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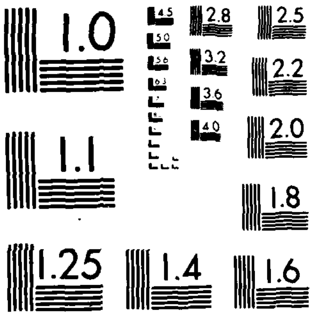

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AFGL-TR-84-0132

A Comparison of Non-Linear Regression and Weighted Least Squares for Predicting Visibility in Germany

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Two methods of parameter estimation for the two parameter Weibill distribution are investigated using data on visibility from Germany. They are: Non-Linear Regression and Weighted Least Squares. Non-Linear Regression gave superior results for both "constants" and "variables" models when measured either by overall RMS or by sample reuse. However, since the Weighted Least Squares gave similar results and utilized only about one-tenth the computer time, the authors recommend that it might be used to investigate other models of visibility for other countries.

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ACKNOWLEDGEMENT

We would like to acknowledge the contribution of Mr. David Van Brackle in the accomplishment of the results of this paper. David was responsible for writing several computer programs and made all of the numerous runs required for this report.

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A Comparison of Non-Linear Regression and Weighted Least Squares for Predicting Visibility in Germany

by

L. A. Franklin, P. N. Somerville, and S. J. Bean
University of Central Florida

1. INTRODUCTION

A goal of the Air Weather Service is to be able to state the probability that a weather element will have a value above a specified threshold for any location at any time. Many models have been developed for such weather elements as visibility, ceiling, sky cover, etc., for locations where records exist. Modeling these elements for locations where there are no records is a more difficult task. Some models have been developed by Bean and Somerville that require only knowledge of elevation of the location in question and the average of the elevations 20 kilometers from the location to estimate visibility probabilities for a specified month and hour period for Germany. These models were developed using the method of non-linear regression. The method of evaluating how well the models predict visibility at data-void locations (i.e., sample reuse) was also developed using non-linear regression. This paper compares the method of non-linear regression with a new method, "weighted least squares," in developing and evaluating these models.

2. BACKGROUND OF PREVIOUS WORK ON VISIBILITY IN GERMANY

Bean and Somerville in AFGL-TR-81-0144 "Some Models for Visibility for German Stations" showed that the Weibull distribution was able to fit visibility data for 30 stations in Germany. For any distance x, the probability F(x) of visibility less than x miles is then given by the Weibull formula

$$F(x) = 1 - e^{-\alpha x^{\beta}}$$

where a different set of α and β values were derived for each month and each 3-hour period.

The values of α and β were estimated by choosing the values for which the Weibull cumulative distribution most closely fits the empirical cumulative distribution. Let $E_j(x_i)$ be the empirical probability (step function) that the visibility is less than x_i miles for the j^{th} station, using data from the RUSSWOs (Revised Uniform Summary of Surface Weather Observations). The values of x_i are the following distances in miles: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, 1, $\frac{5}{4}$, $\frac{3}{2}$, 2, $\frac{5}{2}$, 3, 4, 5 and 6. Let

$$F_j\ (x;\ \alpha_j,\ \beta_j) = 1 - e^{-\alpha_j x^{\beta_j}}$$

be the Weibull distribution at station j. Then the values of α_j and β_j that are chosen are those that minimize the expression

$$\sum_j \sum_i [E_j\ (x_i) - F_j(x_i;\ \alpha_j,\ \beta_j)]^2 \qquad (2.1)$$

That is, the values of α_j and β_j are those that minimize the sum of squares of the difference between the empirical and model probabilities over all stations and all distances for which data is available. This deviation is done for each of the 12 months and 8 three-hour periods of interest.

