TRANSONIC SHOCK INTERACTION WITH A TANGENTIALLY-INJECTED TURBULENT BOUNDARY LAYER (U) WEST VIRGINIA UNIV MORGANTOWN G R INGER ET AL. JAN 84

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Transonic Shock Interaction with a Tangentially-Injected Turbulent Boundary Layer
G.R. Inger and A. Deane, West Virginia Univ., Morgantown, WV
TRANSONIC SHOCK INTERACTION WITH A TANGENTIALLY-INJECTED TURBULENT BOUNDARY LAYER

G. R. Inger* and A. Deane**
West Virginia University, Morgantown, W. Va.

Abstract

A non-asymptotic triple-deck theory of transonic shock turbulent-boundary-layer interaction is described which takes into account the influence of upstream tangential injection on a curved wall. In addition to Reynolds number and the shock strength, the theory is parameterized by arbitrary values of the incoming boundary-layer displacement thickness, wall jet maximum velocity ratio, and the non-dimensional height of the jet relative to the thickness of the incoming turbulent boundary layer. The jet's influence is modeled in the framework of the two-layer theory of turbulent mixing. It is shown that the wall jet significantly reduces the displacement thickness of the incoming boundary layer, while increasing the upstream and downstream fluid friction levels, these effects also reduce the minimum jet core size and thus promote the onset of incipient separation at the shock foot.

Nomenclature

A  skin friction coefficient, $A = \frac{\tau_w}{\rho_0 V^2}$
B  jet-to-incoming turbulent boundary-layer displacement thickness ratio
C  skin friction coefficient, $C = \frac{\tau_w}{\rho_0 V^2}$
D  jet-to-incoming turbulent boundary-layer thickness ratio
E  turbulent kinetic energy, $E = \frac{1}{2} \rho_0 V^2$ (intercept parameter)
F  jet-to-incoming turbulent boundary-layer momentum thickness ratio
G  jet-to-incoming turbulent boundary-layer momentum thickness ratio
H  jet-to-incoming turbulent boundary-layer momentum thickness ratio
I  jet-to-incoming turbulent boundary-layer momentum thickness ratio
J  jet-to-incoming turbulent boundary-layer momentum thickness ratio
K  jet-to-incoming turbulent boundary-layer momentum thickness ratio

Subscripts

a adiabatic wall
b undisturbed inviscid values ahead of incident shock
i incident conditions at the boundary layer edge
inc incompressible value
j undisturbed jet disturbance solution value
max maximum jet disturbance solution value
jet jet disturbance solution value
w velocity profile maximum due to jet
x undisturbed incoming boundary layer properties
y undisturbed incoming boundary layer properties
z undisturbed incoming boundary layer properties

1. Introduction

The use of tangential slot-injection to influence and control turbulent boundary layer behavior has been extensively studied in various types of low-speed external and internal aerodynamic flow fields (e.g., on circulation-controlled airfoils, slotted slaps, in film cooling applications and for separation control in inlets and diffusers). In recent years, many applications of such injection have arisen in supercritical transonic flow fields where local shock wave is present; however, little is presently available to provide a basic understanding of how the resulting shock-boundary-layer interaction (SBLI) alters the influence of tangential injection. Conversely, in such supercritical flows it may be of interest to know how the effects of SBLI may be altered by the use of injection. The present paper addresses these questions for the case of steady non-separating 2-D turbulent boundary layers on adiabatic surfaces of small-to-moderate longitudinal curvature.

The primary objectives of our work are to develop a fundamental theory of a transonic SBLI region occurring downstream of a tangentially-injected turbulent boundary layer on a curved wall (Fig. 1) and then to present the results of a parametric study of this theory showing the

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relationship between the dominant physical parameters, the injection and the physics of the SBLI zone. In Section 2, we briefly outline the non-asymptotic triple-deck theory of a SBLI zone on a curved surface without tangential injection. Then by taking the SBLI zone sufficiently far downstream of the injection slot for mixing of the wall jet and overlying turbulent boundary layer to have produced a well-defined "jet-bulged" boundary layer profile, the interactive perturbation field caused by normal shock interaction with this profile is analyzed in Section 3 by an extension of the aforementioned SBLI theory. This is followed in Section 4 by presentation and discussion of the results of a parametric study of this extended solution for the interactive pressure, displacement thickness and skin friction effects.

2. Brief Outline of the Basic SBLI Theory

2.1. The Triple-deck Model

It is well-known experimentally that when separation occurs, the disturbance flow pattern associated with normal shock-boundary layer interaction is a very complicated one involving a bifurcated shock pattern, whereas the unreported case pertains to turbulent boundary layers up to $M = 1.3$ has instead a much simpler type of interaction pattern which is more amenable to analytical treatment (Fig. 2). The flow consists of a known impinging turbulent boundary layer profile $M = 1.3$ subjected to a small transverse perturbation due to an impinging wall normal shock. In the practical Reynolds number range of interest here ($Re_x = 10^5$), we propose an asymptotic triple-deck flow model to the boundary layer profile $M = 1.3$. It is supported by large body of transonic and supersonic interaction data. The resulting flow model, Fig. 2, consists of an inviscid boundary layer problem surrounding a shock discontinuity and modified by a thin shear stress-disturbance sublayer that contains the upstream influence and skin friction perturbations. An approximate analytic solution is further achieved by assuming small linearized disturbances ahead of and behind the nonlinear shock jump plus neglect of the detailed shock structure within the boundary layer, which gives accurate predictions for all the properties of engineering interest when $M > 1.05$. The resulting equations can be solved by operational methods yielding the interactive rise, displacement thickness growth and the skin friction behavior upstream and downstream of the shock foot. This solution contains all the essential global features of the mixed transonic viscous interaction flow and detailed comparisons with experiment 3, have Navier-Stokes numerical solutions that have shown that it gives a very good account of all the important engineering features of non-separating interactions over a wide range of Mach-Reynolds number conditions.

