This report discusses the development of an approach to automate major aspects of the personnel assignment process and to integrate the personnel assignment and allocation processes. Success in implementing this integrated system would not only make the assignment process efficient but would also make distribution policy evaluation and analysis feasible.
NETWORK FORMULATION OF MULTIPLE-CRITERION PROBLEMS FOR DEVELOPING AN INTEGRATED PERSONNEL DISTRIBUTION SYSTEM IN THE NAVY

NAVY PERSONNEL RESEARCH AND DEVELOPMENT CENTER
San Diego, California 92152
NETWORK FORMULATION OF MULTIPLE-CRITERION PROBLEMS
FOR DEVELOPING AN INTEGRATED PERSONNEL
DISTRIBUTION SYSTEM IN THE NAVY

Timothy T. Liang

Reviewed by
Joe Silverman

Approved by
Martin F. Wiskoff

Released by
J. W. Renard
Captain, U.S. Navy
Commanding Officer

Navy Personnel Research and Development Center
San Diego, CA 92152
FOREWORD

This report was prepared as part of work unit ZF66-512-001.013 (Multiple-criterion Optimization Techniques). It discusses the development of an idea and a methodology to automate major aspects of the personnel assignment process and to integrate the personnel assignment and allocation processes as an interdependent function of the Navy's personnel distribution system. It provides the theoretical underpinning necessary for the development of an operational model.

Advances in this large-scale, multiple-criterion optimization approach can be applied to solve longstanding Navy personnel distribution problems. Success in integrating personnel allocation and assignment processes would not only make the assignment process efficient but would also make distribution policy evaluation and analysis feasible. This could open opportunities for other major improvements in the Navy's personnel distribution system.

Acknowledgments are due to Professors Shao-ju Lee of California State University at Northridge and Glenn W. Graves of the University of California at Los Angeles for their assistance in the technical development.

J. W. RENARD
Captain, U.S. Navy
Commanding Officer

J. R. TWEEDDALE
Technical Director
SUMMARY

Problem

Although military personnel distribution is an important function of personnel management in the Navy, little research has been done in this area. The major functions of personnel distribution are allocation control and assignment control. Allocation is the process of estimating and controlling numerical allocations of personnel to major Navy units; and assignment, of identifying and assigning individuals to specific jobs. Basically, personnel distribution requires policy planning at the aggregate level and policy execution at the individual level.

In the current distribution system, personnel allocation and assignment are two independent and largely manual processes. There are doubts about the capability of the current manual assignment process in determining thousands of possible assignments and selecting the best one among all the alternatives, especially with regard to multiple policy objectives. Because of the complexities of the current personnel distribution system, the quantitative techniques required, and the computer skills involved, relatively little research effort has been devoted to the development of a methodology to quantify the individual information and policy information.

Currently, the Navy allocates and assigns thousands of personnel to job vacancies every week. In mathematical terms, it is a large-scale, multiple-criterion optimization problem. Conventional linear programming techniques are not capable of solving this type of problem efficiently. Although the newly developed network codes may offer a plausible alternative, developing a network formulation to integrate allocation and assignment processes is complicated. A number of technical problems related to simultaneously quantifying allocation goals, assignment goals, and the assignment procedure would have to be overcome. Research is needed to derive a methodology to solve these technical problems.

Objective

The objective of this research was to develop an adequate theoretical basis for an integrated allocation and assignment system. Such a system could be used for both planning and operation in the personnel distribution area and could be applied to personnel distribution for nonrated enlisted personnel, rated enlisted personnel, or officers.

Approach

A theoretical network transshipment model was formulated by using the preemptive multiple-criterion optimization technique and large-scale network codes. No measures of relative weights for the multiple objectives are needed. The assignment process related to matching individual qualifications to job requirements was modeled as a modified assignment network, and the aggregate goals related to allocation policies were formulated into a general transshipment network. Special emphasis was directed toward developing a capability to link the flexible allocation goals and the proportional allocation goals to the assignment process.

Results and Conclusions

This research identified the importance of integrating allocation and assignment. The approach developed to automate the overall personnel distribution system is general
enough to be used for all types of military personnel skill groups in the Navy. Because it integrates both the individual information and the aggregate information, this approach not only makes the operational system efficient, but also makes policy planning and execution feasible. Using this theoretical approach as a basis, an empirical model could be developed. A successful empirical model would not only provide the Navy with its first automated system to match people to authorized job vacancies, but would also offer a different direction for further research to solve other personnel distribution problems.

The technique of modeling the Navy's personnel allocation and assignment problems into a network formulation is efficient in terms of computer time and computer memory in solving large-scale problems. It is realized that network formulation is a complicated procedure. Continuing research in developing a simplified formulation technique is always desirable. However, any simplified formulation technique should be developed without significantly increasing the resource requirements or complicating current operations.
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INTRODUCTION

Problem

Personnel management in the Navy includes the major functional areas of recruiting, training, retention, and distribution. While much research has been performed in the first three areas, considerably less has been devoted to personnel distribution.

Every month, a large number of military personnel are available for new assignments; that is, they are scheduled to rotate from one job to another and from one region to another. Every rotation action creates a vacancy that needs to be filled by the personnel available for new assignment. In addition to personnel rotation, many personnel leave the Navy, creating additional job vacancies. Also, after new personnel receive training from proper schools, they become assets for assignment. On the average, the Navy makes over 20,000 assignments every month. These assignments are made by the "wholesale" allocation of personnel via quotas to major Navy units distinguished by regions, types of duties, etc., followed by the assignment or "detail" of individuals to jobs within those unit groups. Figure 1 illustrates the current personnel distribution process.

