K-SHELL YIELD SCALING LAW FOR CONVENTIONAL PRS LOADS

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J. P. APRUZESE AND J. DAVIS

Plasma Radiation Branch
Plasma Physics Division

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**K-Shell Yield Scaling Law for Conventional PRS Loads**

The simplified equation of motion for a Z-pinch implosion is combined with an empirical scaling law for K-shell yield as a function of temperature and density, and also with power law fits to K-shell atomic data to predict K-shell yields as a function of atomic number and generator power. Assuming the efficiency of future machines to approximate that of existing generators leads to the conclusion that argon will be a prolific PRS with generator powers of \( \sim 25 \) TW while titanium will require \( \sim 40 \) TW. Conventional iron and krypton loads will require superclass generators in the 50-100 TW range to be efficient K-shell radiators.
CONTENTS

I. INTRODUCTION ............................................. 1
II. BASIC MODEL ELEMENTS .................................... 1
III. BASIC ATOMIC PHYSICS ENERGETICS ...................... 6
IV. CONCLUSIONS ................................................ 9
ACKNOWLEDGMENTS .............................................. 11
REFERENCES ..................................................... 11
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I. INTRODUCTION

An interesting figure of merit often used to characterize the performance of Z-pinch devices such as BLACKJACK 5 at Maxwell Laboratories, Inc., and PITHON, of Physics International Corp. is the energy released in K-shell radiation during a discharge with a given element as the plasma load. On both PITHON and BLACKJACK 5, K-shell yields have been experimentally measured for load elements ranging from neon (Z=10) to krypton (Z=36). To date, BLACKJACK 5, and DOUBLE EAGLE with a maximum power of approximately 8-10 TW, represent the leading edge in the technology of such devices.

For theoretical analyses of the effects produced by these generators, a variety of tools and strategies have been developed, ranging from simple "zero-dimensional" implosion models\(^1\) to detailed MRHD (Magneto Radiation HydroDynamic) codes containing atomic ionization dynamics and radiation transport capabilities.\(^4,5\) While the latter detailed models are indispensable toward achieving an understanding of the complicated physics issues, it is oftentimes appropriate to appeal to simplified models in order to gain physical insight and order of magnitude estimates of the various effects in a noninteractive mode. In this Memo Report it is our intent to bridge the gap between the overly complicated and the overly simplistic approaches by considering a simple dynamic model for the implosion coupled with atomic physics and empirical data based on experimental observation. Our principal result is the establishment of a predictive capability which will serve as a guide for estimating generator power requirements for substantial K-shell radiation for conventional PRS loads.

II. BASIC MODEL ELEMENTS

The equation of motion\(^1\) describing the implosion of a plasma annulus when the magnetic pressure greatly exceeds the plasma pressure is

\[
m \frac{d^2 r}{dt^2} = -10^{-7} \frac{I(t)^2}{r}
\]  

(1)

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where \( m \) is the annulus mass per unit length (kg \( \text{m}^{-1} \)), \( r \) is the radius (m), and \( I(t) \) is the time-varying current (amps). For a wide variety of machines from GAMBLE-II to BLACKJACK 5, the main current pulse is adequately described by a linear ramp \( I(t) = I_0 t \) where \( I_0 \) is the current rise slope in amps sec\(^{-1}\). Fig. 1 shows the actual current pulse for BLACKJACK 5 shot 177 and the corresponding excellent linear fit to \( I(t) \) during the main run-in phase. Equation (1) with the linear current fit becomes

\[
m \frac{d^2 r}{dt^2} = \frac{-10^{-7} I_0^2 t^2}{r}.
\]

Of course, the resisting plasma pressure late in the run-in will render this simple picture inaccurate. However, considerable evidence from various machines\(^{1-3}\) strongly indicates that Eq. (2) obtains until the plasma has collapsed to about 10% of its initial radius, at which time the implosion abruptly ceases and the main radiation pulse commences. Our description is based upon this empirical observation. Eq. (2) may be universalized by adopting the following dimensionless variables, where \( R_o \) = initial radius in meters.

