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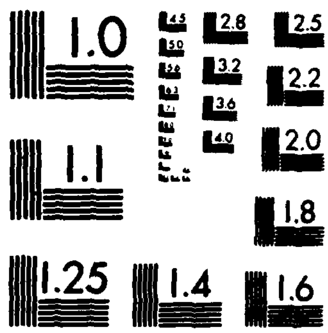
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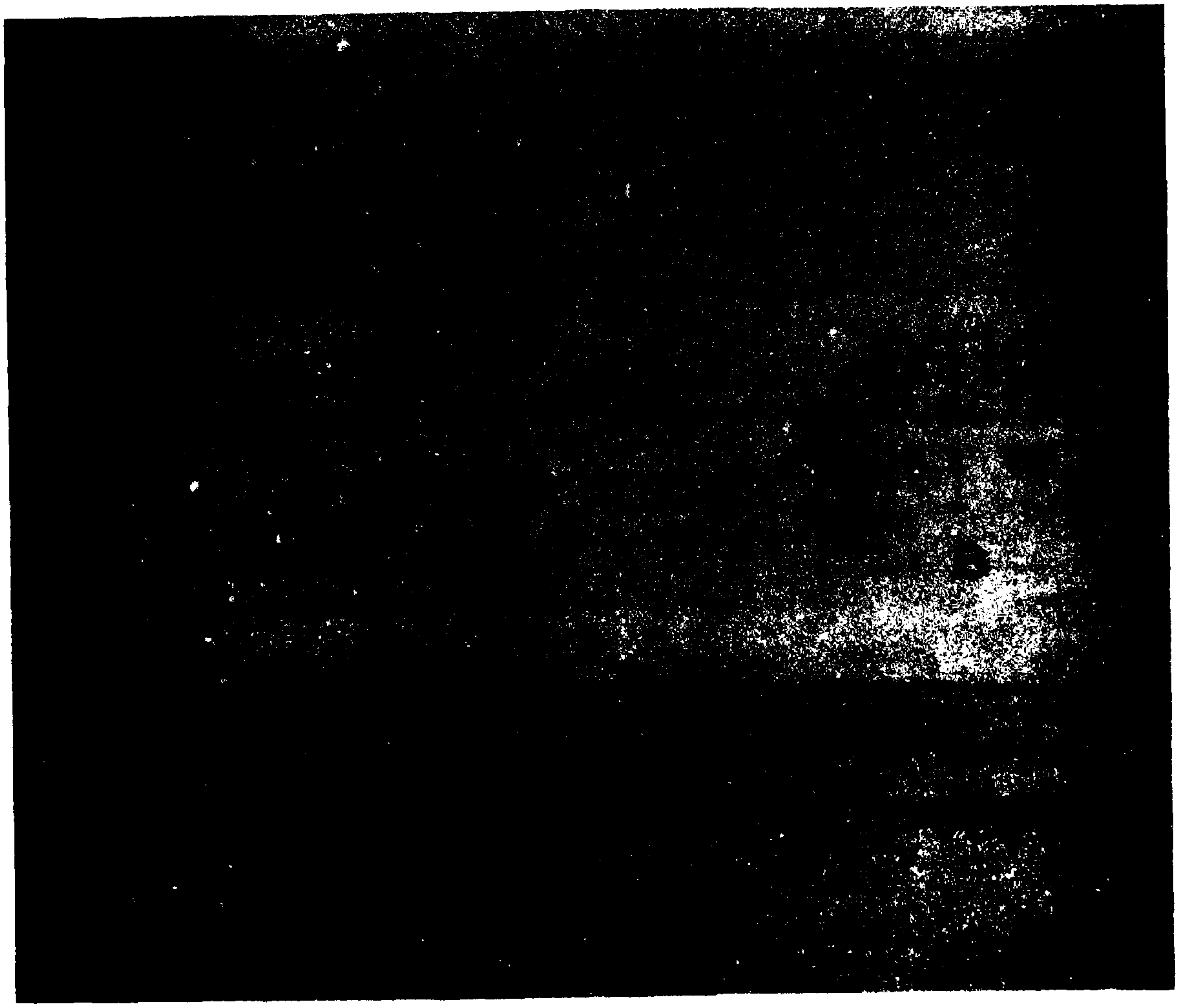

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by

Michael C. Loui and Gianfranco Bilardi

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THE CORRECTNESS OF TISON'S METHOD FOR GENERATING PRIME IMPLICANTS

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Abstract. Tison devised a generalized consensus method to generate the prime implicants of a switching function. We present a complete, rigorous proof of its correctness.