In matching people to jobs, the Navy has to consider not only whether a person is qualified for a job, but also whether alternative assignments would better satisfy the Navy's and the individual's needs. The person/job matching process presents a problem with a very large number of choices. The Navy must first determine all the possible choices and then select the best combination among all the possible alternatives, while considering the relative importance of various policy criteria.

Currently, person/job matching is basically a manual process. Questions have been raised as to (1) its efficiency in terms of time and cost, (2) its current ability to identify all possible choices and select the best one, and (3), perhaps most serious, its ability to execute allocation and assignment policies properly. The Navy has been concerned about deficiencies in the manual process and has devoted continuing efforts to computerize personnel and job information. Although these efforts have resulted in various data
retrieval systems that are used to help make assignment decisions, use of these systems has not changed the fundamental personnel assignment process. The Navy is still matching people to jobs manually on a daily basis. Without increasing automation, it is difficult to make the assignment decision equitable, efficient, and satisfactory to the Navy and its personnel simultaneously, or to make tradeoffs among multiple and conflicting goals.

To remedy many of the deficiencies cited, a number of technical problems, particularly the ability to quantify individual and policy information, must be overcome. There are numerous rules, regulations, and policy goals in the personnel distribution process, some of which conflict with others. Certain goals are changeable with respect to personnel supply and demand. For example, the goals involving allocation of personnel to major Navy units change frequently when the personnel levels change. An optimized allocation and assignment system could be developed that would include distribution policy objectives, rules, and regulations, as well as personnel eligibility criteria. With the addition of these multiple goals to an already large assignment system, the problem becomes enormous. The capability and cost of solving this multiple-criterion optimization problem is one of the major concerns of this research. The conventional linear-programming approach is not efficient for solving this type of problem. Rather, an acceptable allocation and assignment system should include the capability of solving a large-scale, multiple-criterion optimization problem at reasonable time and cost.

Background

Assignment Models

In 1968, the Navy's Bureau of Personnel (BUPERS) encouraged research to increase use of the computer to improve personnel assignment in the Navy, particularly to reduce the workload of detailers in making assignment decisions. In the early 1970s, Malone, Thorpe, Tate, and Pehl (1974) developed a prototype computer-assisted distribution and assignment system (CADA), using a composite, multiple-objective linear programming approach. Although CADA was tested on the BUPERS computer, it was never implemented. Butterworth, Gibfried, and Marshall (1975) evaluated CADA and pointed out several factors that contributed to its demise. The most detrimental factor is that it requires a large amount of computer memory and time to run CADA for a small sample. In 1977, Glover, Karney, and Klingman developed a prototype model to improve the mathematical algorithm for the assignment problem. Their approach requires significantly less computer resources, but it was never expanded to become an operational model.

The Air Force currently has an automated assignment system, basically a sort-match process, which was developed a decade ago. Although a transportation model developed by Beatty (1978) was incorporated into this system, it does not appear that the multiple-criterion aspect of assignment has been addressed sufficiently.

Hatch, Nauta, and Pierce (1973) developed an enlisted distribution model for the Marine Corps that, like CADA, employed a linear-programming approach with a composite multiple-objective function. The problem of using this type of approach is the difficulty in determining the relative weights for the criteria. In most cases, determining the weights is subjective. Naturally, results from the model differ when different sets of weights are selected. This type of approach is discussed below.
Multiple-goal Approaches

Goal programming, first defined by Charnes and Cooper (1977), is a variation of linear programming that allows for consideration of multiple and conflicting goals. In handling multiple goals, the decision maker must specify an ordinal ranking of goals. Thus, decision makers are forced to give careful consideration to the relative importance or priority of their goals. They must use postoptimal analysis to assess the effect of changing the priorities of the multiple goals.

There are two ways of dealing with multiple objectives—the weighting method and the preemptive method. Using the weighting method, decision makers would first have to identify the tradeoff relationships among the objectives. Goal programming allows the different deviational penalties to be aggregated into a composite objective function. The coefficients of the goal deviation variables are the penalties or weights in the objective function. Although this type of approach is advantageous because of its capability of demonstrating policy tradeoffs, it is very difficult to find proper and meaningful tradeoff relationships among the various goals. When controversy exists regarding the relative importance of goal attainment, the validity of a solution is in doubt. In an attempt to improve the accuracy of the weights for the objectives, various researchers (Geoffrion, Dyer, & Feinberg, 1972; Steuer, 1977; Zionts & Wallenius, 1983) developed algorithms to generate various sets of weights for the multiple-objective function. Decision makers can judge these weights subjectively in terms of attainment of the various goals and choose the one that appears to be most "reasonable." However, the process requires repeated analysis of the weights attributed to various alternatives. For a large operational problem like the Navy's assignment system, the detailers cannot be expected to have the time or capability to analyze different weighting schemes daily.

The preemptive method is a sequential elimination method that requires ranking of the objectives in terms of their relative importance. Optimization begins by considering the highest priority objective. Successive optimizations are obtained by selecting a solution from alternative solutions. As more preemptive criteria are considered, less alternate solutions remain. The sequential elimination process stops when no alternative solutions can be selected to better satisfy goals. The major advantage of the preemptive method is that different objectives are satisfied sequentially in the order of the relative importance of the objectives. There is no need to estimate the numerical weights and combine the incomparable goals into one weighted index. The disadvantage of this preemptive method lies in ranking the objectives, and the assumption that an ordinal ranking of goals is sufficient to describe the relationship among goals. Different rankings lead to different optimal solutions. One way to reduce the rigidity of the ordering is to experiment with various plausible rankings or to classify the objectives into compatible groups and to rank the groups in order. Objectives within the group may be denoted by different weights.

