REDUCING GRATING LOBES DUE TO SUBARRAY AMPLITUDE TAPERING (U) ROME AIR DEVELOPMENT CENTER GRIFFISS AFB NY
R L HAUP'T APR 84 RADIC-TR-84-94
UNCLASSIFIED
REDUCING GRATING LOBES DUE TO SUBARRAY AMPLITUDE TAPERING

Randy L. Haupt, Capt., USAF

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ROME AIR DEVELOPMENT CENTER
Air Force Systems Command
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Reducing Grating Lobes Due to Subarray Amplitude Tapering

17. COSATI Codes

<table>
<thead>
<tr>
<th>FIELD</th>
<th>GROUP</th>
<th>SUB GROUP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

- Phased arrays
- Amplitude taper
- Low sidelobes
- Large aperture
- Subarrays

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Subarray amplitude tapering is a simple, lower cost method to generate low sidelobes in an antenna's far field pattern. Unfortunately, this simple technique also generates unwanted grating lobes. Placing the exact amplitude taper at the element outputs produces the desired far field pattern, but the architecture is complicated and expensive.

This report describes an alternative to these two techniques. A trade-off exists between sidelobe performance and simplicity of design. This trade-off consists of amplitude tapering the subarray outputs and the element outputs in such a way that the element amplitude tapers are identical for every subarray. In this way, the amplitude taper approximates the desired taper much better than subarray tapering alone, yet all the subarrays are identical. Thus, the design remains very simple.
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I. INTRODUCTION

Many communications and radar systems require large aperture antennas. In the past reflector antennas fulfilled this role. Phased arrays were (and still are) too expensive. Today, however, many applications need an antenna with low sidelobes, wide bandwidth, wide scan angles, adaptive pattern control, and the ability to conform the antenna to a structure. Reflectors cannot meet all these specifications. Consequently, phased arrays have become a necessary, as well as an expensive, part of many electronic systems.

The cost of a phased array is proportional to the number of elements in the aperture. Thus, large high performance phased arrays are still extremely expensive to build. Lossy components, bandwidth limitations, and tight manufacturing tolerances postpone the advent of the cheap phased array. We need to develop new techniques and components that will reduce the cost of building a phased array to an acceptable level.

Designing and testing a low sidelobe feed network is an expensive step in constructing a phased array. Theoretically, the low sidelobes result from modifying the signal amplitudes at each element. In practice, the amplitude weights are due to the various coupling coefficients of the power divider in the feed. The feed network becomes simple when all the elements have the same amplitude weight. A

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uniform amplitude taper will not produce low sidelobes in the far field pattern, though.

Often a large phased array is divided into contiguous subarrays as shown in Figure 1. Normally the low sidelobe amplitude taper is the product of the amplitude weight at the element ($A_{mn}$) and its corresponding subarray weight ($b_m$). Ignoring any errors, this weighting scheme exactly replicates any low sidelobe amplitude distribution such as Taylor or Chebychev. In a real phased array, the element weights are due to a power dividing feed network in the subarray. The subarray weights may be individual transceivers or also a result of a power dividing feed network with a single transmitter/receiver at the array output.

![Diagram of a Linear Array With Contiguous M Subarrays and N Elements per Subarray](image)

Figure 1. Model of a Linear Array With Contiguous M Subarrays and N Elements per Subarray

This type of low sidelobe amplitude taper is very expensive to design, build, and test. Every subarray has a different feed network, except for subarrays that are at symmetric locations with respect to the array center. If every subarray were identical, then mass production techniques become possible.
One way to keep all the subarrays identical, but still maintain low sidelobes, is to put the amplitude taper only at the subarray outputs. Now the feed networks are identical for every subarray. The effective amplitude weight at each element in a subarray is the amplitude weight at that subarray output. Thus the effective amplitude taper looks like a quantized version of the desired taper. This periodic amplitude quantization produces grating lobes in the far field pattern. These grating lobes make subarray tapering an unacceptable means of generating low sidelobes.