Index terms: Tison's method, generalized consensus, prime implicant, switching function.

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Tison [3,4] developed novel methods for determining all prime implicants and all irredundant sums for a switching function. These methods employ a generalized notion of consensus and do not require the minterms of the function, unlike the well known Quine-McCluskey tabulation procedure. Neither Tison's published paper [4] nor the textbook by Muroga [2] includes a complete proof of correctness. The available correctness proofs are either inaccessible [3] or lengthy [4]. Nevertheless, we refer the reader to thse two works for a comprehensive treatment of the methods.

In this note we establish the correctness of Tison's method for prime implicants; the correctness of the method for irredundant sums follows by a similar argument. We review a portion of Tison's paper [4] that shows that every generalized consensus can be formed via a sequence of consensus operations of order 2. The statement and proof of Theorem 2 below are new, however; they enable us to demonstrate that each biform variable can be treated _just once_. Because this note is self-contained, the reader need not have studied Tison's paper.

Let $S = xS_0$ be a product term (a conjunction of literals without duplication) with a variable x and $T = \bar{x}T_0$ be a product term with $\bar{x}$. If there is no variable y such that y appears in one of S_0, T_0 and $\bar{y}$ in the other, then S and T have _consensus_ $S_0 T_0$ _with respect to x_. (Duplicate occurrences of literals are omitted from the consensus.)

Let (P,Q,...,R) be a list of terms. The variable x is _monoform_ in this set if either x or $\bar{x}$ appears among the literals in P,Q,...,R, but not both x and $\bar{x}$. The variable x is _biform_ in this set if both x and $\bar{x}$ appear among P,Q,...,R.

Tison's Method

Let P + Q + ... + R be a sum-of-products expression for a function f. This procedure produces a list of all prime implicants of f.

Step 1. Initially, let L be the list (P, Q, ..., R). Throughout the computation, L is a list of implicants of f called the implicant list. At the completion of the computation, L is the list of all prime implicants of f.

Step 2. For each biform variable x in (P, Q, ..., R) perform Steps 2.1 and 2.2.

Step 2.1. For every pair of terms S, T on L, add to L the consensus of S, T, with respect to x, if such a consensus exists.

Step 2.2. Delete from L all terms S such that S implies another term on L.

Example. Let $\bar{w}x + \bar{y}z + wxy + xy\bar{z} + w\bar{x}yz$ be an expression for f(w,x,y,z). Initially,

$$L = (\bar{w}x,\ \bar{y}z,\ wxy,\ xy\bar{z},\ w\bar{x}yz).$$

We show the terms generated when the variables are processed in the order w,x,y,z.

w: The only consensus with respect to w is xy, which covers wxy and $xy\bar{z}$. (Recall that a product term P covers another product term Q if and only if every literal in P occurs in Q, equivalently, Q implies P.)

$$L = (\bar{w}x,\ xy,\ \bar{y}z,\ w\bar{x}yz).$$

x: The only consensus with respect to x is wyz, which covers $w\bar{x}yz$.

$$L = (\bar{w}x,\ xy,\ \bar{y}z,\ wyz).$$

y: The consensi with respect to y are xz and wz, and wz covers wyz.

$$L = (\bar{w}x,\ wz,\ xy,\ xz,\ \bar{y}z).$$

z: There are no consensi with respect to z on this last list L, which is the list of prime implicants of f.

According to the Consensus Theorem, if R and S have consensus T, then $R + S = R + S + T$, and T implies $R + S$. Thus, Tison's Method generates implicants of a function f. We prove that it produces precisely the set of prime implicants of f.

Let $T_1,\ldots,T_p$ be product terms. For each i let

B_i = product of literals of T_i that are biform in $(T_1,\ldots,T_p)$,

X_i = product of literals of T_i that are monoform in $(T_1,\ldots,T_p)$.

By definition, $T_i = B_i X_i$. Set

$$X = X_1 \ldots X_p, \tag{1}$$

omitting duplicate occurrences of literals. Call X the <u>generalized consensus</u> of $T_1,\ldots,T_p$ if

$$B_1 + \ldots + B_p = 1 \quad \text{(irr.)} \tag{2}$$

holds irredundantly. Write $X = GC(T_1,\ldots,T_p)$, and call p the <u>order</u> of X in $T_1,\ldots,T_p$.