\[
x = \frac{r}{R_o}
\]

\[
q = 1.78 \times 10^{-2} \left( \frac{I_0}{R_o} \right)^{1/2} m^{-1/4} t
\]

Eqs. (3a) and (3b) combined with Eq. (2) lead to

\[
x \frac{d^2 x}{dq^2} = -q^2
\]

The boundary conditions applied to Eq. (4) are: at \( t=0 \), \( x=1 \), and all \( q \)-derivatives of \( x \) are 0, since \( I(t)=0 \) at \( t=0 \). To our knowledge, no analytic solution of Eq. (4) exists, but a power series solution is obtainable. This series solution converges too slowly to be of real practical value

\[
x(q) = 1 - \frac{q}{12} - \frac{8}{672} - \frac{q}{15,650} - \ldots
\]
Fig. 1. Current trace for BLACKJACK 5 shot 177, along with the linear fit adopted to describe the major run-in phase. Current trace provided courtesy of Maxwell Laboratories, Inc.
The exact numerical solution of Eq. (4) is presented in Fig. 2. At our "magic" $r/R_o = x = 0.1$, where the implosion is deemed to cease and thermalization begins, the time variable $q = 1.70$, and the implosion velocity $dx/dq = 3.3$. Key physical variables at this point are the kinetic energy per ion and the implosion time. Since $q = 1.70$ the implosion time (sec) is easily obtained from Eq. (3b) as

$$t_{\text{imp}} = 95.6 \left( \frac{R}{R_0} \right)^{1/2} \text{ m}^{1/4} \text{ sec.} \quad (6)$$

To obtain the kinetic energy per ion at $x = 0.1$, note the atomic weight $A \approx 1.32Z^{1.15}$ where $Z$ is the element's atomic number. Straightforward algebraic manipulation of the variables leads to

$$\text{(KE)}_{\text{ion}} = \frac{1.64 \times 10^{-12} R_0 I_0 A (dx)^2}{\sqrt{m}} \text{ eV} \quad (7a)$$

or, since $dx/dq = 3.3$ at $x = 0.1$, and $A = 1.32Z^{1.15}$

$$\text{(KE)}_{\text{ion}} = \frac{2.36 \times 10^{-11} R_0 I_0 Z^{1.15}}{\sqrt{m}} \text{ eV} \quad (7b)$$

Conventional loads are usually designed$^{2,3}$ so that the implosion time, Eq. (6), matches the time of peak current, $I_p$. In some cases the match is only approximate, but even a 10-20% deviation does not substantially change our results. The current slope $I_o = I_p/t_{\text{imp}}$. Using Eq. (6) for the implosion time, $t_{\text{imp}}$, leads to a relationship between $I_o$ and $I_p$

$$I_o = \frac{1.09 \times 10^{-4} I_p^2}{R_0 \sqrt{m}} \text{ amps sec}^{-1}. \quad (8)$$

Substituting for $I_o$ in Eq. (7b) we finally obtain the following expression for kinetic energy per ion at 10% of the initial radius, where we assume that thermalization and conversion to radiation of the kinetic energy occurs, viz.
Fig. 2. The exact numerical solution $x(q)$ to Eq. (4) is plotted.
III. BASIC ATOMIC PHYSICS ENERGETICS

If the K-shell of a given element is to be excited, an expended minimum of energy which must be imparted to each imploding ion is that needed to strip all but one or two electrons from the ion plus the energy needed to maintain each stripped electron at an appropriate temperature conducive to K-shell dominance. Of course inefficiencies and losses—particularly radiative—imply that in general much more energy than this is needed. However, much insight into basic Z-scaling may be obtained by consideration of these basic quantities.