Network Models

The network flow algorithm was first developed decades ago. In the mid 1970s, there was a breakthrough in the computational capability of network optimization techniques. Bradley, Brown, and Graves (1977), and Klingman and Russel (1975) made significant contributions to the development of the new network codes, enabling the network method to solve large-scale integer problems with a reasonable amount of computer memory and computer time. However, network models lack flexibility in handling variables and equations. They do not accept nonnetwork types of constraints or additional objectives functions directly. Any assignment problem with additional nonnetwork types of constraints or multiple-objective functions might have to be converted into a standard
transportation or transshipment problem. The conversion is not a straightforward procedure and may require sophisticated formulations. Currently, Graves is developing an advanced algorithm and codes for general mathematical programs capable of solving large-scale integer problems without going through the complicated network formulation procedure.

**Allocation Procedures**

The personnel allocation function of personnel distribution management has become increasingly important. The major task of personnel allocation is to estimate the number of personnel to allocate to major Navy units to meet various policies. These allocation quotas are then used as goals for the assignment process. Personnel assignment offices assign individuals based on those allocations goals, in addition to other assignment goals and procedures. At the present time, allocation quotas serve as one of the most important criteria guiding personnel assignment. However, due to a lack of feedback between these two processes, personnel allocation and personnel assignment are two independent operations.

Recently, Blanco, Liang, Habel, and Ritter (1984) developed an automated allocation process that enables allocation offices to use the daily manning levels (i.e., the percent of positions filled) and minimum and maximum manning requirements to estimate personnel allocation quotas directly from available personnel. Figure 2, a simplified version of the Blanco et al. process, shows an example with only two allocation levels—duty type and region. Duty type is broken down into sea duty and shore duty, and region is broken into Pacific, Atlantic, and Continental United States (CONUS), for a total of six Navy unit groups. The total available personnel are allocated (1) between the duty groups and (2) from a given duty group to the various regions. For each step, there might be a set of maximum and minimum manning requirements controlling allocations. It involves a technique in handling flexible goals and proportional goals.

![Figure 2. An example of the allocation procedure.](image)

Figures 3 and 4 show the allocation procedure for the two types of duties and three regions. In describing this procedure, let:

- \( F_1 \) and \( F_2 \) represent duty types 1 and 2 respectively,

- \( n_1 \) and \( n_2 \) represent manning for \( F_1 \) and \( F_2 \) respectively, including all previous allocations,

- \( k_1 \) and \( k_2 \) represent manning for \( F_1 \) and \( F_2 \), including all previous and current allocations.
\( s_1 \) represent minimum manning for \( F_1 \),
\( g_2 \) represent maximum manning for \( F_2 \),
\( R_1, R_2, \) and \( R_3 \) represent regions 1, 2, and 3 respectively, and
\( m_1, m_2, \) and \( m_3 \) represent manning for \( R_1, R_2, \) and \( R_3 \) respectively.

Figure 3. Allocation procedure for duty type balance.

Figure 4. Allocation procedure for regional balance.
Then, starting from duty allocation, corresponding to Figure 3, each additional available person should be allocated according to the following criteria:

1. Allocate personnel to $F_1$ to satisfy $s_1$, if $s_1 > n_1$.
2. Allocate personnel to $F_1$ to raise $n_1$ to $n_2$ if $k_1 < n_2$.
3. Allocate personnel to $F_2$ to raise $n_2$ to $n_1$ if $n_2 < k_1$.
4. Allocate personnel to $F_1$ if $n_1 < n_2$ and $k_1 < g_2$.
5. Allocate personnel to $F_2$ if $n_2 < n_1$ and $k_2 < g_2$.
6. Allocate personnel to $F_1$, if $k_1 > g_2$.

The available personnel can be iteratively allocated to the two duty types one by one until all personnel is allocated.

After the personnel quotas for each duty are determined, the next step is to distribute those quotas into regions, as shown in Figure 4. To keep the three regions' manning levels proportionally equal, Blanco et al. proposed that each additional person be allocated to the region that has lower manning than other regions. For example, if $m_2 < m_1 < m_3$, the first available person should be allocated to $R_2$, which would increase $m_2$. If the condition of $m_2 < m_1 < m_3$ remains unchanged, the second available person should also be allocated to $R_2$. However, if the relationship becomes $n_1 < n_2 < n_3$ after the second person is allocated, the third available person should be allocated to $R_1$. In general, the allocation procedure for regional balance is to allocate personnel one by one to the region with the lowest manning percentage.

This allocation procedure is an heuristic process using only aggregate information. No detailed information about individuals or jobs is involved. Without a quantitative linkage between the aggregate information related to allocation and the individual information related to assignment, it is difficult for decision makers to assess (1) the impact of conflicting objectives, (2) whether the goals have been met, or (3) how allocation policies could be modified to better satisfy both the Navy's needs and the individual's needs. The personnel allocation and assignment systems must be integrated to develop such linkages. Decision makers could use this integrated system as a tool for policy planning and evaluation as well as execution. The development of an optimization technique for this large-scale, multiple-criterion problem is required.

Objective

The objective of this research is to develop a modeling technique that can be used to support an integrated personnel allocation and assignment system. This modeling technique should be capable of:

1. Automating the distribution process, including the nomination of people to jobs in an operational environment.
2. Quantifying multiple, conflicting, and changeable goals.
3. Integrating policy planning and assignment execution.
4. Solving large problems with integer solutions.

