Influence of Magnetic Shear on the Collisional Current Driven Ion Cyclotron Instability

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The effects of magnetic shear on the collisional current driven ion cyclotron instability are discussed by incorporating the self-consistent magnetic shear produced by the currents driving the instability itself. It is found that large currents ($J \geq \text{ma/m}^2$) are needed to significantly modify the properties of the local instability. The wave packet localizes around $k_x/k_y \approx 1$, thereby indicating that the mode is propagating almost perpendicular to the magnetic field. Under auroral conditions ($J \sim 10 \mu\text{A/m}^2$, $L_x \sim 500 \text{ km}$) the mode is confined to a region of a few hundred meters along the direction of the magnetic field and its local growth rate is unaffected by the magnetic shear.
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16. SUPPLEMENTARY NOTATION (Continued)

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INFLUENCE OF MAGNETIC SHEAR ON THE COLLISIONAL CURRENT DRIVEN ION CYCLOTRON INSTABILITY

I. INTRODUCTION

Intense field aligned currents observed in the auroral E and F regions (Park and Cloutier, 1971, Cahill et al., 1974, Kelley et al., 1975) were conjectured to be responsible for the generation of electrostatic ion cyclotron waves. Since the ionosphere is collisional in the E and F regions the field aligned currents give rise to negative energy waves which grow due to the dissipation associated with electron-neutral collisions (Chaturvedi, 1976). These are electrostatic waves and lead to density irregularities in the medium. In fact, radar measurements in the E region detect irregularities collocated with auroral electrojets (Greenwald et al., 1975), and a.c. field measurements (Kelley et al., 1975) have been identified as being due to unstable ion cyclotron waves (Drummond and Rosenbluth, 1962—collisionless domain; Kindel and Kennel, 1971—weakly collisional domain). Sounding rockets launched from Syowa station, detected transversely propagating electrostatic plasma instabilities of scale sizes around 200 km in association with strong field aligned currents (Ogawa et al., 1981). Chaturvedi (1976), using a local fluid analysis, showed that, in a partially ionized plasma, field aligned currents can support almost transversely propagating ion cyclotron waves owing to resistivity experienced by electron parallel motion (electron-neutral collisions), while the ion-neutral collisions were found to have a mild stabilizing effect. The field aligned currents observed are usually ~ 1-10 μA/m² (Kelley et al., 1975), although recently Burke et al. (1983) reported observations of large currents ~ 135 μA/m². These field aligned currents can generate magnetic shear which in turn affect the mode structure. This is especially so in the case of collisionless current driven ion cyclotron waves (Ganguli and Bakshi, 1982).

In this brief report we investigate the effects of a self-consistent magnetic shear on the collisional current driven ion cyclotron instability (CIC). We find that the magnetic shear has no noticeable stabilizing effects on the current driven ion cyclotron instability in the domains of interest pertaining to auroral conditions (characterized by \( V_d \sim 0.5 - 5 \) km/s, corresponding to currents of order 10 - 100 μA/m², and \( v_e \sim 10^4 \) s⁻¹, where
V_d is the electron drift velocity along the magnetic field and \( v_e \) is the electron-neutral collision frequency. This is because the self-consistent magnetic shear produced by the currents observed in the auroral region is very small. This result is in contrast to the major effects found by Ganguli and Bakshi (1982) in the collisionless domain. Their kinetic treatment shows that, due to the nonlocal boundary conditions, magnetic shear affects the collisionless current driven ion cyclotron instability in two ways: One, due to an explicit shear dependent term of \( O(p_s/L_s) \) which vanishes for \( p_s/L_s \to 0 \). Two, due to a shear independent term containing the derivatives of the potential function defined in equation 18. In the fluid example, the second term happens to be zero (since the derivative of \( Q \), defined in Eq. 18, vanishes at \( x_0 \) which is defined in Eq. 19) and hence the lack of the significant effects in the \( p_s/L_s \to 0 \) limit. Furthermore, for \( p_s/L_s \ll 1 \) magnetic shear localizes the wave packet around \( k_z/k_y \ll 1 \), indicating that the mode is almost perpendicularly propagating, and stronger magnetic shear stabilizes the instability. The paper is organized as follows. In the next section we give the theory and derive the nonlocal mode structure equation. In the third section we present the results, and in the last section we apply the results to the auroral ionosphere and discuss future work.