The coefficients α_j and β_j may themselves be dependent upon other variables. In fact, Bean and Somerville in AFGL-TR-82-0335 "Some New Practical Models for Visibility for Germany Locations Where No Records Exist" found that elevation of the location of interest and relative elevation at 20 kilometers around the location of interest provided an improvement in the model. The coefficients can be written as

$$\alpha_j = \gamma_0 + \gamma_1 * EL_j + \gamma_2 * AE_j$$

$$\beta_j = \delta_0 + \delta_1 * EL_j + \delta_2 * AE_j$$

where EL_j and AE_j are transformed and scaled values for the elevation at station j and the average elevation at 20 equispaced locations on a circle of radius 20 kilometers and centered at the station. EL is the cube of the elevation in feet, divided by 10^9. AE is the cube of the average elevation in feet, divided by 10^9. The constants γ_0, γ_1, γ_2 and δ_0, δ_1, δ_2 are determined by minimizing expression (2.1). The presence of all six constants comprises what has been called the "variables model" using non-linear regression. If we have only $\alpha_j = \gamma_0$ and $\beta_j = \delta_0$ the model has been called the "constants model" using non-linear regression and effectively fits only one model $\alpha = \delta_0$ and $\beta = \delta_0$ for all stations, ignoring any geographical features at the stations. The "constants model" has been evaluated and discussed for 30 stations in Germany in AFGL-TR-81-0313 "Modeling Visibility for Locations in Germany Where No Records Exist." The "variables model" has been evaluated and discussed for 60 stations in Germany in AFGL-TR-82-0335.

The method of minimizing expression (2.1) has been the method of non-linear regression and has been discussed in detail in AFGL-TR-80-0362 "Least Squares Fitting of Distribution Using Non-Linear Regression." The technique involves an iterative solution incorporating an initial estimate of the parameters. While the method has been extremely successful in fitting models and displays very robust features, it is time consuming to execute even on a mid-sized computer.

The method used to evaluate the models formulated is called sample reuse and has been discussed in AFGL-TR-82-0335. Briefly, sample reuse takes a single station and uses all the other stations to build the model and then calculates how well the model predicts visibility at the omitted station measured in the usual root mean square sense (i.e., RMS). This is done for each station for which you have data and hence, for n stations, results in n times as many non-linear regressions as needed to fit the original model to all n stations at once.

It was in the interest of trying to find a new method of estimating the models that several methods were investigated in AFGL-TR-83-0248 "A Comparison of Several Alternatives to Maximum Likelihood for the Weibull Distribution." In that simulation study, the method of non-linear regression seemed more robust in that it was able to give better models when the data was contaminated or when the underlying true distribution was not the form of the distribution chosen to model it. However the method of weighted least squares showed promise in that it gave good results (although not as good as non-linear regression) and used only a small fraction of the time that non-linear regression needed. This method was initially suggested by Major Al Boehm, USAF, and it was the purpose of this report to compare this technique with non-linear regression on actual visibility data in Germany.

3. METHODOLOGY AND RESULTS

3.1 The Constants Model

The resulting RMS of fitting the "constants model" by both non-linear regression and weighted least squares to the first group of 30 stations in Germany is recorded in Exhibit 3.1 for all month and hour periods. In that exhibit for the "constant model," non-linear regression always produced smaller RMS than the weighted least squares. The difference was as small as .002 for the period 06-08 LST of October to as large as .112 for period 00-02 LST of January where RMS for weighted least squares was three times the value for non-linear regression.

In AFGL-TR-82-0187, "Evaluation of An Observation-Based Climatology Model for Predicting Visibility for Data-Void Locations in Germany," the constants model was expanded to a total of 60 stations in Germany and RMS from sample reuse based on non-linear regression was utilized to measure its ability to predict visibility. The overall results of Exhibits 2.3 and 2.4 of that report are included in Exhibits 3.2 and 3.3 of this report along with the RMS from sample reuse based on weighted least squares. Hour periods 00-02 LST and 03-05 LST are omitted since data from these pre-dawn hours was frequently missing. For the stations, the weighted least squares method produced a larger RMS than non-linear regression at each station, usually between 2 and 3 times as large. The average RMS for non-linear regression was .098 while for weighted least squares, it was .177. Similar results hold for the monthly and hourly RMSs.

3.2 <u>The Variables Model</u>

The variables model was fit to all 60 stations in Germany and the coefficients for the α and β terms as well as overall fit RMS and sample reuse RMS were reported in AFGL-TR-82-0335 and displayed as Exhibit 2.4 of that report. It is included in Exhibit 3.4 along with comparable RMS results of overall fit and sample reuse using weighted least squares. Whether one compares RMS of the overall fitted "variables model" done by non-linear regression and by weighted least squares or whether one compares RMS from sample reuse from the two methods the results are the same. In virtually every instance non-linear regression gives a smaller RMS, usually by a factor of 2 to 3. Noteworthy exceptions occur at Grosser Falk and Bad Kreuznach where weighted least squares does marginally better. However both stations have previously presented problems of modeling and have unusually large RMS values under non-linear regression.

