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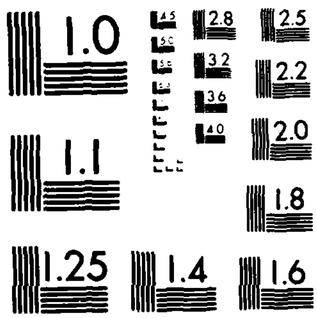

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MODIFIED 5-POINT AND 9-POINT SCHEMES FOR THE SOLUTION OF THE VORTICITY-TRANSPORT EQUATION WITH CROSS DERIVATIVES

S. Abdallah

Technical Memorandum
File No. TM 84-28
6 February 1984
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The method of Ref. [1] is modified to include cross derivatives in the vorticity-transport equation. Two modified versions of the 5-point and 9-point schemes are derived for elliptic and parabolic type partial differential equations.

From: S. Abdallah

Subject: Modified 5-Point and 9-Point Schemes for the Solution of THE Vorticity-Transport Equation with Cross Derivatives

References: See p. 8

Abstract: The method of Ref. [1] is modified to include cross derivatives in the vorticty-transport equation. Two modified versions of the 5-point and 9-point schemes are derived for elliptic and parabolic type partial differential equations.

INTRODUCTION

Cross derivatives are generated in the vorticity-transport equation when it is transformed to non-orthogonal coordinates. The presence of cross derivatives in the equation cause some difficulties in the method of Ref. [1]. More specifically, they arise in the integration of the vorticity diffusion form over the control volumes. These difficulties are eliminated by modifying the diffusion form of the equation and the integrating factor K. This treatment is suitable for both elliptic and parabolic type partial differential equations. Details of the derivation are given in the following sections.

ANALYSIS

The vorticity-transport equation in non-orthogonal coordinates can be written in the following form:

$$\bar{a}\omega_{xx} + \bar{b}\omega_{xy} + \bar{c}\omega_{yy} - \bar{d}\omega_{x} - \bar{e}\omega_{y} = 0 \quad , \tag{1}$$

where ω is the vorticity, $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ and $\bar{e}$ are functions of x and y. For both elliptic and parabolic partial differential equations, $\bar{b}^2 - 4\bar{a}\bar{c} \leq 0$.

Direct application of the method of Ref. [1] to Eq. (1) yields the following diffusion form:

$$\left(Ka\omega_x\right)_x + \left(Kc\omega_y\right)_y + Kb\omega_{xy} = 0 \quad , \tag{2}$$

where

$$K = -\frac{d}{a}x - \frac{e}{c}y \text{ , a and c} \neq 0 \quad . \tag{2a}$$

a, b, c, d and e are the linearized values of the coefficients $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ and $\bar{e}$ respectively.

The integration of the third term in Eq. (2) over the control volume of Fig. 1 does not yield a suitable form for the 5-point and 9-point schemes [1].

The diffusion form, Eq. (2), is modified to eliminate the integration difficulties encountered in the cross derivatives term.

The first and second order derivatives with respect to x are combined together in one term using the integrating factor in the x-direction.

$$\text{Exp}\left(\frac{d}{a}x\right)\left[\text{Exp}\left(-\frac{d}{a}x\right)a\omega_x\right]_x + b\omega_{xy} + c\omega_{yy} - e\omega_y = 0 \quad . \tag{3a}$$

Equation (3a) is rewritten as

$$\mathrm{Exp}\left(\frac{d}{a}x\right)\left[\mathrm{Exp}\left(-\frac{d}{a}x\right)\left(a\omega_x + b\omega_y\right)\right]_x + \frac{d}{a}b\omega_y + c\omega_{yy} - e\omega_y = 0 \quad . \quad (3b)$$

The last three terms in Eq. (3b) are combined together in one term using the integrating factor in the y-direction.

$$\mathrm{Exp}\left(\frac{d}{a}x\right)\left[\mathrm{Exp}\left(-\frac{d}{a}x\right)\left(a\omega_x + b\omega_y\right)\right]_x +$$

$$\mathrm{Exp}\left(\frac{ea-db}{ac}y\right)\left[\mathrm{Exp}\left(-\frac{ea-db}{ac}y\right)c\omega_y\right]_y = 0 \quad . \qquad (3c)$$