II. THEORY

We consider a partially ionized plasma such as the one encountered in the auroral E region. The geometry used in this analysis is as follows. The magnetic field is aligned with the z-direction along which flow the currents that drive the CIC. Because of the self-consistent shear produced by the field aligned currents, the magnetic field acquires a component along the y-direction as \( x \) is varied,

\[
B(x) = B \left( \hat{z} + x/L_s \hat{y} \right).
\]

This implies that the parallel wave vector becomes a function of \( x \),

\[
k_z = k_z^0 + k_y(x/L_s),
\]

(2)
where $L_s$ is the shear length. The self-consistent shear is calculated from the Maxwell's equation

$$\nabla \times B = \frac{4\pi}{c} j = \frac{4\pi}{c} (n e V_d)$$

where $V_d$ is the drift velocity parallel to the z-direction, and is given by

$$L_s = \alpha^{-1} \rho_s (c_s / V_d)$$

where $c_s = (T_e / m_i)^{1/2}$, $\rho_s = c_s / \Omega_i$, $\Omega_i$ is the ion cyclotron frequency, and

$$\alpha = \left( c_s / c \right)^2 \left( M / m \right) (\omega^2 / \Omega_e^2)$$

where $\omega$ and $\Omega$ are the electron plasma and cyclotron frequencies, respectively, $c$ is the speed of light, and $M$ and $m$ are the ion (NO+) and electron masses, respectively.

The basic equations describing the problem are as follows. The electron and ion continuity equations are given by

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha V_\alpha) = 0$$

where $\alpha$ indicates the species. The equation of motion for the ions is,

$$M n \frac{d}{dt} V_i = - \nabla p_i + e n (\nabla \phi + \frac{V_i \times B}{c}) - M n v_i V_i,$$

while the equation of motion for electrons is,

$$0 = - \nabla p_e - e n (\nabla \phi + \frac{V_e \times B}{c}) - m n v_e V_e,$$

where $v_e$ and $v_i$ are electron-neutral and ion-neutral collision frequencies, respectively, $p_\alpha = n_\alpha T_\alpha$ is the pressure of species $\alpha$, and $d/dt = \partial / \partial t + V \cdot \nabla$. In this paper we ignore the electron inertia, and assume that $v_e / \Omega_e \ll 1$ and $v_i / \Omega_i \ll 1$. We also ignore the viscosity and thermal conductivity terms because these terms only introduce a numerical constant that multiplies the growth rate and do not introduce any additional $k_z / k_y$ terms (see, for example, Chaturvedi and Kaw, 1975). We also ignore the electromagnetic effects and consider only to electrostatic perturbations. We derive the mode structure equation assuming that the perturbed quantities
vary as \( f \sim f(x) \exp \left[ -i(\omega t - k_y y - k_z z) \right] \). We note that we do not Fourier analyze in the \( x \) direction since the magnetic field is assumed to vary in that direction.

From Eq. (8), we obtain the perpendicular and parallel (to \( B \)) components of the electron velocity

\[
V_{el} = \frac{1}{n_e} \frac{1}{1 + \nu^2 / \Omega_e^2} \left[ \left( \frac{V_{pe}}{m} - e \frac{V_{i \phi}}{m} \right) \times \hat{z} \right]
- \left( \frac{V_{pe}}{m} - e \frac{V_{i \phi}}{m} \right) \frac{e}{n_e} \frac{v}{e}
\]  
\( V_{ez} = -\frac{1}{v_e} \left[ \frac{\nu_p e}{m} - e \frac{V_{i \phi}}{m} \right]. \]  

From Eq. (10) we define

\[
V_d \equiv \nu_{ez} = \frac{\Omega_e}{v_e} \left( \frac{C}{B} \nu_{z \phi}^0 \right).
\]

