INTERIM SCIENTIFIC REPORT ON GRANT AFOSR-82-0305 ENTITLED,
"SEQUENTIAL DECISION MODELS IN RELIABILITY," FOR
THE PERIOD ENDING SEPTEMBER 30, 1983
As originally proposed, we have studied the problem of the order in which to inspect a system to determine its state and the cause of its possible failure. Our results to date are for a series system, and $k$ types of inspections, $k \geq 1$. This generalizes the direct search results of Matula [1964] and Stone [1975] where $k$ is fixed at 1. Our model also allows multiple inspections in the same period.

As with Matula and Stone, our model is described in terms of searching for a lost object. In reliability applications the lost object represents a failed component. There is a known (prior) probability $p_i$ that the object is in location $i$. Searcher $j$, $1 \leq j \leq k$, is available for a total of $m_j$ searches, and the cost of assigning him to location $i$ is $c_{ij}$. If searcher $j$ is assigned to location $i$ and the lost object is in location $i$, the probability that the object will be found is $1 - \beta_{ij}$. In reliability applications $\beta_{ij}$ corresponds to the probability that a malfunctioning component would pass inspection.

This model has been examined using two different objective functions. The first is to maximize the probability that the object is found before a predetermined time expires. In this case it is evident that when the searchers are identical, the optimal search strategy is a direct generalization of the well known myopic search policy for the case of a single searcher. When the searchers are not identical we have established that there is no myopic policy that can be optimal so that a computationally simple marginal allocation approach cannot be guaranteed to yield the optimal solution. Numerical experiments have been carried out to measure the nonoptimality of the marginal allocation approach in this case.

The second objective function is to minimize the total expected searcher-days until the object is found. For this criterion we have established that the optimal search strategy never uses more than one searcher on any given day.
Therefore the problem can be solved by considering only the best searcher for each location and then solving a one-searcher problem over the n locations.

The second part of our current research looked at the question of the level of spare components to stock. These components replace the failed components determined by the methods described above. The planning horizon is finite on an interval \([0, T]\), and the ordering decisions for spares are made at times \(0, 1, \ldots, t_n, \ldots, t_N = T\). Thus the interval \([0, T]\) corresponds to the period in the production phase when spare parts can be procured.

From a general point of view this is a sequential decision model under uncertainty. An interesting and important property of many sequential decision problems under uncertainty was noted by Marshak and Nelson [1962, p. 43], "Although before time \(t_n\) the decision maker is uncertain as to what the world will be like at time \(t_n\), he is less uncertain at times closer to \(t_n\) than he was at times farther away. The decision maker acquires additional information - he learns about future states of the world - as time goes by." In the case of spare parts procurement, the uncertainty is uncertainty about activity levels and especially about failure rates. Clearly uncertainty about activity levels and failure rates at time \(t_n\) decreases as time passes from 0 to \(t_1\) on up to \(t_{n-1}\) so that the observation of Marshak and Nelson applies in a marked way to our problem.

In order to capture this important Marshak-Nelson effect of additional information becoming available after time 0, our research proposal suggested a Bayesian formulation of the spare parts problem. This formulation requires an additional state variable in the dynamic programming equation of optimality. This second state variable makes the problem much more time consuming to solve, especially when many different items (possibly in the low thousands) are involved. We proposed to develop a convenient bound on the improvement that could be obtained using the Bayesian formulation as opposed to a simpler formulation.

Our research to date on the problem has taken a different direction than we
envisioned at the time of the proposal. Rather than a Bayesian formulation, our approach has been to describe the demand for spare parts by an exponential smoothing formula. Exponential smoothing is perhaps the most prevalent method of demand prediction. The exponential smoothing formulation also requires an additional state variable in the equation of optimality, and therefore appears to have the same computational difficulties as the Bayesian formulation. An important result that we have obtained is that the two state variable exponential smoothing dynamic program can be solved by solving a specially constructed one state variable dynamic program. This, of course, results in a great computational saving. We do this by applying the same approach Scarf [1960] used to reduce a Bayesian Inventory model with a gamma demand from two state variables to one.

We used this same exponential smoothing model to examine the idea of Marshak-and Nelson [1962] called decision flexibility. A second result has been to show that the exponential smoothing model always orders less than or equal the amount a comparable model which does not capture the Marshak-Nelson effect of additional information becoming available orders. In terms of flexibility, the exponential smoothing model always takes at least as flexible a decision as the comparable model.

The results described above are available upon request as a paper by Miller (1983) and by Subelman (1984). In addition the paper by Katy Azoury entitled "Bayes Solution to Dynamic Inventory Models Under Unknown Demand Distribution" was supported by this contract. The paper of Scarf (1960) cited earlier shows how a Bayesian inventory model can be reduced from two state variables to one in the case of the gamma demand distribution. Azoury shows how the same simplification can be obtained in the case of the uniform, Weibull, and normal demand distributions.
The investigators have studied the problem of the order in which to inspect a system to determine its state and the cause of its possible failure. Results to date are for a series system, and \( k \) types of inspections, \( k \leq 1 \). This generalizes the direct search results of Matula [1964] and Stone [1975] where \( k \) is fixed at 1. The model also allows multiple inspections in the same period.
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