The temperature at which copious K-shell radiation has been observed in Z-pinches is given approximately by

\[ T = 1300 \left( \frac{Z}{18} \right)^2 \text{eV} . \]  

\( (9) \)

We approximate the number of stripped electrons as equalling Z. Therefore, to maintain the stripped electrons from an ion of atomic number Z at this temperature requires,

\[ E_m = 4 Z^3 \text{eV/ion}. \]  

The energy needed to strip the electrons from each ion is obviously also important and non-negligible. We shall assume that the plasma is 50% hydrogen-like and 50% helium-like. The ionization potentials for each element are available from many references; we have used the data of Carlson et al.\(^6\) The energy needed to strip an ion of atomic number Z to a mean charge state of Z-1.5 is given by the following fit—which for the approximate treatment of this report has negligible error.

\[ E_s = 3.83 Z^{2.64} \text{eV/ion} \]  

\( (12) \)
Obviously, the expended minimum of energy per ion required is given by $E_m + E_s$. We now inject another empirical observation. A critical question is, in practice, how much energy per ion is required for copious K-shell radiation in Z-pinches? BLACKJACK 5 shot 177, an aluminum wire array implosion which produced ~20 kJ of K-line radiation was one of the best diagnosed shots in the BLACKJACK 5 series. From the current trace, spectroscopic diagnosis, and known characteristics of the load, it was determined that $3.6 \times 10^5$ eV was needed, on average, to produce a K-shell ion in this shot. This number is 30 times $E_m + E_s$. We now make the assumption that for all Z, 30 times $(E_m + E_s)$ will be required for efficient K-shell production. Such a large factor is plausible when one considers the numerous sinks for generator energy other than production of K-shell radiation. Such processes include joule heating early in the implosion, radiative losses other than K-shell, thermal conduction losses, and inductive losses. This is not unreasonable as a first approximation since it allows radiative and other losses to scale upward with Z in the same fashion as the kinetic and ionization energies, which is intuitively reasonable. This factor of 30 can be viewed optimistically, since it indicates much room for improvements in efficiency of these devices. This assumption means that

$$(KE)_{ion} = 30 \left(4Z^3 + 3.83 Z^{2.64}\right) \approx 187 Z^{2.95} \text{ eV/ion} \quad (13)$$

where, in Eq. (13) the additional simplification of the power-law sum to a single power law does not result in substantial error for present purposes. Perhaps the most important single step in our development now follows, namely equating Eq. (13), the required kinetic energy per ion, to Eq. (9) the actually imparted kinetic energy per ion

$$\frac{2.5 \times 10^{-15} \, I_p^2 \, Z^{1.15}}{m} = 187 \, Z^{2.95} \quad (14)$$

which yields

$$m = 1.34 \times 10^{-17} \, I_p^2 \, Z^{-1.8} \, \text{kg/m} \quad (15)$$
for the mass per unit length which can be stripped to the K-shell for an
element of atomic number Z with a peak machine current $I_p$ amps.

An ion density may be obtained from Eq. (15) if the radius of
implosion is known. No systematic variation of the pinch diameter with Z
has been seen; we adopt 1.34 mm as an approximate average which was
observed on successful shots using pinhole imagery, and assume future
machines will produce similar pinches. Given the total mass, the mass per
ion ($2.19 \times 10^{-27} \, Z^{1.15}$ kg), and the cylinder radius the K-shell ion density
is obtained easily as

$$N_I = 4.33 \times 10^9 \frac{I_p^2}{Z} \, \text{cm}^{-3}.$$  \hspace{1cm} (16)

From extensive spectroscopic density and temperature diagnostics, we
have determined that the yield on previous shots obeys the following
function of temperature $T$(eV) and ion density $N_I$(cm$^{-3}$).

$$Y(kJ) \approx 142\left(\frac{Z}{18}\right)^2 \left[\frac{N_I}{3.7 \times 10^{19} Z^{2}}\right]^{1.9} \exp \left[-2 \times 10^3 \left(\frac{Z}{18}\right)^2 /T\right] \min \left[1.0, 2 \times 10^3 \left(\frac{Z}{18}\right)^2 /T\right].$$  \hspace{1cm} (17)