This report describes how to eliminate the grating lobes due to subarray tapering. The technique is simple. First, the subarray taper remains the same. Next, the subarray feed networks are all identical, but they have an element amplitude taper other than uniform. The product of the element weights and the subarray weights result in a close approximation to the desired amplitude taper. Grating lobes no longer appear because the amplitude taper is not quantized. In addition, since all the subarrays are identical, the cost of building the array becomes less.

Equation (1) gives the far field pattern for a linear array of isotropic elements with the mainbeam pointing at broadside.

\[
F(u) = \sum_{m=1}^{M} b_m \sum_{n=1}^{N} a_{mn} e^{jkd_i \sin \theta}
\]

where

- \( b_m \) = amplitude weight at subarray \( m \),
- \( M \) = number of subarrays,
- \( a_{mn} \) = amplitude weight at element \( n \) of subarray \( m \),
- \( N \) = number of elements per subarray,
- \( k \) = \( 2\pi/\lambda \),
- \( \lambda \) = wavelength,
- \( d_i \) = distance of element \( i \) from the center of the array (in wavelengths),
- \( i = (m-1)N + n \).

When the values for \( b_m \) and \( a_{mn} \) are 1.0, the array has a uniform amplitude taper. The first sidelobes in the pattern are about 13 dB below the mainbeam peak. Low sidelobes occur from weighting the amplitude of the received signals in such a way that the Fourier Transform of the weights result in the desired sidelobe level. Many formulas exist to derive low sidelobe amplitude tapers for a predetermined beamwidth and sidelobe characteristics. Taylor, Chebychev, triangular, and cosine are a few. The amplitude taper may appear either at the elements (\( a_{mn} \)), the subarrays (\( b_m \)), or both.
Figure 2a shows a 30 dB, $n = 4$ Taylor amplitude distribution for a 70 element linear array of isotropic elements spaced one-half wavelength apart. The corresponding far field pattern appears in Figure 2b. This exact amplitude taper results from the amplitude weights at the subarrays ($a_m$) and elements ($a_{mn}$).

Low sidelobe distributions have different amplitude weights at every element in the array (except at symmetric locations). Subarray tapering simplifies the architecture by having element weights of 1.0. This design has several advantages over an amplitude taper applied at the individual elements:

1. Easier to design,
2. Easier to build,
3. Easier to test,
4. Easier to maintain.

These advantages are due to the fact that all subarrays are identical. As a result, the subarrays require only one design, can be mass produced and tested, and are easy to replace for maintenance.

Amplitude weighting at the subarray ports simplifies the antenna architecture, but degrades the sidelobe performance. All the elements in a given subarray appear to have the same weight, because the effective weight at an element is a product of the subarray amplitude and element amplitude. The resulting quantized amplitude taper causes the far field pattern to have grating lobes of the height and the angles predicted by theory. Location of the grating lobes are given by

$$u_p = \frac{\rho}{N}$$

where

- $u_p = \sin \theta$,
- $\theta = \text{direction of grating lobe}$,
- $N = \text{number of elements per subarray}$,
- $d = \text{element spacing in wavelengths}$,
- $\rho = \pm (1, 2, \ldots)$.

Equation (3) yields the peaks of the grating lobes (GP) derived in Eq. (2)

$$GP = \frac{B^2}{M^2 N^2 \sin^2(\pi \rho/N)}$$

where

- $B = \text{beam broadening factor}$. It is the ratio of the 3 dB beamwidth of the tapered array to that of a uniformly illuminated array,
- $M = \text{number of subarrays}$.