5. Being used without estimating the numerical weights for the multiple objectives.

6. Being applied to nonrated enlisted personnel, rated enlisted personnel, or officers.

**APPROACH**

The general approach was to develop transshipment flows for the allocation goals and to integrate allocation and assignment into one network model.

**General Form of a Single-criterion Assignment Model**

In operations research, the assignment model is a special case of the transportation model. The typical example is the assignment of n persons to n different jobs in such a way as to minimize or maximize some objective function. The mathematical statement of the standard form of the assignment model is

\[
\text{minimize (maximize)} \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}x_{ij} \\
\text{subject to} \quad \sum_{j=1}^{n} a_{ij}x_{ij} = 1, \quad i = 1, 2, \ldots, n, \\
\sum_{i=1}^{n} a_{ij}x_{ij} = 1, \quad j = 1, 2, \ldots, n, \\
x_{ij} = 0 \text{ or } 1
\]  

where

- \(x_{ij}\) represents the assignment of the \(i\)th person to the \(j\)th job,
- \(c_{ij}\) represents the coefficients of benefits or "cost" of assigning the \(i\)th person to the \(j\)th job, and
- \(a_{ij}\) represents the eligibility of the \(i\)th person for the \(j\)th job.

The coefficient \(a_{ij}\) in the constraints is given a value of 0 if person \(i\) is not eligible for job \(j\) or a value of 1 if \(i\) is eligible for job \(j\). The first \(n\) constraints assure that each person is assigned to one job only; and the next \(n\) constraints, that each job is assigned to
one and only one person. The decision variables are required to have an integer solution of either 0 or 1.

For each assignment period, the number of available Navy personnel may not equal the number of available jobs. In addition, a person might not be eligible for any job and a job might not be suitable for any person. The general assignment model for n persons and n jobs must be modified. If it is assumed that there are m persons and n jobs, a set of m+1 variables would be created to represent "unspecified" persons and a set of n+1 jobs, to represent "unspecified" jobs. This implies an increase of m+n+1 variables. The model will then be able to include both assigned and unassigned personnel and jobs. It is to

minimize (maximize)

\[ z = \sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij} x_{ij} \]  

subject to

\[ \sum_{j=1}^{n+1} a_{ij} x_{ij} = 1, \quad i=1,2,...,m, \]
\[ \sum_{i=1}^{m+1} a_{ij} x_{ij} = 1, \quad j=1,2,...,n, \]
\[ x_{ij} = 0 \text{ or } 1. \]

Figure 5 shows the eligibility matrix for a_{ij}. a_{ij}=1 indicates that the ith person is eligible for the jth job and a_{ij}=0, that the ith person is not eligible for the jth job. Assume that P represents people and V represents jobs. The ith person may or may not be eligible for the jobs from V_1 to V_n. He or she is always eligible to be assigned to the unspecified job V_{n+1}. This implies that a_{i(n+1)} for all is always equal to 1. Although a person may be eligible for many jobs, including the unspecified job V_{n+1}, he or she can be assigned to only one job. From another viewpoint, a job j may or may not be suitable for persons from P_1 to P_m, but it is always suitable for the unspecified person P_{m+1}. For example, a_{ij}=1 indicates job j is suitable for person i and a_{ij}=0 indicates job j is not suitable for the ith person. a_{(m+1)j}=1 for all j indicates that job j is always suitable for the unspecified person, which means that the job can always be left vacant. Although job j may be suitable for many people, including the unspecified person, it can actually be filled by only one person. The best choice of assigning persons to jobs depends upon the values of the coefficients for the decision variables in the objective function.
General Form of the Preemptive Multiple-criterion Optimization

The basic concept of the preemptive approach to the multiple-criterion problem is to optimize the multiple-objective functions one after another. The optimized objective function for the most important criterion is used as a constraint in the problem, with the criterion next in importance. Multiple policy criteria related to personnel distribution often conflict with one another. For example, one criterion might specify that personnel be reassigned in the same area to save moving costs, while a second criterion might encourage the assignment of personnel according to their preferences to maintain morale and, by implication, personnel retention. To accommodate these types of conflicting objectives, the preemptive multiple-criterion optimization approach is considered to be superior to other weighting approaches. Users of the model do not have to estimate the weights for each criterion. They only need to prioritize the relative importance of the criteria. The results from a low priority criterion should never violate the higher priority criteria.

Assume that there are two criteria involved in the assignment problem. The mathematical expression of the first-stage criterion optimization is

minimize (maximize)

\[
\begin{align*}
z^1 & = \sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij} x_{ij}
\end{align*}
\]  

(3)
subject to

\[ \sum_{j=1}^{n+1} a_{ij} x_{ij} = 1, \quad i = 1, 2, \ldots, m, \]

\[ \sum_{i=1}^{m+1} a_{ij} x_{ij} = 1, \quad j = 1, 2, \ldots, n, \]

\[ x_{ij} = 0 \text{ or } 1. \]