By multiplying and dividing the first term in Eq. (3c) by $\mathrm{Exp}\left(-\frac{ea-db}{ac}y\right)$ and the second term by $\mathrm{Exp}\left(-\frac{d}{a}x\right)$, one obtains

$$\left[K\left(a\omega_x + b\omega_y\right)\right]_x + \left[Kc\omega_y\right]_y = 0 \quad , \qquad (4)$$

where

$$K = -\frac{d}{a}x - \frac{ea-db}{ac}y \quad . \qquad (4a)$$

The modified diffusion form, Eq. (4), is then integrated over the control volume shown in Fig. 1 to obtain Eq. (5).

$$\int_{y=-\Delta y/2}^{y=\Delta y/2} K\left(a\omega_x + b\omega_y\right)\Big|_{x=\Delta x/2} dy - \int_{y=-\Delta y/2}^{y=\Delta y/2} K\left(a\omega_x + b\omega_y\right)\Big|_{x=-\Delta x/2} dy$$

$$+ \int_{x=-\Delta x/2}^{x=\Delta x/2} Kc\omega_y\Big|_{y=\Delta y/2} dx - \int_{x=-\Delta x/2}^{x=\Delta x/2} Kc\omega_y\Big|_{y=-\Delta y/2} dx = 0 \quad . \qquad (5)$$

The Modified 5-Point Scheme

The vorticity derivatives ω_x and ω_y at $x = \pm\ \Delta x/2$ and $y = \pm\ \Delta y/2$ are approximated using central finite difference equations. The vorticity derivatives ω_y at $x = \pm\ \Delta x/2$ are approximated as follows:

$$\omega_y\Big|_{\Delta x/2} = (\omega_{i+1,j+1} + \omega_{i,j+1} - \omega_{i+1,j-1} - \omega_{i,j-1})/4\Delta y \tag{6a}$$

$$\omega_y\Big|_{-\Delta x/2} = (\omega_{i,j+1} + \omega_{i-1,j+1} - \omega_{i,j-1} - \omega_{i-1,j-1})/4\Delta y \quad . \tag{6b}$$

Upon substitution into the integrated form of Eq. (4), i.e., Eq. (5), one obtains

$$(B\omega)_{i+1,j} + (B\omega)_{i-1,j} + (B\omega)_{i,j+1} + (B\omega)_{i,j-1} + (B\omega)_{i+1,j+1}$$

$$+ (B\omega)_{i+1,j-1} + (B\omega)_{i-1,j+1} + (B\omega)_{i-1,j-1} = (B\omega)_{i,j} \tag{7}$$

where

$$B_{i+1,j} = a\ [\text{Exp}\ (-\ p - q) - \text{Exp}\ (-\ p + q)/(2\Delta x^2 q) \tag{7a}$$

$$B_{i,j+1} = c\ [\text{Exp}\ (-\ p - q) - \text{Exp}\ (p - q)]/(2\Delta y^2 p)$$

$$+\ b\ [\text{Exp}\ (-\ p) - \text{Exp}\ (p)]\ [\text{Exp}\ (-\ q) - \text{Exp}\ (q)/(8\Delta x\Delta y q) \tag{7b}$$

$$B_{i+1,j+1} = b\ [\text{Exp}\ (-\ p - q) - \text{Exp}\ (-\ p + q)]/(8\Delta x\Delta y q) \tag{7c}$$

$$B_{i,j} = a\ [\text{Exp}\ (-\ p) + \text{Exp}\ (p)]\ [\text{Exp}\ (-\ q) - \text{Exp}\ (q)]/(2\Delta x^2 q)$$

$$+\ c\ [\text{Exp}\ (-\ q) + \text{Exp}\ (q)]\ [\text{Exp}\ (-\ p) - \text{Exp}\ (p)]/(2\Delta y^2 p) \tag{7d}$$

$$p = \frac{d\Delta x}{2a} \qquad (7e)$$

$$q = \frac{(ea - db)\Delta y}{2ac} \quad . \qquad (7f)$$

The rest of the coefficients in the above Eq. (7) can be obtained by *permutations of* (7a) through (7f).