Figure 2a. 30 dB Taylor Amplitude Taper

Figure 2b. Far Field Pattern of the 70 Element Array With a 30 dB Taylor Amplitude Taper
As an example of the effects of the subarray amplitude tapering, consider applying the low sidelobe taper shown in Figure 2a at the subarray ports. Figure 2b shows the resulting far field pattern. Two different cases were tried: one with 14 subarrays of 5 elements per subarray and the other with 10 subarrays of 7 elements per subarray. The beam broadening factor for a 30 dB, \( n = 4 \) Taylor distribution is 1.25.

Case 1: \( M = 14 \) and \( N = 5 \)

<table>
<thead>
<tr>
<th>Location in Degrees from Eq. (2)</th>
<th>Sidelobe Level in dB below the Main Beam From Eq. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ± 23.1</td>
<td>30.3</td>
</tr>
<tr>
<td>2 ± 53.1</td>
<td>34.5</td>
</tr>
</tbody>
</table>

Case 2: \( M = 10 \) and \( N = 7 \)

<table>
<thead>
<tr>
<th>Location in Degrees from Eq. (2)</th>
<th>Sidelobe Level in dB below the Main Beam From Eq. (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ± 16.6</td>
<td>27.7</td>
</tr>
<tr>
<td>2 ± 34.8</td>
<td>32.8</td>
</tr>
<tr>
<td>3 ± 59</td>
<td>34.7</td>
</tr>
</tbody>
</table>

The effective element weights and their associated far field patterns for Cases 1 and 2 are shown in Figures 3a to 4b. Grating lobes appear at the angles and at the heights predicted by theory.

Two techniques are available for generating low sidelobes in the far field pattern of a large array. On the one hand, an amplitude taper at the individual elements produces the best sidelobes, but at the cost of complex feed architectures. On the other hand, an amplitude taper only at the subarray outputs provides a simple, cost effective way to implement the taper, but causes grating lobes to form. Rather than using either of these techniques, a trade-off is possible between simplicity of design and performance. This trade-off consists of an amplitude taper at the subarray outputs in conjunction with an identical element amplitude taper for every subarray. In other words, the element amplitude taper within a subarray is identical from subarray to subarray. In addition, there are amplitude weights at the subarray outputs. The new amplitude taper maintains the advantages of having identical subarrays in addition to reducing the grating lobes.
Figure 3a. Effective Element Amplitude Distribution Due to a 30 dB Taylor Amplitude Taper Applied at the Outputs of 14 Subarrays.

Figure 3b. Far Field Pattern Resulting From the Approximate Taper in Figure 4a.
Figure 4a. Effective Element Amplitude Distribution Due to a 30 dB Taylor Amplitude Taper Applied at the Outputs of 10 Subarrays

Figure 4b. Far Field Pattern Resulting From the Approximate Taper in Figure 5a
2. LOW SIDELOBE SUBARRAY AMPLITUDE TAPERS

Amplitude tapering only at the subarray output would be desirable if grating lobes were not formed. Grating lobes form because of the periodic amplitude quantization at the elements. All the elements in the same subarray have the same amplitude weight. Hence, the quantized amplitude taper is a poor approximation of the desired amplitude taper as shown in Figures 4a and 5a. This approximation improves when the elements within a subarray are also given an appropriate amplitude taper. In turn, the far field pattern becomes more acceptable.

The approximation becomes exact when

\[ b_{mn} = \text{desired amplitude weight at element } i \]

where

\[ i = (m - 1)M + n \]

for

\[ m = 1, 2, \ldots, M \]

\[ n = 1, 2, \ldots, N \]

The exact solution has different amplitude weights at each element. Only the symmetric elements and subarrays have corresponding identical amplitude weights. Thus, the exact solution produces the desired far field pattern, but has \( M/2 \) different element tapers within the subarrays. In turn, \( M/2 \) different subarray feeds must be designed, manufactured, and tested.