<u>Theorem 1</u>. For all product terms $X, T_1, \ldots, T_p$, $X = GC(T_1,\ldots,T_p)$ if and only if

(i) $X \le T_1 + \ldots + T_p$;

(ii) the deletion of any literal in X invalidates (i); and

(iii) the deletion of any T_i invalidates (i) (the sum is irredundant).

<u>Proof</u>. We employ the notations B_i and X_i defined above.

<u>Necessity</u>.

(i) If $X = 1$, then every $X_i = 1$, hence every $T_i = B_i$, and $T_1 + \ldots + T_p = B_1 + \ldots + B_p = 1$.

(ii) Let $X = uY$ for a literal u, and suppose u occurs in $T_1,\ldots,T_m$. We assert that $Y \not\le T_1 + \ldots + T_p$. When $Y = 1$ but $u = 0$,

$$T_1 + \ldots + T_p = B_{m+1} + \ldots B_p \ne 1$$

because equation (2) holds irredundantly.

(iii) Suppose, to the contrary, $X \leq T_2 + \ldots + T_p$. Then when $X = 1$,

$$T_2 + \ldots + T_p = 1$$

implies

$$B_2 + \ldots + B_p = 1.$$

<u>Sufficiency</u>. Suppose X satisfies (i), (ii), (iii).

<u>Claim</u>: No variable in X is biform in $(T_1, \ldots, T_p)$. Suppose $X = xY$, $T_1 = \bar{x}T_1{}^*$ and $T_2 = xT_2{}^*$. If $X = 1$, then $x = 1$ and $T_1 = 0$. It follows from (i) that

$$X \leq T_2 + \ldots + T_p,$$

a contradiction of (iii).

<u>Claim</u>: X has all the monoform variables in $(T_1, \ldots, T_p)$. Suppose that the monoform variable x appears in $T_1, \ldots, T_m$ but not in X. Then when $X = 1$ and $x = 0$,

$$T_1 + \ldots + T_p = T_{m+1} + \ldots + T_p,$$

hence

$$X \leq T_{m+1} + \ldots + T_p,$$

again contravening (iii).

By (ii), X has no variables that do not occur among $(T_1, \ldots, T_p)$. Since X has precisely the monoform variables in $(T_1, \ldots, T_p)$, we can write X in the form (1). If $X = 1$, then every $X_i = 1$ and $T_i = B_i$, hence by (i),

$$B_1 + \ldots + B_p = 1.$$

The sum in this equation is irredundant, because if

$$B_2 + \ldots + B_p = 1,$$

then

$$X \leq T_2 + \ldots + T_p,$$

violating (iii). □

We employ the notations that lead to (1) and (2). Let $X = GC(T_1,\ldots,T_p)$. Let x be a biform variable in $(T_1,\ldots,T_p)$. Renumbering terms if necessary, suppose x occurs in $B_1,\ldots,B_m$, literal $\bar{x}$ in $B_{n+1},\ldots,B_p$, and neither x nor $\bar{x}$ in $B_{m+1},\ldots,B_n$, where $1 \le m \le n \le p-1$. Rewrite (2):

$$x \sum_{i=1}^{m} (B_i/x) + \sum_{i=m+1}^{n} B_i + \bar{x} \sum_{i=n+1}^{p} (B_i/\bar{x}) = 1 \quad \text{(irr.)}, \tag{3}$$

where B_i/x denotes the product of all literals of B_i except x. For $x = 1$ in (3),

$$\sum_{i=1}^{m} (B_i/x) + \sum_{i=m+1}^{n} B_i = 1, \tag{4}$$

and for $x = 0$ in (3)

$$\sum_{i=m+1}^{n} B_i + \sum_{i=n+1}^{p} (B_i/\bar{x}) = 1 \quad .$$

We contend that canceling redundant terms in these two equations yields (with renumbering of terms)

$$\sum_{i=1}^{m} (B_i/x) + \sum_{i=m+1}^{n'} B_i = 1 \quad \text{(irr.)}$$

$$\sum_{i=m'}^{n} B_i + \sum_{i=n+1}^{p} (B_i/\bar{x}) = 1 \quad \text{(irr.)}$$

with $m < m' \le n'+1$ and $n' \le n \le p-1$. For if to the contrary, some B_j/x were redundant in (4), then B_j would be redundant in (2). Furthermore, if to the contrary $m' > n'+1$, then $B_{n'+1}$ would be redundant in (2). Define

$$\begin{aligned} Y &= xX_1 \ldots X_{n'} = GC(T_1,\ldots,T_{n'}) \\ Z &= \bar{x}X_{m'} \ldots X_p = GC(T_{m'},\ldots,T_p) \ . \end{aligned} \tag{5}$$