From Eq. (7) we obtain for the ions

\[
V_{il} = \frac{i\omega / \Omega_i}{1 + (i\omega + \nu_i^2 / \Omega_i^2)} \left[ \left( \frac{C}{B} \nu_{i \phi} + \frac{1}{n_i} \frac{V_{pe}}{M} \right) \times \hat{z} \right]
\]  
and

\[
V_{iz} = -\frac{1}{i\omega} \left[ \frac{\nu_p e}{m} - e \frac{V_{i \phi}}{m} \right]
\]

where we have assumed \( T_e \sim T_i \), and \( \phi = \phi^0 + \hat{\phi} \) (\( \hat{\phi} \) being the perturbed potential). Substituting Eqs. (9) thru (13) in Eq. (6) we obtain, after subtracting the electron continuity equation from the ion continuity equation, the nonlocal mode structure equation

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{k_z^2}{c_s^2} \phi = 0,
\]

where \( k_z = k_z^0 + k_y x / L_s \) and where quasi-neutrality has been assumed. The local limit of Eq. (14) \((L_s \rightarrow \infty \text{ and } \partial / \partial x \ll k_y)\) yields \( Q(x) = 0 \), where \( Q(x) \)
are the second two terms in the above equation. This gives the results obtained previously by Chaturvedi (1976). The linear local growth rate is

$$\gamma = -\frac{2}{2M} \frac{k_y^2}{k_z^2} \left( 1 - \frac{k \cdot v_d}{\omega} \right) v_e - \frac{v_1}{2}$$  \hspace{1cm} (15)$$

where

$$\omega^2 = \frac{\omega_1^2}{r} + k_y^2 c_s^2.$$ 

$$\omega = \omega_1 + i\gamma$$

Eq. (15) leads to the threshold condition

$$k \cdot v_d > \omega(1 + \frac{M}{m} \frac{k_z^2}{k_y^2} \frac{v_1}{v_e}).$$  \hspace{1cm} (16)$$

Note that the nonlocal treatment allows for the formation of a wave packet in the x-direction compared with the plane wave [exp(ikx)] solutions of Chaturvedi (1976).

We solve Eq. (14) analytically and numerically to determine the eigenvalues and eigenfunctions. We follow the techniques developed by Ganguli and Bakshi (1982) to solve Eq. (14) as

$$\frac{d^2 \phi}{dx^2} + Q(x) \phi = 0,$$  \hspace{1cm} (17)$$

where

$$Q(x) = \frac{1}{k_y c_s} \left[ (\omega^2 - \omega_1^2 - k_y^2 c_s^2) + i\omega_1 \frac{k_x^2}{k_y} \frac{x^2}{L^2} + i\frac{M}{m} \frac{k_z^2}{k_y} \frac{v_1}{v_e} (\omega - k_y V_d x/L_s) \frac{k_z^2}{k_y c_s x^2/L^2} \frac{x^2}{L^2} \right.$$ 

$$\left. - i\frac{M}{m} \frac{k_z^2}{k_y} \frac{v_1}{v_e} (\omega - k_y V_d x/L_s) \right] \frac{k_z^2}{k_y c_s x^2/L^2} \frac{x^2}{L^2}.$$  \hspace{1cm} (18)$$

We expand Q(x) around $\xi = x - x_o = 0$ to $O(\xi^2)$ where $x_o$ is the value of x for which the local growth rate is a maximum,

$$\frac{x_o}{\rho_s} = \frac{2 \omega}{k_y V_d}$$  \hspace{1cm} (19)$$
Now, we have

\[ \gamma^2 \frac{\partial^2 \phi}{\partial \eta^2} + \left( a + \frac{Q''_0}{2} \eta^2 \right) \phi(\eta) = 0 \tag{20} \]

where \( \eta = \xi + Q''_0/Q_0'' \) and \( a = Q_0 - Q''_0/2Q_0'' \). The primes indicate the derivatives with respect to \( x \) evaluated at \( x = x_0 \).