It is possible to improve the approximation while at the same time having all the element tapers within the subarrays identical. Assume that

\[ a_{1n} = a_{2n} = \cdots = a_{MN}, \quad n = 1, 2, \ldots, N \]

and

\[ a_{m1} \neq a_{m2} \neq \cdots \neq a_{mN}, \quad m = 1, 2, \ldots, M \]

Multiplying the tapered element amplitude weights \( a_{MN} \) by their subarray amplitude weight \( b_m \) produces a closer approximation to the desired amplitude distribution than the uniformly weighted elements. Since every subarray has an identical
amplitude taper at its elements, all the subarrays are interchangeable, and the advantages of subarray tapering remain. At the same time, the far field pattern is a closer approximation to the desired far field pattern, than in the case of tapering at the subarray outputs alone.

Figure 3b shows the far field pattern resulting from a 30 dB Taylor amplitude taper at the output ports of 14 subarrays with 5 elements per subarray. We want to find the element amplitude weights \( a_{mN} \) that give a clear approximation to the Taylor distribution. The element weights within the subarray are unknown. Since the desired amplitude taper and the subarray amplitude weights \( b_m \) are known, the unknown element weights \( a_{mN} \) can be found. We assume that every subarray has identical element amplitude weights represented by \( a_{m1}, a_{m2}, a_{m3}, a_{m4}, \) and \( a_{m5} \). With this information a set of 5 equations is formed for each subarray.

\[
\begin{align*}
\text{Subarray 1} & : & a_{m1} &= 0.254 & a_{m2} &= 0.247 & a_{m3} &= 0.254 & a_{m4} &= 0.266 & a_{m5} &= 0.281 \\
\text{Subarray 2} & : & a_{m1} &= 0.345 & a_{m2} &= 0.345 & a_{m3} &= 0.345 & a_{m4} &= 0.345 & a_{m5} &= 0.345 \\
\text{Subarray 3} & : & a_{m1} &= 0.496 & a_{m2} &= 0.496 & a_{m3} &= 0.496 & a_{m4} &= 0.496 & a_{m5} &= 0.496 \\
\text{Subarray 4} & : & a_{m1} &= 0.666 & a_{m2} &= 0.666 & a_{m3} &= 0.666 & a_{m4} &= 0.666 & a_{m5} &= 0.666 \\
\text{Subarray 5} & : & a_{m1} &= 0.820 & a_{m2} &= 0.820 & a_{m3} &= 0.820 & a_{m4} &= 0.820 & a_{m5} &= 0.820 \\
\text{Subarray 6} & : & a_{m1} &= 0.931 & a_{m2} &= 0.931 & a_{m3} &= 0.931 & a_{m4} &= 0.931 & a_{m5} &= 0.931 \\
\text{Subarray 7} & : & a_{m1} &= 0.983 & a_{m2} &= 0.983 & a_{m3} &= 0.983 & a_{m4} &= 0.983 & a_{m5} &= 0.983 \\
\end{align*}
\]

For our 70 element array these equations are
Only half of the subarrays are evaluated since the other half are mirror images. Seven sets of values for \( a_{m1}, a_{m2}, a_{m3}, a_{m4}, \) and \( a_{m5} \) are found for each subarray by solving the 5 sets of equations. The variables have different values for every subarray. These values represent the exact solution (see Table 1 under Exact Element Taper column). In order to get approximate values for \( a_{m1}, a_{m2}, a_{m3}, a_{m4}, a_{m5} \) that are the same for every subarray, the variables are averaged. Averaging the variables over the 7 subarrays gives the following average values for \( a_{m1}, a_{m2}, a_{m3}, a_{m4}, \) and \( a_{m5} \):

\[
\begin{align*}
a_{m1} &= 0.922 \\
a_{m2} &= 0.959 \\
a_{m3} &= 0.999 \\
a_{m4} &= 1.041 \\
a_{m5} &= 1.085
\end{align*}
\]

Table 1 shows the new configuration for the amplitude weights under the column "Approximation With All Identical Subarrays." Multiplying the subarray weights by the amplitude weights at each element gives a closer approximation to the desired amplitude taper than tapering at the subarray outputs alone.

