Research was performed during this period in four separate areas: (1) Deformation of Solids and Stochastic Flows; (2) Self-Exciting Point Processes; (3) Stability of Dependent Random Variables; and (4) Brownian Motion on Manifolds. This report summarizes progress in these areas.
PROGRESS REPORT

on

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MARKOV PROCESSES

APPLIED TO CONTROL, REPLACEMENT, AND SIGNAL ANALYSIS

for the period

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A. DEFORMATION OF SOLIDS AND STOCHASTIC FLOWS

The aim of this work is to build stochastic models for the nucleation and growth of microcracks in solids. This is intimately related to reliability of materials, since crack nucleation and growth is the major mechanism leading to failure of machine parts subjected to vibratory environments and/or to high accelerations and decelerations.

We are concentrating at the early stages of growth, where the cracks are still of atomic dimensions. This is the difficult part of the problem from the point of view of continuum mechanics, because the methods of continuum mechanics apply well at the later stages, when one crack dominates the others in size and can be regarded as a hole.

We are building two different models for the early stages. The first uses Poisson configuration for nucleation and independent Markov processes for the size changes of cracks, one process per crack. This is theoretically simple and is expected to be a good model for very early stages, where the cracks are small and are widely separated from each other.

As the cracks grow, they alter the stress distribution in larger and larger regions around them. As a result, in computing stress at a point, one needs to take into account influences from more and more cracks.
To handle such "crack interaction", we are building a stochastic flow model. Let $X_{s,t}(p)$ be the position at time $t$ of the material element that was at point $p$ at time $s$. The flow equation expresses the connection between various $X_{s,t}$:

$$X_{t,u}(X_{s,t}(p)) = X_{s,u}(p) \quad s < t < u.$$ 

Mechanical considerations require that both $p + X_{0,t}(p)$ and $t + X_{0,t}(p)$ be continuous. The stochastic process of interest is $\{X_{s,t}: 0 \leq s < t \leq u\}$, which takes values in the space of continuous functions $p \mapsto x(p)$.

In previous literature, there are two main sources: the work of J.M. Bismut on flows of diffeomorphisms handled via stochastic integral equations, and the work of T.E. Harris on Brownian flows. Most of these do not serve our purposes: their models yield time paths $t + X_{0,t}(p)$ that are too turbulent for a solid and spatial pictures $p + X_{0,t}(p)$ that are too homogeneous. In particular, they allow no holes or cracks (that is, in their models, $\{X_{0,t}(p): p \in \mathbb{R}^3\} = \mathbb{R}^3$ again).

Our method is similar to Bismut in spirit, that is, via stochastic integrals. We introduce dependence by making $X_{t,t+dt}$ depend on the configuration $\{X_{0,t}(p): p \in \mathbb{R}^3\}$ seen at time $t$.

Physically, this comes to realizing that the flow of material within body is in response to inhomogeneity of the stress distribution, which is in turn caused by the configuration of cracks and their sizes.
B. SELF-EXCITING POINT PROCESSES

This problem arose from the stochastic flow problem. We describe here the simplest version. Consider shocks occurring over time. Suppose that the "intensity" of shocks during \((t, t+dt)\) is a function of the number and times of the shocks that occurred until time \(t\). Such a process is called self-exciting. We are interested in a complicated version of this where shocks occur over time and at different random locations, and "intensity" is now a random function over space and depends on the locations of the shocks that occurred up to time \(t\).

We are working on ways of representing such self-exciting point processes in terms of Poisson random measures.

C. STABILITY OF DEPENDENT RANDOM VARIABLES

Much of this work is being done by N. BOUZAR, who is being supported by the Grant as a post-doctoral fellow. The present work is a generalization of the Borel-Cantelli lemma to dependent random variables. Starting from some dependent random variables, \(X_1, X_2, \ldots\), let \(S_n = X_1 + \ldots + X_n\) and \(U_n = E(X_1 \mid F_0) + \ldots + E(X_n \mid F_{n-1})\), the latter being a predictor of the former. The question is the asymptotic behavior of the discrepancy \(S_n - U_n\), whether \(S_n - U_n\) is small compared with \(f(U_n)\) for some function \(f\). A note on this is to appear in Advances in Mathematics.
D. BROWNIAN MOTION ON MANIFOLDS

This work is carried by Professor M. PINSKY, who was supported as a faculty associate. It concerns the exit time of Brownian motion from a tubular neighborhood of a submanifold. In particular the following questions are of interest:

(a) To what extent is the mean exit time independent of the imbedding of the submanifold?
(b) To what extent can we recover the intrinsic geometry of the submanifold from the mean exit time?

The interest in (a) is the analogy with the Weyl tube formula, which shows that the volume of a tube in Euclidean space is independent of the imbedding. By contrast we have found that the mean exit time may depend on the imbedding, even when we integrate over the submanifold.

The main tool in the proofs is the "perpendicular laplacian", an ordinary differential operator whose analysis provides the first two terms in the asymptotic expansion of the mean exit time. By considering the special case of plane curves, we show that the higher terms of the mean exit time are not given by the perpendicular laplacian, in general.

In the case of hypersurfaces the method requires no further conditions for its validity. for submanifolds of higher co-dimension it is required that the volume factor be independent of the normal direction. This is satisfied in particular for Kahler
submanifolds of a complex manifold with constant homomorphic sectional curvature.

In case the volume factor is independent of both the base point and the normal direction we obtain an explicit integral formula for the mean exit time. This yields explicit results in the case of hypersurfaces with constant principal curvatures and certain Kahler hypersurfaces, where the integrals can further be expressed in terms of Chern forms.

E. PRESENTATIONS

During June and July 1983, the principal investigator gave three invited addresses based on his work under this grant. The titles of the talks and the meetings involved were as follows.


- Stochastic Flows in Fracture Mechanics. Keynote address: Stochastic Models in Engineering and Biology, organized by