Both Y and Z are generalized consensi of order at most $p-1$, and $X = GC(Y,Z)$ with respect to x.

From this discussion, it is clear that every generalized consensus can be formed via a sequence of consensus operations of order 2. We must demonstrate that in Step 2 of Tison's Method each biform variable can be treated just once.

Theorem 2. Let $T_1,\ldots,T_q$ be implicants of a function and $X = GC(T_1,\ldots,T_p)$ for some $p \le q$. If Tison's Method is started with $(T_1,\ldots,T_q)$, then it generates a product term that covers X.

Proof. During the computation of Tison's Method on $(T_1,\ldots,T_q)$, let $x_1,x_2,\ldots$ be the order in which the biform variables of $(T_1,\ldots,T_q)$ e used. Call the taking of consensi of order 2 with respect to x_i Stage i o ~his computation.

Let t be the smallest nonnegative integer for which all biform variables of $(T_1,\ldots,T_p)$ are among $x_1,\ldots,x_t$. We prove by induction on t that at the end of Stage t and during all subsequent Stages, the implicant list has a term that covers X. If $t = 0$, then $p = 1$ and $X = T_1$, and the result holds trivially.

Suppose $t \geq 1$. In the preceding discussion we established that $X = GC(Y,Z)$ with respect to $x = x_t$ for product terms $Y = GC(T_1,\ldots,T_{n'})$ and $Z = GC(T_{m'},\ldots,T_p)$ of the form (5). Let r and s be the smallest nonnegative integers such that all biform variables of $(T_1,\ldots,T_{n'})$ are among $x_1,\ldots,x_r$ and all biform variables of $(T_{m'},\ldots,T_p)$ are among $x_1,\ldots,x_s$. Because $x = x_t$ is monoform both in $(T_1,\ldots,T_{n'})$ and in $(T_{m'},\ldots,T_p)$ (by definition of m and n), $r < t$ and $s < t$.

By the inductive hypothesis, at the beginning of Stage t, the implicant list has a term Y^* that covers Y and a term Z^* that covers Z.

If Y^* does not contain x_t, then by construction of Y in (5) and X in (1), Y^* covers X. Similarly, if Z^* does not contain $\bar{x}_t$, then Z^* covers X. If Y^* contains x_t and Z^* contains $\bar{x}_t$, then by (1) and (5), the consensus $GC(Y^*,Z^*)$ generated during Stage t covers X. Ergo, at the end of Stage t, the implicant list has a term that covers X. Furthermore, since a term on the implicant list is deleted only when another term that covers it is on the list, after Stage t the implicant list always has a term that covers X. □

Suppose $f = T_1 + \ldots + T_q$ for some implicants $T_1,\ldots,T_q$ of f. Let P be a prime implicant of f. In the inequality

$$P \leq T_1 + \ldots + T_q,$$

redundant terms on the right side can be removed to form an irredundant sum; renumbering terms if necessary, we find

$$P \leq T_1 + \ldots + T_p \quad \text{(irr.)}$$

for some $p \leq q$. Because P is prime, no literal in the left side can be omitted from the left side of this inequality. By Theorem 1, $P = GC(T_1,\ldots,T_p)$. By Theorem 2, Tison's Method, when started on $(T_1,\ldots,T_q)$, generates a term that covers P; since P is prime, it generates P itself. Therefore, all prime implicants are generated. Consequently, every implicant on the list at the completion of the computation must be prime; otherwise, it would have been deleted in Step 2.2. We conclude that Tison's Method generates precisely the set of prime implicants of f.

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[4] P. Tison, Generalized consensus theory and application to the minimization of Boolean functions, IEEE Trans. Electronic Computers EC-16, 4 (Aug. 1967) 446-456.

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