Figure 5a shows the approximate taper superimposed on the desired taper. The approximate values are close to the desired values in the 5 subarrays on the edge. The resulting amplitude tapers at the middle subarrays (subarrays 6 and 7) are poor approximations to the desired tapers. In spite of this crude approximation, the far field pattern in Figure 5a compared reasonably well with the desired pattern in Figure 2b. Sidelobes are somewhat higher than desired, but the grating lobes no longer appear. In general, the antenna pattern in Figure 5b is much more desirable than the antenna pattern due to amplitude tapering at the subarray outputs (Figure 3b).

Since the element amplitude tapers within subarrays 6 and 7 result in a poor approximation to the desired Taylor amplitude taper, they were averaged separate from the other five subarrays. Now, there are two groups of identical subarrays. Group 1 has subarrays 1 to 5 and Group 2 contains subarrays 6 and 7. Instead of averaging the variables \( a_{m1}, a_{m2}, a_{m3}, a_{m4}, a_{m5} \) over all the subarrays, an average is found for each group. The new element weights are shown below.

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_{m1} = 0.904 )</td>
<td>( a_{m1} = 0.966 )</td>
</tr>
<tr>
<td>( a_{m2} = 0.950 )</td>
<td>( a_{m2} = 0.982 )</td>
</tr>
<tr>
<td>( a_{m3} = 1.000 )</td>
<td>( a_{m3} = 0.986 )</td>
</tr>
<tr>
<td>( a_{m4} = 1.055 )</td>
<td>( a_{m4} = 1.007 )</td>
</tr>
<tr>
<td>( a_{m5} = 1.113 )</td>
<td>( a_{m5} = 1.016 )</td>
</tr>
<tr>
<td>( m = 1 ) to ( 5 )</td>
<td>( m = 6 ) or ( 7 )</td>
</tr>
</tbody>
</table>
Table 1. Calculated Element Weights for a 30 dB Taylor Amplitude Taper

<table>
<thead>
<tr>
<th>Subarray</th>
<th>Element</th>
<th>Subarray Weight</th>
<th>Exact Element Amplitude</th>
<th>Approximation With All Identical Subarrays</th>
<th>Approximation With 2 Groups of Identical Subarrays</th>
<th>Approximation With 3 Groups of Identical Subarrays</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>0.957 0.243</td>
<td>0.922 0.234</td>
<td>0.904 0.230</td>
<td>0.898 0.228</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td>0.972 0.247</td>
<td>0.959 0.244</td>
<td>0.950 0.241</td>
<td>0.946 0.240</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.254</td>
<td>1.000 0.254</td>
<td>0.999 0.243</td>
<td>1.000 0.254</td>
<td>1.000 0.254</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>1.047 0.266</td>
<td>1.041 0.264</td>
<td>1.055 0.268</td>
<td>1.061 0.269</td>
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<td></td>
<td>5</td>
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<td>1.106 0.281</td>
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<td>1.126 0.286</td>
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<td>0.867 0.299</td>
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<td>0.898 0.310</td>
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<td>7</td>
<td>0.345</td>
<td>0.928 0.320</td>
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<td>0.950 0.328</td>
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<td></td>
<td>8</td>
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<td>1.000 0.435</td>
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<td>1.078 0.372</td>
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<td></td>
<td>1.059 0.530</td>
<td>1.041 0.516</td>
<td>1.053 0.523</td>
<td>1.061 0.526</td>
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<td>1.085 0.538</td>
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<td>0.950 0.633</td>
<td>0.946 0.630</td>
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<td>0.666</td>
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Figure 5a. 30 dB Taylor Amplitude Taper With Identical Element Amplitude Tapers in Each of the 14 Subarrays

Figure 5b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 6a
Table 1 shows the array configuration for these two groups of subarrays. The approximation to the desired Taylor amplitude taper improves at the cost of having two different types of element tapers within the subarrays instead of one. Figure 6a shows the new approximation superimposed on the desired taper. The resulting far field pattern appears in Figure 6b. No grating lobes are present and the side-lobes are close to the desired levels.