The superscript 1 in the objective function represents the 1st criterion. Let \( z_1^1 \) be the optimal value of \( z_1 \). Then the optimization with the second criterion is to minimize (maximize)

\[ z^2 = \sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij} x_{ij} \]

subject to

\[ \sum_{j=1}^{n+1} a_{ij} x_{ij} = 1, \quad i = 1, 2, \ldots, m, \]

\[ \sum_{i=1}^{m+1} a_{ij} x_{ij} = 1, \quad j = 1, 2, \ldots, n, \]

\[ \sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij} x_{ij} = z_1^1 \]

\[ x_{ij} = 0 \text{ or } 1. \]
The objective function $z^1$, with its optimal value $z^*\_1$, which is used as a constraint here, indicates that the second-stage optimization should never violate the first-stage optimization. However, the optimal variables $x^*\_\text{ij}$ do not have to remain as optimal variables in the second-criterion optimization. Any alternate solution that could keep $z^1$ unchanged and still improve $z^2$ could be introduced into the optimal basis to replace the original optimal variables. This procedure implies that more alternate solutions in the first-stage optimization could provide a chance for better solutions in the second-stage optimization. Because of this consideration, attention should always be paid to the inclusion and creation of the maximum number of alternate solutions for each successive optimization. A relaxation of some rules in one stage of optimization might substantially increase the number of alternate solutions in the following stages of optimization without jeopardizing the quality of solutions.

In general, the preemptive multiple-criterion assignment model for kth stage optimization can be stated as

\[
\begin{align*}
\text{minimize (maximize)} \\
\sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij}^k x_{ij} \\
\text{subject to} \\
\sum_{j=1}^{n+1} a_{ij}^1 x_{ij} = 1, \quad i=1,2,...,m, \\
\sum_{i=1}^{m+1} a_{ij}^k x_{ij} = 1, \quad j=1,2,...,n, \\
\sum_{i=1}^{m+1} \sum_{j=1}^{n+1} c_{ij}^{k-1} x_{ij} = z^*_{k-1}, \quad k > 1, \\
x_{ij} = 0 \text{ or } 1.
\end{align*}
\]

Network Flows for a Multiple-criterion Assignment Problem

It has been shown that the multiple-objective assignment problem is a large 0-1 integer mathematical programming problem. However, due to the capability and the efficiency of the new network codes to solve large-scale problems, it is plausible to cast the multiple-objective assignment problem into a network formulation. Figure 6 shows a
capacitated transportation form of the network model for the multiple-objective assignment problem. Let the set of P nodes \( P_1, P_2, \ldots, P_m \) represent the persons to be assigned and the set of V nodes \( V_1, V_2, \ldots, V_n \), the jobs to be filled. \( P_{m+1} \) denotes the unspecified persons and \( V_{n+1} \), the unspecified jobs. Proceeding from left to right in Figure 6, the arcs between nodes show the eligibilities of persons to jobs. Specifically, the arc between \( P_i \) and \( V_j \) indicates that the \( i \)th person is eligible for the \( j \)th job. If there is no arc connecting \( P_i \) and \( V_j \), it indicates that the \( i \)th person is not eligible for the \( j \)th job. At the bottom of the diagram, \( V_{n+1} \) is used to show the case of nonassignment. Every person is "eligible" for nonassignment to any job. Therefore, there is always an arc connecting each node \( P_i \) to \( V_{n+1} \).

![Figure 6. Network flows for the assignment problem.](image-url)
Take $P_1$ as an example to illustrate the relationships between $P_1$ and $V_j$. In Figure 6, the arc between $P_1$ and $V_1$ indicates that person $P_1$ is eligible for job $V_1$. Similarly, it can be seen that $P_1$ is also eligible for job $V_4$ but not for job $V_2$. In the diagram, $V_{n+1}$ represents the unspecified job. Person $P_1$ is always "eligible" for nonassignment to any of the jobs from $V_1$ to $V_n$ and thus is eligible for $V_{n+1}$.

The first, second, and third numbers in the parentheses represent the lower bound, the upper bound, and the coefficient of the decision variable $x_{ij}$.

1. **Lower bound.** For the assignment problem, the minimum number of $P_i$ that can be assigned to $V_j$ is zero.

2. **Upper bound.** Since each $P_i$ node represents a specific person, the maximum number of $P_i$ that can be assigned to $V_j$ is always 1. The requirement for an integer solution under the condition of $0 < x_{ij} < 1$ leaves only two alternative values for $x_{ij}$ — 0 or 1. There are a number of arcs from each $P$ node to various $V$ nodes. However, the final solution of the model must contain the selection of only one arc from one $P$ node to one $V$ node.

3. **Coefficient.** The coefficient of $x_{ij}$ for criterion $k$ optimization represents the benefit or cost of assigning person $i$ to job $j$. This value affects the selection of a "best" arc between $P_i$ and $V_j$ among all possible alternatives. For unspecified personnel, all outgoing arcs from $P_{m+1}$ have a lower bound of 0 and an upper bound of 1. A positive integer $M$ is given as the coefficient in a minimization problem or a negative integer $M$ is given as the coefficient in a maximization problem, indicating that it is undesirable to leave the job vacant.

Similarly, for the unspecified jobs, all the incoming arcs to $V_{n+1}$ have a lower bound of 0, an upper bound of 1, and a coefficient of $M$.