Exhibit 3.5 presents the overall RMS fit of the variables model to the complete set of 60 stations in Germany using both non-linear regression (the top entry in each cell) and weighted least squares (the bottom entry in each cell) for all 12 months and hour periods 3 through 8. Non-linear regression in every case produces a lower RMS than weighted least squares, sometimes as little as 10 percent lower (for October and hour period 9-11) but usually almost 50 percent lower. The overall average RMS for all months and time periods for non-linear regression was .071 as compared to .095 for weighted least squares.

Exhibit 3.6 presents the RMS for the variables model using sample reuse on the 60 stations in Germany using non-linear regression (the top entry in each cell) and weighted least squares (the bottom entry in each cell). Again the RMS achieved by non-linear regression was always smaller than the RMS achieved by weighted least squares.

4. CONCLUSIONS

The results of investigating weighted least squares as a method to derive visibility models is clear and consistent with other results. Non-linear regression consistently provides better models than weighted least squares, usually dramatically better. This is "better" in the sense of better-fitting models (i.e., overall RMS) and "better" in the sense of better-predicting models (i.e., sample reuse).

However, results from weighted least squares is consistent with non-linear regression in the sense that relatively low RMS values derived by NLR correspond to relatively low RMS values derived by WLS. In particular, stations that are "difficult to fit" or may contain "bad data" remain so under either technique. Thus it seems reasonable that WLS can be used to construct many models of visibility and explore them for (relative) goodness of fit, since it is much faster to compute than NLR. Hence having eliminated a vast array of poor models by WLS, it would be possible to use NLR upon these few models which WLS points to as most promising. NLR can then choose the "best among the good" and give the lowest RMS and most accurate values for the coefficients of α and β in the Weibull Model.

It is hoped that such a two-step method can be employed by the authors on data from Norway to see if it is successful.

5. REFERENCES

Bean, S. J., and P. N. Somerville, "Evaluation of an Observation-Based Climatology Model for Predicting Visibility for Data-Void Locations in Germany," AFGL-TR-82-0187, July 1982, AD A123998.

Bean, S. J., P. N. Somerville and L. A. Franklin, "A Comparison of Several Alternatives to Maximum Likelihood for the Weibull Distribution," AFGL-TR-83-0248, September 1983, AD A139721.

Heuser, M., P. N. Somerville, and S. J. Bean, "Least Squares Fitting of Distributions Using Non-Linear Regression," AFGL-TR-80-0362, September 1980, AD A097039.

Somerville, P. N., and S. J. Bean, "Some Models for Visibility for German Stations," AFGL-TR-81-0144, April 1981, AD A104167.

Somerville, P. N. and S. J. Bean, "Modeling Visibility for Locations in Germany Where No Records Exist," AFGL-TR-81-0313, December 1981, AD A111890.

Somerville, P. N. and S. J. Bean, "Some New Practical Models for Visibility for German Locations Where No Records Exist," AFGL-TR-82-0335, 3 September 1982, AD A126339.

Hour Period (LST)

Month	00-02	03-05	06-08	9-11	12-14	15-17	18-20	21-23
January	.068 .180	.102 .151	.107 .144	.099 .104	.087 .096	.078 .086	.091 .114	.075 .165
February	.082 .178	.093 .135	.078 .081	.064 .067	.057 .060	.053 .057	.072 .088	.090 .164
March	.048 .105	.086 .120	.066 .069	.057 .061	.053 .062	.041 .045	.051 .064	.055 .091
April	.047 .086	.073 .106	.054 .059	.042 .047	.029 .033	.027 .036	.042 .058	.034 .057
May	.034 .064	.064 .090	.045 .051	.029 .032	.016 .019	.013 .018	.021 .029	.028 .050
June	.053 .082	.067 .097	.052 .059	.035 .042	.019 .024	.016 .020	.026 .042	.027 .048
July	.045 .078	.074 .104	.052 .059	.031 .036	.016 .022	.012 .017	.026 .047	.031 .052
August	.040. .072	.076 .113	.076 .091	.047 .058	.025 .034	.015 .019	.023 .030	.071 .460
September	.066 .115	.101 .144	.088 .097	.062 .072	.059 .063	.025 .031	.049 .066	.046 .079
October	.068 .162	.100 .151	.080 .082	.075 .081	.059 .069	.049 .059	.087 .115	.061 .129
November	.062 .163	.091 .136	.070 .073	.066 .070	.059 .065	.063 .069	.087 .103	.061 .147
December	.083 .198	.094 .146	.082 .088	.078 .083	.075 .084	.076 .083	.075 .100	.105 .192

Exhibit 3.1

Values of RMS for "constants Model" for 30 stations in Germany using non-linear regression (top entry) and weighted least square (bottom entry).