We emphasize that Eq. (17) is not based on any hydrodynamics; only on
spectroscopic measurements of $N_I$ and $T$ coupled with knowledge of the yield
on the diagnosed shots. The 1.9 power law of $N_I$ (instead of the usual 2.0)
represents an approximate correction for opacity effects. Letting $T = 1300$
$(Z/18)^2$ eV (Eq. 10), Eq. (17) is greatly simplified to

$$Y = 3.66 \times 10^{-34} N_I^{1.9} Z^{-1.8} \, \text{KJ}.$$  \hspace{1cm} (18)

Substituting for $N_I$ from the zero dimensional hydrodynamics - atomic
energetics model, given by Eq. (16) we obtain

$$Y(kJ) = 7.46 \times 10^{-16} \frac{I_p^2}{Z} \, \text{KJ}.$$  \hspace{1cm} (19)

For BLACKJACK 5 the power in TW ($P$) is given approximately as $3.86 \times 10^{-13} \, I_p^2$, resulting in the following alternative form of Eq. (19).
$Y(KJ) = 2.87 \times 10^8 p^{1.9} z^{-7.405} KJ$. \hspace{1cm} (20)

IV. CONCLUSIONS

We have combined scaling laws for empirically observed yield as a function of spectroscopically diagnosed temperature and density with basic atomic physics energetics and a standard zero-dimensional equation of motion for an imploding Z-pinch load to obtain a prediction for yield for any element as a function of peak current (Eq. 19) or of BLACKJACK 5-type machine power (Eq. 20). The predictions of Eq. 20 are listed in the accompanying table, along with the actual BLACKJACK 5 optimized yields. Clearly this model does a very good job of describing previous results. When it is applied to more powerful machines contemplated for future construction several important implications are seen. First, a 25-TW generator would render a conventional argon load a very prolific PRS, with K-shell yields probably exceeding 50 KJ. Somewhat larger powers in the 40-50 TW range would be needed for similar success with titanium. Finally, conventional loads consisting of iron or krypton will require, for substantial K-shell line radiation, superclass generators in the 50-100 TW range.

Further review of the table shows that by doubling the generator power the K-shell radiative yield quadruples. This suggests that more current would couple into the load to heat the plasma and that the plasma becomes hotter, i.e., the excitation temperature associated with the K-shell emitting region is higher. The question of whether more current actually will couple to the load is somewhat speculative in the sense that there are no direct measurements of current in the vicinity of the load but only measurements of the return current. Also evident from the table and Eq. (20) is that there is a tremendous price to pay in terms of energy expenditure in stripping moderate and high Z-elements down to and maintaining the K-shell plasma. The sequence of events leading to K-shell emission involves both heating and compressing the plasma. Nature is not kind to these schemes. It might be more efficient to devise schemes and design loads that first compress and then heat the plasma.
It is certainly possible that advanced concepts for improving both machine technology and the pinch itself through better understanding of the influence of current pulse duration and crucial plasma instabilities may favorably alter these predictions. For instance, a number of novel experiments involving pulse shaping through such devices as the plasma erosion opening switch are being investigated to determine whether it is possible to enhance K-shell yields in conventional PRS loads. Unless a greater bulk of the plasma can be heated and/or further compressed through pulse shaping it is difficult to conceive how the energy expenditure for stripping and maintaining a K-shell plasma can be reduced from the efficiency factors discussed here.

TABLE: Prediction of Eq. (20) for K-shell yield in kilojoules as a function of atomic number Z and generator power in TW. A BLACKJACK 5-type machine with a load length of 3 cm is assumed. The asterisk indicates where the prediction is probably invalid due to burnthrough of the K-shell. The numbers in parentheses are Maxwell Laboratories' optimized results, courtesy of M. Gersten.

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<th>Z</th>
<th>Element</th>
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<td>11.6</td>
<td>65.9</td>
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<td>5.3</td>
<td>30.2</td>
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<td>*</td>
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<tr>
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<td>Titanium</td>
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<td>2.6</td>
<td>14.9</td>
<td>55.6</td>
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<tr>
<td>26</td>
<td>Iron</td>
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<td>0.75</td>
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<td>5.6</td>
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<td>Krypton</td>
<td>0.02</td>
<td>0.07</td>
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<td>5.4</td>
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ACKNOWLEDGMENTS

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