The network formulation for the assignment problem can be developed by using the nodes $P$ and $V$ and the arcs between $P$ and $V$. By redefining $x_{ij}$ as the arc variable between nodes $P_i$ and $V_j$ the problem may be stated as follows:

\[
\text{minimize (maximize)} \quad z = \sum_{i,j} c_{ij}x_{ij}
\]

subject to

\[
\sum_{i} x_{ij} = 1, \quad j=1,2,...,n
\]

\[
\sum_{j} x_{ij} = 1, \quad i=1,2,...,n
\]
To include multiple conflicting assignment goals in the optimization process, the network formulation can be modified as

$$\sum_{j} x_{ij} = 1, \quad i=1,2,...,m$$

$$0 \leq x_{ij} \leq 1$$

minimize (maximize)

$$z^k = \sum_{i,j} c_{ij}^k x_{ij}$$  \hspace{1cm} (7)

subject to

$$\sum_{i} x_{ij} = 1, \quad j=1,2,...,n$$

$$\sum_{j} x_{ij} = 1, \quad j=1,2,...,m$$

$$\sum_{i,j} c_{ij}^{k-1} x_{ij} = z_{k-1}, \quad k > 1$$

$$0 \leq x_{ij} \leq 1$$

**General Form of Capacitated Transshipment Model**

The network formulation described in the previous section is a type of capacitated transportation model for a multiple-criterion assignment problem. P is a set of origin nodes with only outflows, and V is a set of destination nodes with only inflows. However, when the allocation goals are imposed in the assignment process, the capacitated transportation model is not sufficient to be used to quantify the aggregate policies. The assignment model must be expanded to a capacitated transshipment model.

Nodes P and V and the arcs between P and V in Figure 6 were used to describe the capacitated transportation model. By adding two extra nodes, D and S, to represent demand and supply, the capacitated transportation model for the same problem becomes a capacitated transshipment model. S is the origin node, D is the destination node, and P and V become intermediate nodes. The transshipment flows indicate that the total supply of available personnel S can be disaggregated into individual personnel P. The arcs between S and P show the path of disaggregation. On the right-hand side of the diagram, the individual job V can be aggregated into total demand D. For any intermediate node P or V, the total inflow to a node should always be equal to the total outflow from that node.
The mathematical formulation of a general capacitated transshipment model for a multiple-criterion assignment problem can be developed. By expanding the definition of $x_{ij}$ to represent the arc from any node $i$ to any node $j$ and by relaxing the restriction of the lower bound being 0 and the upper bound being 1, the transshipment form of the assignment model becomes

$$\text{minimize (maximize)}$$

$$z^k = \sum_{all(i,j)} c^k_{ij} x_{ij}$$

subject to

$$\sum_{all(h,j)} x_{hj} - \sum_{all(i,h)} x_{ih} = b_h, \quad h=1,2,...,M$$

$$\sum_{all(i,j)} c^k_{ij} x_{ij} = z_{*}^k, \quad k > 1,$$

$$L_{ij} \leq x_{ij} \leq U_{ij}$$

where

$b_h$ represents supply or demand for node $h$; $b_h = 0$ for all transshipment nodes,

$x_{ih}$ represents inflow from node $i$ to node $h$,

$x_{hj}$ represents outflow from node $h$ to node $j$,

$L_{ij}$ represents lower bound for $x_{ij}$,

$U_{ij}$ represents upper bound for $x_{ij}$,

$c_{ij}$ represents coefficient for $x_{ij}$,

$M$ is the number of nodes in the network, and

$N$ is the number of arcs in the network.

Using matrix form, the model can be rewritten as

$$\text{minimize (maximize)}$$

$$z^k = C^k X$$

subject to

$$AX = B$$

$$C^{k-1}X = z_{*}^{k-1}, \quad k > 1,$$

$$L \leq X \leq U.$$
where

- $X$ represents an $N \times 1$ vector of flow variables,
- $C$ represents an $1 \times N$ vector of coefficients for $X$,
- $A$ represents an $M \times N$ node-arc incidence matrix,
- $B$ represents an $M \times 1$ vector of supplies and demands,
- $L$ represents an $N \times 1$ vector of lower bound for $X$, and
- $U$ represents an $N \times 1$ vector of upper bound for $X$.

Transshipment Flows

For Flexible Allocation Goals

In the discussion of the development of a transshipment model for a multiple-criterion assignment problem, no allocation goal was incorporated. Allocation goals are a set of aggregate targets that cannot be handled directly by those arcs and nodes for individual personnel and individual jobs. New nodes and arcs must be developed.

Figures 2, 3, and 4 illustrated an heuristic allocation procedure. This section shows the development of a network optimization approach for those allocation procedures. The heuristic allocation was based on a set of aggregate numbers of personnel and a set of minimum and maximum manning requirements. The results of the approach are used to guide assignment of people to jobs. However, the allocation quotas are estimated independently of any assignment information. Although the total number of personnel is used to estimate allocation quotas, individual personnel are not identified and, thus, individual qualifications or preferences are not considered. Under these circumstances, the allocation goals become idealistic goals. It is unlikely that the allocation goals can be reached without ignoring some of the other goals. A more effective approach is to make personnel allocation an integrated part of the personnel assignment process so that allocation goals can become more attainable.

The allocation goals related to duty type, shown in Figure 3, are sets of flexible or changeable goals. The allocation goals related to regions, shown in Figure 4, are sets of proportional or equalizing goals. This section describes the techniques used to develop transshipment flows for the flexible goals; and the next, transshipment flows for proportional goals.

The heuristic procedure for priority and duty allocation shown in Figure 3 indicates that there are no fixed allocation goals that can be directly minimized or maximized. How a person should be allocated depends upon the manning percentage after the previous person is allocated. The goal under one condition might be to match one manning percentage to a minimum manning percentage, while the goal under another condition might be to match the manning percentage of a duty with that of another duty. The transshipment representation for these flexible goals is displayed in Figure 7, which expands on the nodes and arcs from Figure 6. New intermediate nodes $E$ and $F$ are added between nodes $V$ and $D$. The original nodes $S$, $P$, $V$, and $D$, and the arcs between them remain unchanged; however, the $E$ nodes represent aggregation of personnel by job priorities and duties and the $F$ nodes, personnel allocation among duties. To illustrate how the flow variables and their coefficients are derived, let

- $E_1$ and $F_1$ denote high-priority duty type 1,
- $E_2$ and $F_2$ denote high-priority duty type 2,
- $E_3$ and $F_3$ denote low-priority duty type 1, and
- $E_4$ and $F_4$ denote low-priority duty type 2.
Figure 7. Transshipment flows for allocation among duties.