Equation (20), which is in the form of a Weber equation, has the solution (for the \( \ell = 0 \) mode)

\[ \phi(x) = \phi \exp\left[ -\frac{1}{2} (x - x_{m0})^2/x_m^2 \right] \exp\left[ i q(x) \right] \tag{21} \]

where \( \ell \) is the order of the mode and \( q(x) \) is a real function. The wavepacket peaks at (Ganguli and Bakshi, 1982)

\[ x_{m0} = x_0 - \frac{\text{Re}[Q''_0/Q_0''(-Q''_0/2)]^{1/2}}{\text{Re}(-Q''_0/2)^{1/2}} \tag{22} \]

The dispersion relation is given by

\[ Q(x_0) = (2\ell + 1) \left( -\frac{Q''_0}{2} \right)^{1/2} \rho_s/L_s + \frac{(Q''_0)^2}{2Q''_0}. \tag{23} \]

It can be easily shown from Eqs. (18) and (19) that \( Q''_0(x_0) = 0 \). Thus, Eq. (23) reduces, for the \( \ell = 0 \) mode, to

\[ Q(x_0) = \left[ -\frac{Q''_0}{2} \right]^{1/2} \rho_s/L_s. \tag{24} \]

Equation (23) is analogous to the dispersion relation obtained by Ganguli and Bakshi (1982). In Eq. (23) the nonlocal effects are contained in the two terms on the right hand side, one of which contains an explicit shear dependent term and the other contains a shear independent nonlocal term. This equation shows that in the \( \rho_s/L_s \rightarrow 0 \) limit the term containing \( Q''_0 \) could significantly modify the growth rate. In fact, this shear independent nonlocal term drastically reduces the growth rate in the collisionless kinetic case. Whereas, Eq. (24) shows that in the fluid case any stabilizing influence by the magnetic shear is due only from the shear dependent term which indicates that large magnetic shear is probably needed
to stabilize the instability. In fact, the results given in the next section support this observation. Equation (24) is solved to obtain the eigenfrequencies and the results are presented in the next section.

III. RESULTS

We solve the dispersion equation given by Eq. (24) numerically in various parameter domains. These results are verified by directly solving Eq. (14) using a numerical shooting code. We present the former results in the following. In Fig. 1 we plot the growth rate normalized to the ion cyclotron frequency \( \nu_s/\Omega_i \) versus the normalized wavenumber \( k_p \) for \( V_{d,c} = 50, \nu_e/\Omega_i = 0.01, \) and \( \nu_i/\Omega_i = 0.01. \) In plotting these results we treated \( \rho_s/L_s \) as an external parameter. The solid line represents the shear-free case and the dashed line represents the case where \( \rho_s/L_s = 0.01 \).

We see from the figure that modes with small \( k_p \) are not strongly affected by the magnetic shear. The growth rate for \( k_p = 0.3 \) reduces from 0.24 to 0.20 for \( \rho_s/L_s = 0.01, \) whereas the growth rate for \( k_p = 0.7 \) reduces by about 30% from 0.75 to 0.55.

In order to throw some light on the magnetic shear required to significantly reduce the growth rate, we plot in Fig. 2, the normalized growth rate versus the normalized shear length \( \rho_s/L_s \), treating \( \rho_s/L_s \) as an external parameter. We choose \( k_p = 0.3, \) and 0.5 and plot the growth rate for the parameters of Fig. 1. This figure shows that moderate to strong magnetic shears, \( \rho_s/L_s > 0.01, \) are required to reduce the growth rate significantly. For example, for \( k_p = 0.3 \) the growth rate is reduced by about 50% for \( \rho_s/L_s = 0.04 \) and the mode is stabilized for \( \rho_s/L_s > 0.065, \) and for \( k_p = 0.5 \) the growth rate is reduced by 50% for \( \rho_s/L_s = 0.04 \) but the mode is stabilized for \( \rho_s/L_s = 0.078. \) However, the theory begins to break down for large shear value \( \rho_s/L_s > 0.1. \) We find that for large shears, the growth rate drops linearly as a function of inverse shear length. For very small shears, \( \rho_s/L_s < 10^{-4} \) the growth rate remains essentially constant and the growth rate decreases noticeably for \( \rho_s/L_s > 0.01. \) In an ionospheric environment \( B = 0.5 \) Gauss and \( \rho_s = 2.5 \) meters, and Eqs. (1) and (3) show that \( \rho_s/L_s = 0.01 \) and 1.0 correspond to currents of 0.1 A/m² and 1 A/m², respectively.
In Fig. 3 we present the wave eigenfunctions for the following parameters: \( V_d = 50 \, c_s \), \( \nu_e/\Omega_e = 10^{-2} \), \( \nu_i/\Omega_i = 10^{-2} \) and for \( k_y s = 0.3 \), \( \gamma/\Omega_i = 0.24 \). The solid line represents the real part of the wave function \( \psi \) and the dashed line \( |\psi| \). Two important features are to be noted in this figure. One feature is that the wavepacket localizes around \( x_0/p_s \ll 1 \), i.e., for small shear lengths \( k_z/k_y \ll 1 \), indicating that the mode is almost perpendicularly propagating. This can also be seen from Eq. (19) which yields