WMO	Station	'at (North)	Long (East)	Non-Linear RMS	Weighted L.S. RMS
10501	Aachen, DL	50.78	6.12	.081	.169
10224	Bremen, GER	53.05	8.80	.048	.172
10305	Lingen, GER	52.52	7.33	.127	.168
10427	Kahler Asten, GER	51.18	8.50	.274	.178
11157	Aigen Ennstal, OS	47.53	14.15	.101	.202
11448	Plezen/Dobra, CZ	49.67	13.30	.145	.159
10900	Bremgarten, GER	47.90	7.63	.050	.164
10948	Oberstdorf, GER	47.40	10.30	.147	.198
10929	Konstanz, GER	47.68	9.20	.051	.170
11120	Innsbruck, OS	47.27	11.37	.081	.198
11150	Salzburg, OS	47.80	13.02	.059	.181
10893	Passau, GER	48.58	13.50	.066	.157
10908	Feldberg, GER	47.87	8.02	.321	.217
10921	Neuhausen, DL	47.98	8.92	.108	.146
10512	Nurburg, GER	50.33	6.97	.134	.108
10515	Koblenz, DL	50.35	7.60	.102	.191
10532	Giessen, GER	50.57	8.72	.076	.194
10542	Hersfeld, DL	50.87	9.72	.135	.180
10658	Kissinlen, DL	50.20	10.10	.126	.179
10671	Coburg, GER	50.27	10.97	.123	.171
10685	Hof, GER	50.32	11.90	.067	.140
10704	Berus, GER	49.27	6.70	.125	.160
10727	Karlsruhe, GER	49.02	8.40	.072	.178
10742	Ohringen, GER	49.20	9.53	.092	.174
10788	Staubing, GER	48.82	12.60	.112	.185
10791	Grosser Falk, GER	49.08	13.30	.275	.161
10805	Lahr, GER	48.37	7.85	.061	.157
10837	Laudheim, GER	48.22	9.93	.094	.159
10953	Kaufbeuren, DL	47.87	10.63	.065	.166
10875	Muhldorf, GER	48.25	12.55	.091	.164
10616	Hahn, AB	49.95	7.27	.048	.147
10610	Bitburg, AB	49.95	6.57	.056	.172
10614	Ramstein, AB	49.43	7.58	.064	.184
10607	Spangdahlem, AB	49.97	6.70	.067	.177
10384	Tempelhof, APRT	52.47	13.40	.074	.192
10755	Ansbach, AAF	49.32	10.63	.071	.174
10544	Fulda, AAF	50.53	9.63	.062	.171
10869	Erding, AS	48.32	11.93	.060	.177
10765	Feucht, AAF	49.38	11.18	.055	.171
10618	Baumholder, AAF	49.65	7.30	.176	.199
10626	Bad Kreuznach, AAF	49.87	7.88	.111	.189
10971	Bad Tolz, AAF	47.77	11.60	.123	.200
10714	Zweibrucken, AB	49.22	7.40	.046	.169
10633	Wiesbaden, AB	50.05	8.33	.050	.165
10633	Finthen, AAF	49.97	8.15	.057	.160
10763	Furth, AAF	49.50	10.95	.090	.187
10642	Hanau, AAF	50.17	8.95	.046	.171

(continued)