One further step was taken to improve the approximation to the amplitude taper. The subarrays were divided into 3 groups. Group 1 had subarrays 1 to 4, group 2 had subarrays 5 and 6, and group 7 was subarray 7. Table 1 shows the resulting amplitude taper. This taper along with the desired taper is shown in Figure 7a. The resulting far field pattern appears in Figure 7b. As expected, the side-lobes are closer to the desired side-lobes than the other approximations.

The techniques were then tried on a 70-element linear array with a $30\,\text{dB}$, $\bar{n} = 4$ Taylor amplitude distribution and 10 subarrays. Figures 8a and 8b show the approximation and far field pattern resulting from having all the subarrays identical. Next, the subarrays were divided into two groups of element amplitude tapers. The first group had subarrays 1 to 3 and the second group had subarrays 4 and 5. Figures 9a and 9b show the amplitude taper and resulting far field pattern respectively. Finally, subarrays 1 to 3 were placed in group 1, 4 was a group 2, and 5 was in group 3. This grouping produced excellent results (Figures 10a and 10b). Results for the 10 subarray case were similar to the results for the 14 subarray case. The more subarray groups, the better the amplitude taper approximation becomes, hence, the far field pattern comes closer to the desired far field pattern. In the limiting case of 70 subarrays, the approximation and desired tapers are the same.
Figure 6a. 30 dB Subarray Amplitude Taper With Two Groups of Identical Subarrays (Subarrays 1 to 5, and 6 and 7)

Figure 6b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 7a
Figure 7a. 30 dB Subarray Amplitude Taper With Three Groups of Identical Subarrays (Subarrays 1 to 4, 5 and 6, and 7)

Figure 7b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 8a
Figure 8a. 30 dB Subarray Amplitude Taper with Identical Element Amplitude Tapers in Each of the 10 Subarrays

Figure 8b. Far Field Pattern Resulting from the Approximate Amplitude Taper in Figure 9b
DASHED LINE IS DESIRED TAPER
SOLID LINE IS APPROXIMATE TAPER

Figure 9a. 30 dB Subarray Amplitude Taper With Two Groups of Identical Subarrays (Subarrays 1 to 3, and 4 and 5)

Figure 9b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 10b
Figure 10a. 30 dB Subarray Amplitude Taper With Three Groups of Identical Subarrays (1 to 3, 4, and 5)

Figure 10b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 11a
3. EXTENDING THE TECHNIQUE TO LOWER SIDELOBE LEVELS

One might expect that subarray amplitude tapering becomes more of a problem as sidelobe levels get lower. Equations (2) and (3) quickly verify this suspicion. Equation (2) does not depend upon the aperture amplitude tapers at all. Thus, the grating lobes always appear at the same locations, independent of the sidelobe levels. On the other hand, the grating lobe peaks do depend upon the amplitude taper. Equation (3) shows that the peaks are directly proportional to the beam broadening factor, $B$. In turn, $B$ gets larger as the sidelobe levels get lower. Although $B$ does change with sidelobe level, the change is relatively small. For instance $B = 1.25$ for the 30 dB Taylor taper and $B = 1.50$ for the 50 dB Taylor taper. This change results in an increase in grating lobe height of 1.59 dB for the 50 dB taper.

Figures 11a and 11b show the amplitude taper and associated far field pattern of a 50 dB, $n = 12$ low sidelobe Taylor distribution. The next two figures (Figures 12a and 12b) show the results of placing the amplitude taper at the subarray outputs for 14 subarrays. As previously predicted, the grating lobe locations are the same as the 30 dB Taylor far field pattern. Grating lobe peaks are slightly higher in the 50 dB Taylor taper.