\[
\frac{k_z}{k_y} \sim 2(\omega k_y V_d)(\rho_s/L_s) \ll 1 \quad \text{for} \quad \rho_s/L_s \sim 10^{-7}.
\]

The second feature is that the width of the wave packet is of the order 200 \( \rho_s \) suggesting a localization region of 500 meters for \( \rho_s = 2.5 \) meters (corresponding to \( \text{NO}^+ \) ions in the ionosphere).

The effect of self-consistent magnetic shear is understood by plotting the normalized growth rate versus the normalized drift velocity. For this purpose, we use Eq. (4) to express the shear length in terms of the drift velocity and solve Eq. (24). In figure 4, we plot the growth rate for \( a = 10^{-6}, 10^{-5}, \) and \( 10^{-4} \) (\( \omega^2/\omega_p c_e^2 = 10^2, 10^3, \) and \( 10^4 \), respectively; \( c_s = 500 \, \text{m/s} \)) and for \( \nu_e/\Omega_e = 0.01, \nu_i/\Omega_i = 0.01, \) and \( k_y \rho_s = 0.3 \). We also give the shear-free local growth rate (curve A) to compare with the growth rate with shear. Three points are to be noted in this figure: (1) \( a = 10^{-6} \) corresponds to ionospheric parameters. We find that the growth rate is not too different from the shear-free case. In fact, the growth rate curve overlaps curve A and eventually turns around for \( V_d/c_s > 500 \). (2) Curve B, which gives the growth rate for \( a = 10^{-5} \), shows that the growth rate drops significantly beyond drift velocities \( > 150 \, c_s \); and (3) curve C, which represents the growth rate for \( a = 10^{-4} \), shows that the optimum drift velocity (the drift velocity for which the growth rate is a maximum), 80 \( c_s \), is much smaller for this case. This leads to the conclusion that the optimum drift velocity decreases as the parameter \( a \) is increased. At large drift velocities, the self-consistent magnetic shear produced is large enough that the mode is stabilized and the growth rate tends to zero.
IV. DISCUSSION

We now apply the results to the high latitude ionospheric E region. We choose \( \nu_i/\Omega_i = 10^{-2} \), \( \nu_e/\Omega_e = 10^{-2} \), and \( V_d \sim 20 \, c_s \). For \( V_d = 20 \, c_s \) the corresponding shear length is about 1000 km (NO\(^+\) ions, \( n \sim 10^5 \, \text{cm}^{-3} \)). We find that the growth rate is \( 0.9 \, \text{s}^{-1} \), for modes with wavelengths of 50 meters (\( k_y \rho_s = 0.3 \), and \( \rho_s = 2.5 \, \text{meters} \)), which is comparable with the local growth rate. From the plots of the wave functions we see that the mode is localized in the north-south direction in a region of \( \Delta x \sim 200 \, \rho_s = 500 \, \text{m} \). Moreover, we find that \( k_y > k_x (k_x \sim 1/\Delta x) \) which indicates the strong two dimensional structure of the mode in the plane perpendicular to the magnetic field during the linear phase of the instability.

We conclude that the magnetic shear corresponding to normal auroral conditions with field aligned currents of the order of \( \mu A/m^2 \) is small (\( L_s \sim 1000 \, \text{km} \)) and thus does not appreciably alter the current driven collisional ion cyclotron mode discussed by Chaturvedi (1976). Even strong currents under some disturbed conditions, \( \sim 135 \, \mu A/m^2 \), reported by Burke et al. (1983) do not produce significant shear (\( L_s \sim 300 \, \text{km} \)) in the magnetic field to have any effect on the CIC. Currents of the order \( mA/m^2 \) or more possibly produce significant effects.