WMO	Station	Lat (North)	Long (East)	Non-Linear RMS	Weighted L.S. RMS
10852	Gablingen, AAF	48.45	10.87	.109	.192
10653	Giebelstadt, AUX AF	49.67	9.88	.082	.186
10687	Grafenwohr, AAF	49.70	11.95	.073	.169
10734	Heidelberg, AAF	49.40	8.64	.041	.167
10752	Illesheim, AAF	49.47	10.38	.149	.205
10659	Kitzingen, AAF	49.75	10.20	.066	.189
10763	Nurnberg	49.50	11.08	.066	.191
10729	Coleman, AAF	49.57	8.47	.114	.182
10657	Wertheim, AAF	49.77	9.48	.153	.177
10745	Schwaebisch Hall, AA	49.17	9.78	.097	.179
10712	Sembach, AB	49.52	7.87	.037	.169
10862	Siegenberg Gunnery	48.75	11.80	.177	.219
10738	Echterdingen, ARPT	48.68	9.22	.057	.183
	Overall			.098	.177

Exhibit 3.2

Overall RMS for Each of 60 Stations for the Constants Model Utilizing Sample Reuse Based on Non-Linear Regression and Weighted Least Squares

Hour Period
(LST)

	06-08	09-11	12-14	15-17	18-20	21-23	all
January	.126 .156	.119 .144	.114 .159	.113 .161	.124 .169	.166 .210	.128 .167
February	.117 .147	.105 .137	.099 .159	.099 .167	.111 .175	.152 .217	.115 .167
March	.103 .145	.097 .160	.087 .198	.083 .205	.094 .211	.116 .253	.097 .195
April	.101 .176	.089 .214	.074 .226	.072 .226	.080 .235	.089 .238	.085 .219
May	.093 .167	.076 .225	.060 .209	.051 .183	.060 .201	.077 .227	.071 .202
June	.094 .171	.073 .212	.054 .184	.045 .159	.056 .182	.070 .190	.067 .183
July	.088 .164	.066 .194	.049 .170	.039 .137	.047 .146	.065 .177	.061 .165
August	.093 .126	.070 .156	.051 .159	.046 .155	.055 .167	.080 .202	.068 .161
September	.106 .123	.085 .131	.064 .163	.053 .161	.074 .190	.097 .212	.082 .163
October	.100 .107	.097 .112	.082 .142	.078 .157	.100 .157	.134 .176	.100 .142
November	.112 .137	.108 .140	.104 .174	.106 .175	.121 .181	.151 .206	.118 .169
December	.113 .143	.107 .136	.106 .153	.108 .151	.115 .156	.167 .207	.121 .158
all	.104 .147	.092 .161	.082 .175	.079 .170	.090 .181	.119 .210	.096 .174

Exhibit 3.3

RMS Over All 60 Stations by Month and Hour Period for the Constants Model Utilizing Sample Reuse Based on Non-Linear Regression (top entry) and Weighted Least Squares (bottom entry).