The arcs between V and E represent paths for job V being aggregated into E categories. There is only one arc from each V to E. For example, if job $V_1$ is a high-priority duty 1 job, an arc should be drawn from $V_1$ to $E_1$. If job $V_5$ is a high-priority duty 2 job, an arc should be drawn from $V_5$ to $E_2$. Every single job V has a single path to only one of the E nodes. Under normal conditions, there is insufficient personnel to fill every single job; thus, scarce personnel resources need to be distributed among jobs. The distribution of personnel among E nodes must be optimized. In a minimization problem, a value of 0 can be given to the coefficients of the variables for the high-priority jobs; and a value of 1, to the coefficients of the variables for the low-priority jobs. During the optimization process, these coefficients will direct personnel distribution to the high-priority jobs first. The remaining personnel will go to the low-priority jobs.
The arcs between V and E and their coefficients are used to direct distribution among various job priorities. No personnel distribution among duties is involved. The optimization procedure for the allocation among duties is described by the arcs between nodes E and F.

The allocation procedure in Figure 3 shows that the personnel are allocated one after another according to the changing manning levels. When this allocation procedure is imposed on the assignment problem, modifications of the network formulation must be made. It was found to be beneficial to allocate personnel among duties based on the increment of 1 percent in the manning levels. Since there is a large number of personnel in the Navy, 1 percent manning represents quite a number of people. The increment of 1 percent instead of one person might affect the accuracy of the estimates; by using percentages, the number of the arcs needed to represent the transshipment flows can be substantially reduced, as well as computer memory and time. If the same values could be logically given to the coefficients for certain variables, it would improve the chance of generating more alternate solutions. For a multiple-criterion problem, more alternate solutions can increase the possibility of producing better final solutions.

In allocating personnel among duties, the use of multiple arcs between each pair of nodes E and F is important. The lower bounds, the upper bounds, and the coefficients associated with those arcs must also be developed. However, since the goals for allocation are flexible with respect to the number of available personnel, the current manning percentages, or the minimum or maximum manning requirements, it is impossible to predetermine a fixed set of arcs. Also, the sequence of arcs among duties could not be fixed. This could cause complications in determining the values for the variables.

To illustrate, use the arcs and nodes for \( E_1, F_1, E_2, \) and \( F_2 \) from Figure 7 as an example and suppose that 900 people are available for assignment. Multiple arcs should be developed from E to F to show personnel flows. The best way to handle this type of flexible goal would be to generate heuristically the maximum number of arcs between E and F. Although the maximum number of arcs for one problem may be different from another, the number of arcs for any specific problem can be predetermined. Also, an equal number of arcs for each duty should be created to handle the uneven flows. Upper bounds are used to control the existence of the arcs. A value of 0 is given to the upper bound of an arc if the arc is not needed. It makes the number of arcs unequal for different duties. In addition, the values of the coefficients of the variables are given values equal to the sequential order of the arcs. Thus, the coefficients of the variables for the same sequence of arcs in different duties are given the same values. Let

- \( h \) represent the duty with a minimum manning percentage requirement and no maximum,
- \( t \) represent the duty with a maximum manning percentage requirement and no minimum,
- \( u_{wh} \) represent the upper bound of the \( h \)th arc for duty \( w \),
- \( u_{th} \) represent the upper bound of the \( h \)th arc for duty \( t \),
- \( c_{wh} \) represent the coefficient of the \( h \)th arc for duty \( w \),
- \( c_{th} \) represent the coefficient of the \( h \)th arc for duty \( t \), and
- \( n \) represent the total number of arcs for each duty.
The detailed procedure for iteratively generating arcs and their upper bounds and coefficients is described as follows:

1. The total number of personnel, the minimum manning requirement, and the current manning are used to compute an estimated number of people required to satisfy the minimum manning requirement. \( u_{w1} \) is the largest integer less than that estimated number and is the upper bound of the first arc for duty with a minimum manning requirement. If \( u_{w1} < 0 \), set \( u_{w1} = 0 \). \( u_{t1} \) is always equal to 0.

2. The second stage is an attempt to generate the second set of arcs to make the manning percentages for both duties equal. If the manning percentage including \( u_{w2} \) for duty \( w \) is less than the manning percentage for duty \( t \), add an integer \( u_{w2} \) to duty \( w \) to approximately match the manning percentage for duty \( t \). \( u_{t2} \) is set equal to 0. If the manning percentage for duty \( w \) is greater than that for duty \( t \), give an integer \( u_{t2} \) to duty \( t \) to raise its manning percentage to match the manning percentage for duty \( w \). In this case, \( u_{w2} \) is set equal to 0.

3. \( u_{w3} \) and \( u_{t3} \) can be generated by estimating the number of people equivalent to 1 percent increase in manning for \( w \) and \( t \) respectively. Repeat this procedure until the addition of 1 percent of manning would make the manning exceed the maximum manning limit for duty \( t \).

4. \( u_{w(n-1)} \) and \( u_{t(n-1)} \) are integer residuals to represent the number of people needed to raise the manning percentage to the maximum manning limit for duty \( t \).