Figures 13a to 18b show different approximations to the 50 dB Taylor amplitude distribution for 14 subarrays. The 50 dB sidelobe levels are quite sensitive to the accuracy of the approximation. Figures 13a and 13b clearly show the inadequacy of the approximation when all the subarrays have identical element amplitude tapers. The accuracy of the approximation improves when 2 or 3 different groups of subarrays having identical elements amplitude tapers are found (Figures 14a - 17b).

Finally, Figures 19a and 19b show the approximate amplitude taper and associated far field pattern for a 40 dB Taylor amplitude taper. The Taylor distribution was approximated by three groups of identical subarrays (subarrays 1 to 4; 5 and 6; and 7). This approximation produced an excellent far field pattern.
Figure 11a. 50 dB, \( \pi = 12 \) Taylor Amplitude Taper

Figure 11b. Far Field Pattern of a 70 Element Array With a 50 dB, \( \pi = 12 \) Taylor Amplitude Taper
DASHED LINE IS DESIRED TAPER
SOLID LINE IS APPROXIMATE TAPER

Figure 12a. Effective Element Amplitude Distribution Due to a 50 dB Taylor Amplitude Taper Applied at the Outputs of 14 Subarrays

Figure 12b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 13a
DASHED LINE IS DESIRED TAPER
SOLID LINE IS APPROXIMATE TAPER

Figure 13a. 50 dB Subarray Amplitude Taper With Identical Element Amplitude Tapers in Each of the 14 Subarrays

Figure 13b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 15a
Figure 14a. 50 dB Subarray Amplitude Taper With Two Groups of Identical Subarrays (Subarrays 1 to 4, and 5 to 7)

Figure 14b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 16a
Figure 15a. 50 dB Subarray Amplitude Taper With Two Groups of Identical Subarrays (Subarrays 1 to 5 and 6 and 7)

Figure 15b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 17a
Figure 16a. 50 dB Subarray Amplitude Taper With Three Groups of Identical Subarrays (Subarrays 1 to 4, 5 and 6, and 7)

Figure 16b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 16a
Figure 17a. 50 dB Subarray Amplitude Taper With Three Groups of Identical Subarrays (Subarrays 1 to 3, 4 and 5, 6 and 7)

Figure 17b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 17a
Figure 18a. 50 dB Subarray Amplitude Taper With Four Groups of Identical Subarrays (Subarrays 1 to 3, 4 and 5, 6 and 7)

Figure 18b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 20a
Figure 19a. 40 dB Subarray Amplitude Taper With Three Groups of Identical Elements (Subarrays 1 to 4, 5 and 6, and 7)

Figure 19b. Far Field Pattern Resulting From the Approximate Amplitude Taper in Figure 24a
4. DISCUSSION OF RESULTS

Amplitude tapering only at the subarray outputs produces unwanted grating lobes in the far field pattern. In order to eliminate these grating lobes, an amplitude taper must be applied to the individual elements in the subarray as well. The element amplitudes can be adjusted in such a way that the combined element and subarray tapers produce the desired amplitude distribution. Now, the sidelobes are at the desired levels, but every subarray has different elements amplitude tapers. Consequently, the antenna architecture is more complicated than when the taper was only at the subarray outputs.

This report described a technique that uses a subarray taper and identical element tapers within the subarrays to approximate the desired amplitude taper. The technique works well for a 30 dB Taylor taper. As the sidelobes get lower, the approximation is not accurate enough. Therefore, groups of identical subarrays must be used to arrive at a good enough approximation to the desired amplitude taper. A trade-off exists between sidelobe performance and simplicity of design. The more groups of identical subarrays, the more complicated the array design becomes, but the better the sidelobe performance becomes, too. This approximation technique is not limited to linear phased arrays. In fact, the savings has a greater potential for a planar phased array.
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Rome Air Development Center

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