WMO	Station	Lat (North)	Long (East)	Elev. (ft.)	Rel. Elev. (ft.)	Non-Linear Regression Sample Reuse RMS	Non-Linear Regression Overall Fit RMS	Weighted Least Squares Sample Reuse RMS	Weighted Least Squares Overall Fit RMS
10616	Hahn, AB	49.95	7.27	1650.	1267.	.062	.048	.172	.070
10610	Bitburg, AB	49.95	6.57	1228.	1363.	.044	.042	.092	.091
10614	Ramstein, AB	49.43	7.58	780.	710.	.051	.052	.109	.108
10607	Spangdahlem, AB	49.97	6.70	1196.	1330.	.057	.055	.101	.100
10384	Tempelhof, APRT	52.47	13.40	164.	164.	.058	.058	.115	.114
10755	Ansbach, AAF	49.32	10.63	1542.	1181.	.068	.064	.100	.099
10544	Fulda, AAF	50.53	9.63	1010.	1650.	.045	.046	.085	.085
10869	Erding, AS	48.32	11.93	1522.	1525.	.064	.062	.098	.097
10765	Feucht, AAF	49.38	11.18	1265.	1407.	.046	.045	.086	.086
10618	Baumholder, AAF	49.65	7.30	1408.	1538.	.153	.154	.156	.153
10626	Bad Kreuznach, AAF	49.87	7.88	355.	825.	.071	.071	.120	.119
10971	Bad Tolz, AAF	47.77	11.60	2360.	2688.	.121	.102	.120	.117
10714	Zweibrucken, AB	49.22	7.40	1132.	938.	.041	.040	.091	.090
10633	Wiesbaden, AB	50.05	8.33	470.	617.	.036	.036	.083	.082
10633	Finthen, AAF	49.97	8.15	769.	948.	.048	.047	.082	.081
10763	Furth, AAF	49.50	10.95	1000.	1225.	.085	.085	.112	.111
10642	Hanau, AAF	50.17	8.95	377.	408.	.037	.036	.089	.087
10852	Gablingen, AAF	48.45	10.87	1530.	1550.	.084	.082	.120	.120
10653	Giebelstadt, AUX AF	49.67	9.88	985.	950.	.060	.060	.112	.111
10687	Grafenwohr, AAF	49.70	11.95	1370.	1510.	.063	.066	.088	.087
10734	Heidelberg, AAF	49.40	8.64	369.	36[illegible].	.034	.034	.086	.084
10752	Illesheim, AAF	49.47	10.38	1060.	1250.	.084	.084	.135	.135
10659	Kitzingen, AAF	49.75	10.20	699.	899.	.051	.052	.112	.111
10763	Nurnberg	49.50	11.08	1053.	1053.	.062	.063	.114	.113
10729	Coleman, AAF	49.57	8.47	334.	350.	.078	.077	.111	.110
10657	Wertheim, AAF	49.77	9.48	1120.	1075.	.095	.096	.167	.116
10745	Schwaebisch Hall, AA	49.17	9.78	1303.	1632.	.046	.046	.093	.092
10712	Sembach, AB	49.52	7.87	1052.	1315.	.037	.037	.082	.082
10862	Siegenberg Gunnery	48.75	11.80	1325.	1450.	.140	.140	.164	.163
10738	Echterdingen, ARPT	48.68	9.22	1306.	1200.	.056	.054	.105	.105
10501	Aachen, DL	50.78	6.12	673.	755.	.081	.080	.099	.097
10224	Bremen, GER	53.05	8.80	502.	250.	.043	.043	.091	.090

(continued)

						Non-Linear Regression		Weighted Least Squares	
WMO	Station	Lat (North)	Long (East)	Elev. (ft.)	Rel. Elev. (ft.)	Sample Reuse RMS	Overall Fit RMS	Sample Reuse RMS	Overall Fit RMS
10305	Lingen, GER	52.52	7.33	549.	250.	.040	.040	.086	.085
10427	Kahler Asten, GER	51.18	8.50	2822.	1563.	.204	.188	.207	.174
11157	Aigen Ennstal, OS	47.53	14.15	2139.	3350.	.056	.053	.110	.097
11448	Plezen/Dobra, CZ	49.67	13.30	1194.	1400.	.130	.131	.109	.105
10900	Bremgarten, GER	47.90	7.63	699.	1438.	.044	.041	.077	.076
10948	Oberstdorf, GER	47.40	10.30	2664.	3350.	.097	.080	.157	.102
10929	Konstanz, GER	47.68	9.20	1368.	1725.	.037	.036	.082	.081
11120	Innsbruck, OS	47.27	11.37	1962.	3400.	.069	.074	.102	.089
11150	Salzburg, OS	47.80	13.02	1463.	2338.	.048	.047	.080	.078
10893	Passau, GER	48.58	13.50	1335.	1613.	.056	.055	.075	.074
10908	Feldberg, GER	47.87	8.02	4898.	2550.	.280	.069	.355	.078
10921	Neuhausen, DL	47.98	8.92	2648.	2463.	.072	.063	.087	.081
10512	Nurburg, GER	50.33	6.97	2064.	1425.	.085	.077	.059	.056
10515	Koblenz, DL	50.35	7.60	318.	900.	.066	.068	.116	.115
10532	Giessen, GER	50.57	8.72	640.	825.	.059	.060	.119	.118
10542	Hersfeld, DL	50.87	9.72	738.	1200.	.059	.060	.099	.098
10658	Kissinlen, DL	50.20	10.10	704.	1215.	.039	.040	.095	.094
10671	Coburg, GER	50.27	10.97	1106.	1338.	.038	.039	.085	.084
10685	Hof, GER	50.32	11.90	1864.	1938.	.048	.057	.064	.062
10704	Berus, GER	49.27	6.70	1204.	1050.	.056	.054	.088	.087
10727	Karlsruhe, GER	49.02	8.40	394.	600.	.041	.041	.099	.098
10742	Ohringen, GER	49.20	9.53	909.	1075.	.049	.049	.092	.091
10788	Staubing, GER	48.82	12.60	1155.	1350.	.084	.085	.117	.116
10791	Grosser Falk, GER	49.08	13.30	4291.	2475.	.259	.064	.098	.058
10805	Lahr, GER	48.37	7.85	509.	800.	.074	.073	.077	.075
10837	Laudheim, GER	48.22	9.93	1765.	1938.	.053	.056	.074	.073
10953	Kaufbeuren, DL	47.87	10.63	2388.	2513.	.069	.062	.080	.076
10875	Muhldorf, GER	48.25	12.55	1319.	1625.	.053	.053	.077	.076