The Modified 9-Point Scheme

The vorticity flux components ω_x and ω_y are approximated on the control volume surfaces using first order polynomials in x and y [1]. Also, the vorticity derivatives ω_y at $x = \pm\ \Delta x/2$ are approximated using Eqs. (6a) and (6b). Upon substitution in the integrated form, Eq. (5), we obtain the following equation:

$$(A\omega)_{i+1,j} + (A\omega)_{i-1,j} + (A\omega)_{i,j+1} + (A\omega)_{i,j-1} + (A\omega)_{i+1,j+1}$$

$$+ (A\omega)_{i+1,j-1} + (A\omega)_{i-1,j+1} + (A\omega)_{i-1,j-1} = (A\omega)_{i,j} \qquad (8)$$

where

$$A_{i+1,j} = a \text{ Exp } (-\ p) \frac{g(q)}{\Delta x^2} + 2c \cosh(q) \frac{f(p)}{\Delta y^2} \qquad (8a)$$

$$A_{i,j+1} = c \text{ Exp } (-\ q) \frac{g(p)}{\Delta y^2} + 2a \cosh(p) \frac{f(q)}{\Delta x^2} +$$

$$b \left[\text{Exp } (-\ p) - \text{Exp } (p)\right] \left[\text{Exp } (-\ q) - \text{Exp } (q)\right] / (8\Delta x \Delta y q) \qquad (8b)$$

$$- A_{i+1,j+1} = a \, \mathrm{Exp}\,(-p)\,\frac{f(q)}{\Delta x^2} + c \, \mathrm{Exp}\,(-q)\,\frac{f(p)}{\Delta y^2} \tag{8c}$$

$$- b\,[\mathrm{Exp}\,(-p-q) - \mathrm{Exp}\,(-p+q)]/(8\Delta x \Delta y q)$$

$$A_{i,j} = 2\,\left[\cosh(q)\,\frac{g(p)}{\Delta y^2} + \cosh(p)\,\frac{g(q)}{\Delta x^2}\right] \tag{8d}$$

$$f(t) = [(1+t)\,\mathrm{Exp}\,(-t) - 1]/(2t)^2 \tag{8e}$$

$$g(t) = [(1-t)\,\mathrm{Exp}\,(-t) + (1+t)\mathrm{Exp}(t) - 2]/(2t)^2 \quad . \tag{8f}$$

$g(t)$ is an even function in t; $g(t) = g(-t) > 0$ and $|g(t)| \geq |f(t)|$ for for all values of t.

REFERENCE

1. Abdallah, S., "Two finite difference schemes for the solution of the vorticity-transport equation at high Reynolds number," ARL/PSU Internal Memorandum, Applied Research Laboratory, The Pennsylvania State University (1984).

NOMENCLATURE

$\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$, $\bar{e}$	Coefficients defined in Eq. (1).
a, b, c, d, e	Linearized coefficients $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ and $\bar{e}$, respectively.
f, g	Functions defined in Eqs. (8e) and (8f).
K	Function defined in Eqs. (2) and (4).
p, q	Parameters defined in Eqs. (7e) and (7f).
x, y	Non-orthogonal coordinates.
ω	Vorticity.
Δx, Δy	Grid spacing in x and y directions, respectively.

Subscripts

x, y	Refer to derivatives in x and y directions, respectively.
i, j	Refer to grid lines in x and y directions, respectively.

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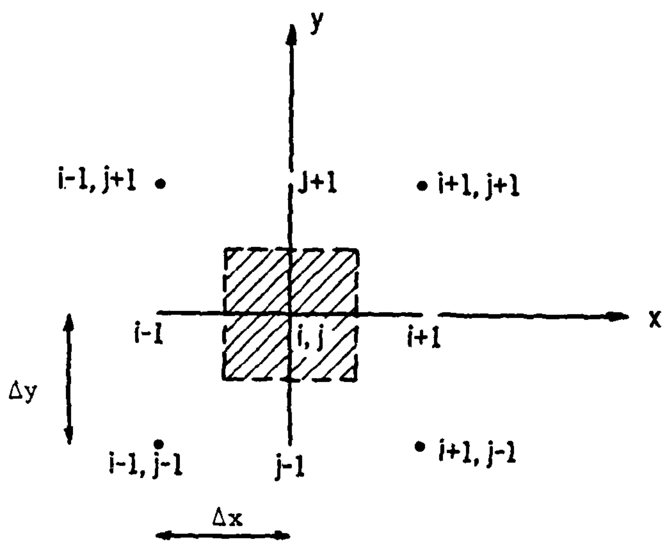

Figure 1. Schematic diagram for grid geometry and control volume.

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