5. The last set of arcs is used to represent everyone else. Remaining personnel are assigned to duty \( w \). \( u_{tn} \) is the upper bound of the last arc for duty \( t \). \( u_{tn} \) is set equal to zero. The total number of the arcs can be heuristically determined. There are precisely \( n \) arcs for each duty.

In a minimization problem, the variable coefficients can be given as \( c_{wh} = c_{th} = h \).

For Proportional Allocation Goals

Another type of allocation criterion relates to personnel allocation among regions, shown in Figure 4. The goal of the allocation is to make the manning percentages for the regions proportionally equal, which leads to an expansion of the network formulation.

Figure 8 is a continuation and extension of Figure 7. Using similar notations and concept, nodes G and R and their arcs can be derived to represent allocation among regions. The basic idea is to create an equal number of arcs for every region. An upper bound of 0 implies that the arc does not actually exist. The coefficients of the arcs for different regions are given a set of identical values corresponding to the sequential order of the arcs. Assuming there are three regions, let

- \( s \) represent the region with the smallest manning percentage,
- \( g \) represent the region with the largest manning percentage, and
- \( f \) represent the region with the manning percentage between the largest and the smallest.
Using notation similar to the previous section, a detailed iterative approach can be developed as follows:

1. The first set of arcs between nodes $G$ and $R$ are drawn for each region. The percentage difference between the manning for regions $f$ and $s$ can be converted into a difference in persons. $u_{s1}$ is the integer upper bound of the first arc for region $s$. Adding $u_{s1}$ persons to region $s$ will raise its manning percentage to match the manning percentage for region $f$. The upper bounds of the first arcs for other regions are $0$ (i.e., $u_{f1} = 0$ and $u_{g1} = 0$). The coefficients of the first set of arcs are given a value corresponding to the order of the arcs (i.e., $c_{s1} = c_{f1} = c_{g1} = 1$).
2. The second step is to raise the manning percentages for regions s and f proportionally by 1 percent. \( u_{s2} \) and \( u_{f2} \) represent the number of people equivalent to 1 percent of their respective mannings. At the same time, let \( u_{g2} = 0 \), and \( c_{s2} = c_{f2} = c_{g2} = 2 \). Repeat this procedure to develop the third set of arcs, the fourth set of arcs, etc., for all three regions until increase of manning percentage for regions s and f by 1 percent would make their manning percentages greater than the manning percentage for region g.

3. This stage is used to generate a set of arcs to accommodate residuals representing the number of people needed to raise the manning percentage for regions s and f to match the manning for region g.

4. The manning percentages for the three regions are raised proportionally by 1 percent each time. This procedure is used repeatedly to generate new arcs until the sum of these upper bounds and all the previous upper bounds exceeds or is equal to the number of available personnel. The variable coefficients are determined by \( c_{sh} = c_{fh} = c_{gh} = h \), disregarding whether any of their upper bounds is 0.

RESULTS

Using the approach developed to determine arcs and their coefficients, upper bounds, and lower bounds, the allocation goals, the assignment goals, and the assignment procedures could be all integrated into one system. Figure 9 shows the network flows for the integrated personnel distribution system.

The multiple-criterion problem is a mixture of minimization and maximization problems. When a network model is formulated, the mixed problem must be converted to a minimization problem. If the problem is converted to a minimization problem, the overall goal is to minimize the total cost of flows. Using equation (9) and its notation as a base, the general network approach of the integrated multiple-criterion model is to

\[
\text{minimize} \quad z^k = C^k X
\]

subject to

\[
AX = B
\]

\[
C^{k-1} X = z^{k-1} \quad k > 1,
\]

\[
L^k \leq X \leq U^k
\]

where \( C^k \) represents coefficients for the variables related to the kth criterion.

In solving the kth stage problem, post-optimal adjustment of its preceding problem is necessary. The optimal basis for the preceding problem is used as the starting solution to the kth stage optimization. With the preemptive approach, at the kth stage, the preceding optimal basis satisfies objective functions 1 through k-1. Sequential elimination of alternative optimal solutions directs the subsequent optimization process to more and more focused solution space. The solution process terminates either when all objective criteria have been satisfied and an optimal solution is obtained or when, at stage k, a unique optimal solution results.
CONCLUSION AND RECOMMENDATIONS

The personnel distribution in the Navy employs a decision-making process using both individual and aggregate information. The model developed in this research provides a theoretical basis to quantify individual and aggregate information simultaneously and can be used to improve the efficiency of the distribution process. It would also aid decision makers in evaluating the impacts of the existing and proposed policies.

Allocation goals are a set of policies for distributing personnel to maintain equitable fleet manning. These are flexible goals that change when other conditions change. This research develops a technique to quantify this type of multiple and changeable criterion.

The overall approach for modeling the personnel distribution system is a network formulation. Job requirements and individual qualifications are considered as constraints to represent personnel eligibility. Assignment goals and allocation goals are considered objective functions. A transshipment model was developed to show how the goal variables and their coefficients can be derived in the network text. The main reasons for developing a network formulation are its capability to solve large-scale integer problems and its efficiency in using computer resources. The capability to solve large-scale
problems efficiently is crucial to a large system like personnel distribution, which involves both policy planning and execution. At present, it is suggested that the theoretical model developed in this research be expanded to an empirical model. If the empirical model is acceptable, it could be prepared for implementation. However, it is realized that the effort needed to develop a network formulation is extraordinary. Whenever a new distribution policy is introduced, model modifications may be necessary and modification of a network model might not be a simple process. Research in developing a simplified formulation should be continued. Currently, researchers are developing efficient codes in the integer programming area that could be used as an alternative to the network approach for the Navy's multiple-criterion personnel distribution problems.
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