Exhibit 3.4

RMS of Sample Re-use and Overall Fit RMS by Station Over all Month and Hour Periods 3 through 8 for the Variables Model Based on Non-Linear Regression and Weighted Least Squares

Hour Period
(LST)

Month	06-08	09-11	12-14	15-17	18-20	21-23	all
January	.106 .123	.089 .111	.086 .103	.083 .105	.100 .121	.140 .169	.104 .122
February	.088 .104	.074 .087	.061 .080	.060 .081	.079 .102	.120 .155	.083 .102
March	.075 .092	.066 .086	.056 .085	.049 .080	.060 .095	.085 .140	.066 .096
April	.069 .089	.058 .081	.041 .076	.035 .078	.042 .088	.053 .121	.051 .089
May	.068 .092	.048 .094	.031 .072	.026 .066	.030 .086	.048 .122	.044 .089
June	.070 .093	.048 .088	.031 .087	.026 .064	.032 .080	.047 .105	.095 .086
July	.071 .099	.046 .082	.029 .065	.023 .052	.030 .067	.047 .103	.044 .078
August	.082 .098	.053 .067	.036 .063	.028 .043	.035 .062	.063 .170	.053 .084
September	.095 .109	.063 .075	.050 .104	.036 .038	.054 .089	.073 .115	.065 .088
October	.087 .094	.071 .078	.057 .074	.052 .074	.081 .106	.111 .143	.079 .095
November	.082 .094	.076 .087	.064 .086	.065 .088	.087 .105	.115 .143	.083 .101
December	.089 .106	.087 .101	.079 .102	.080 .103	.090 .112	.142 .171	.097 .116
All	.083 .099	.068 .086	.055 .083	.051 .073	.065 .093	.094 .138	.071 .095

Exhibit 3.5

RMS by Month and Hour for Overall Fit of All 60 Germany Stations Using the Variables Model by Non-Linear Regression (top entry) and Weighted Least Squares (bottom entry).

Hour Period
(LST)

Month	06-08	09-11	12-14	15-17	18-20	21-23	all
January	.118 .130	.111 .119	.100 .113	.099 .114	.115 .131	.150 .176	.117 .131
February	.102 .113	.089 .096	.081 .094	.081 .094	.097 .112	.132 .162	.097 .112
March	.092 .101	.086 .098	.079 .099	.074 .094	.084 .106	.098 .149	.086 .112
April	.089 .099	.078 .095	.066 .090	.064 .086	.072 .094	.080 .128	.075 .099
May	.085 .108	.060 .124	.040 .094	.038 .078	.052 .104	.069 .136	.060 .107
June	.085 .106	.058 .114	.041 .120	.046 .088	.057 .100	.074 .122	.062 .108
July	.087 .115	.055 .113	.036 .096	.041 .071	.056 .086	.074 .123	.061 .101
August	.099 .106	.069 .091	.056 .105	.050 .062	.062 .078	.088 .172	.073 .102
September	.105 .118	.079 .096	.067 .138	.060 .085	.079 .107	.097 .126	.083 .112
October	.096 .102	.085 .084	.078 .094	.078 .091	.102 .117	.126 .150	.096 .107
November	.094 .104	.090 .097	.074 .099	.083 .100	.102 .114	.125 .150	.096 .111
December	.104 .117	.102 .114	.096 .116	.097 .115	.107 .124	.151 .179	.111 .128
All	.097 .110	.082 .104	.071 .105	.070 .090	.085 .106	.109 .148	.087 .111

Exhibit 3.6

RMS by Month and Hour for Sample Reuse of All 60 German Stations Using the Variables Model by Non-Linear Regression (top entry) and Weighted Least Squares (bottom entry).

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