,3\) these effects were considered further and, besides the smearing of the emission angle,
it was noted that the radiated power may be larger for a finite medium.

The theory is based on Ref. 1, Eq. (A13), which gives the energy radiated per unit solid angle in the frequency range $d\omega$:

$$\omega (\omega, \kappa) d\omega = \frac{1}{16\pi^3} \frac{u}{c} \omega^2 \sin^2 \theta^2 \left( \frac{L}{\nu} \right)^2 \frac{\sin^2 \theta}{u^2} q^2 \mathcal{F}(\kappa)$$  \hspace{1cm} (1)

where $u$ is defined below, $L$ is the length of the medium, $\kappa$ is the wave number of the emitted radiation, and $\mathcal{F}$ is the form factor for the bunch.

**Threshold of Cerenkov Radiation**

The mechanism allowing the smearing of the angle and the increase in power for a finite medium is relaxation of the phase matching between the electron and the wave. If the wave is emitted at an angle $\theta \neq \theta_C$, the electron and wave will be only slightly out of phase at the end of a finite path $L$. In fact from 2, the null of the radiation pattern occurs for

$$u = \frac{\kappa L}{2} \left( \frac{c}{v} - \cos \theta \right) = v$$  \hspace{1cm} (2)

Thus we have radiation from $\theta = 0$, to $\theta_C$ (where $u = 0$) and beyond, to $\theta_n$ (where $u = \pi$). Now note that, if $v < c$, there is no Cerenkov angle ($\cos \theta_C = c/v$ has no solution) but $\theta_n$ may exist, and radiation occurs below the usually accepted threshold.
This effect was investigated in the optical region \(^4,5,6\), both theoretically and experimentally by Kobzev and Frank. We may calculate how much the usual Cerenkov threshold could be lowered by noting that the radiation will disappear for all practical purposes when \(v\) decreases such that \(n - 0\). A simple calculation gives:

\[
\frac{1}{2\gamma^2} (\text{threshold}) = \frac{\lambda}{L} + \Delta, \quad (3)
\]

where \(1/1-\Delta\) is the relative index of refraction.

**Numerical Example**

Example: \(\Delta = 2.68 \times 10^{-4}\) for air. Let \(kL = \infty\). Then

\[
\gamma = 43.
\]

Now let \(L = 1m, \lambda = 1 cm\). Then \(\gamma = 7.07\). The change in threshold is indeed dramatic. No attempt has been made to include effects other than abrupt termination of the ideal, infinite uniform medium. This could be accomplished by abruptly stopping the beam, which is assumed to have constant velocity before the stop.

**REFERENCES**


Figure Caption:

Qualitative illustration of diffraction effects in Cerenkov radiation associated with a finite length of path. In the upper curve $v > c$ and the radiation is spread about the Cerenkov angle (shaded area). In the lower curve, $v < c$ but the same diffraction function allows radiation of occur.
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