Finally, we discuss the limitations of the present theory. (1) The mode structure equation (Eq. 18) and the subsequent dispersion relation (Eq. 23) were derived under the assumption that \( k_z/k_y < 1 \) based on the premise that the ion parallel motion \([V \cdot (V_{ii}) \text{ term in the ion continuity equation}]\) may not be important. Thus the results presented do not contain ion parallel motion effects. However, we extended the theory to include the ion parallel motion and found no significant changes in the way the magnetic shear affects the mode. The local growth rate (in the limit \( \rho_s/L_s \sim 0 \)) with the ion parallel motion is smaller by about 10%; the growth rate with magnetic shear is consistently smaller but has similar behaviour as shown earlier. (2) For larger shears, \( \rho_s/L_s > 0.1 \), the parabolic expansion of the potential function (Eq. 23), used to derive the analytical dispersion relation, may not be adequate. This is due to the fact that the wave function spreads out and samples non-parabolic part of the potential. Numerical solutions of (Eq. 18) do confirm the spreading of the wave packet. Furthermore, the wave packet localizes in the region where the effective \( k_z/k_y \sim 1 \) because magnetic shear introduces an effective \( k_z \). (3)
We have done an MHD analysis and arrived at the mode structure equation with a potential term, $Q(x)$ [Eq. (18)], whose derivative vanishes at $x_0$, whereas, in the collisionless case the derivative of $Q(x)$ at $x_0$ is finite, thus drastically affecting the collisionless kinetic ion cyclotron instability (Ganguli and Bakshi, 1982). To further examine the above three aspects we will present the kinetic analysis of the collisional ion cyclotron waves in the presence of magnetic shear in a future paper. For higher drift velocities (comparable to the $E \times B$ drift velocities) where the self-consistent magnetic shear plays a significant role in determining the mode structure, a complete treatment is needed which includes an electric field along the $y$-direction and the related velocity shear.

In conclusion, we have examined the influence of magnetic shear on the collisional current driven ion cyclotron instability. Self consistent magnetic shear corresponding to moderate drift velocities ($V_d \sim 0.5 - 5$ km/s) near the threshold velocity for the instability does not have significant effects on the instability. We find that the mode is almost perpendicularly propagating and is localized in a region extending up to a few hundred kilometers in the direction of the magnetic field under auroral conditions. However, in domains where the plasma density is high such that the parameter $\omega^2_{pe}/\omega^2_{ce}$ is large, we find that the growth rate of a particular mode maximizes at an optimum drift velocity much larger than the sound speed. Finally, we find that strong shears ($\rho_s/L_s > 0.05$) significantly reduce the growth rate and stabilize the collisional current driven ion cyclotron instability. This strong shear corresponds to large parallel drift velocities ($V_d \gg c_s$), as seen in fig. 4.

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Figure 1. Plot of normalized growth rate ($\gamma/\Omega_1$) versus the normalized wave number ($k_y \rho_s$). The parameters used are $\nu_e/\Omega_e = 0.01$, $\nu_i/\Omega_i = 0.01$, and $V_d/c_s = 50$. The solid line represents the shear free case, and the dashed line represents the case where $\rho_s/L_s = 0.01$. 
Figure 2. Normalized growth rate ($\gamma/\Omega_1$) versus $\rho_s/L_s$ treating $\rho_s/L_s$ as an external parameter for $v_d/c_s = 50$, $k_y \rho_s = 0.3$, and 0.5, and for $v_e/\Omega_e = 0.01$, and $v_1/\Omega_1 = 0.01$. 
Figure 3. Plot of wave eigenfunctions for the same parameters as Fig. 1 and for $k_y \rho_s = 0.3$, $\gamma/\Omega_L = 0.25$, and $\rho_s/L_s = 0.001$. 
Figure 4. Growth rate versus the drift velocity curves with and without self-consistent magnetic shear for the following parameters: $k \rho_s = 0.3, \nu/e_\Omega = 0.01$, and $\nu_i/e_\Omega = 0.01$. Curve A represents the shear-free case and $\alpha = 10^{-6}$. Curves B and C are for $\alpha = 10^{-5}$ and $10^{-4}$, respectively, and these correspond to $\omega_{pe}/e_{ce} = 10^3$ and $10^4$. 
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