An important and unique feature of this interaction theory is that it employs for the incoming turbulent boundary layer velocity profile a very general Composite Law of the Wall-Law of the Wake profile model due to Zarlenga, which is characterized not only by the shock Mach number, $M$ and the boundary layer thickness Reynolds number $Re_x$, but also by arbitrary nonequilibrium values of the incompressible shock layer thickness, $H$. The resulting predictions, such as typically illustrated in Fig. 3, show that $H$ has a very large effect on the local and downstream interactive properties that is important to account for in practical applications. By thereby accommodating a wide range of possible upstream histories of pressure gradient, heat and mass transfer, the theory has found wide-spread success as an interactive module in global composite viscous-inviscid flow field analysis programs on supercritical aircraft and projectiles, while also proving adaptable to the accommodation of new effects.

2.2. Wall Curvature and Shock Obliquity Effects

Since SBLI with tangential injection often arises in flows on curved surfaces, it is desirable to account for wall curvature effects in the foregoing interaction theory. For the small to moderate curvatures usually encountered ($K = 10^{-2}$), details analysis of the transonic small disturbance flow in the outer deck shows that while the explicit new curvature terms in the perturbation equations for the boundary layer profile $M = 1.3$, the interactive viscous displacement effect from the underlying deck eliminates the well-known inviscid jump shock singularity while slightly altering the shock into a collapse configuration. Detailed examination of the middle-deck region shows that any new terms in the inviscid rotational disturbance equations are of the negligible order $K^2$ only the curvature effect on the undisturbed-boundary-layer-velocity and eddy-viscosity profiles is of possible significance. Here again, the explicit $K$ terms in the governing equations of this inviscid flow are all negligible; however, curvature can moderately influence (10-20%) the eddy-viscosity terms, with a concomitant effect on the boundary-layer profile in the form of a skin friction reduction and shape factor increase is described approximately by the relationship:

$$C_f = 1.0 - 200K^2, C_{(f),}$$

where to this order of accuracy the corresponding effect on $C_f$ is negligibly small. Note, for example, that the typical value $K = 0.01$ yields a reduction in $C_f$ of 10 and 5%, respectively, upon use of $M = 1.3$ and $K = 10^{-2}$, with the wall velocity profile model and $K = 10^{-2}$, in addition small insert parameter provides a good engineering account of the moderate curvature effects on the middle-deck interaction solution. Within the very thin inner disturbance shear stress deck it is found that again that the explicit curvature effects on the various inertia, pressure gradient, and laminar viscous terms in the disturbance flow equations are altogether negligible. Moreover, because of the extreme inner-deck thinness, the eddy viscosity curvature effects can safely be neglected for the high Reynolds number conditions typifying most practical external aero-dynamic flows.
Predictive results for the typical influence of \( R_e \) on SBLI properties, which agree with experimental observations, may be found in Ref. 9, where their curvature effect slowly spreads out the interaction, weakening the adverse pressure gradient along the wall, and is the primary to the increase in shape factor. Since the curvature effect slightly reduces the inviscid boundary-layer velocity profile fullness and spreads out the interaction, it further acts to advance the downstream boundary layer while slowly increasing the level of shock bow out to the reduced flow velocity. The prediction outlined here allows the inviscid velocity profile shape to be analytically modeled in a manner appropriate to the SBLI solution (see below), and the effect is no longer the adverse pressure gradient along the wall. The flow is then studied using inviscid analysis. When the viscous effect is included, the boundary-layer thickness increases, and therefore it is equivalent to normal shock at the effective lower shock Mach number.

\[
M_{\text{eff}} = \frac{\sqrt{1 - (\alpha^2 - 1)}}{\sqrt{1 - (\alpha^2 - 1)}},
\]

and the shock is treated as a normal shock.

\begin{itemize}
\item \textit{Extension to Include Potential Interaction:}
\end{itemize}

We recall that the inviscid analysis theory leads to a shown shock strength and measured inviscid turbulent velocity profile characterized by the overall parameter \( R_e \), \( \alpha \), and \( \beta_1 \). In the present application we have a new unique shape of velocity profile that exists due to downstream influence. In this section we will be concerned with modeling such a profile and its associated wall-region eddy viscosity behavior by a convenient set of parameters that characterize the essential physical features and yet are flexible. It is specific for the particular case to be considered in Appendix A, and it is assumed that the boundary layer profile is:

\begin{align}
\alpha &= \frac{u_{\text{max}}}{u_{\text{inlet}}}, \\
\beta_1 &= \frac{V}{u_{\text{inlet}}}
\end{align}

where the total profile is expressed as

\begin{align}
u_{\text{max}} &= C_{\text{Max}}(\alpha^3 - \alpha), \\
V &= \frac{u_{\text{inlet}}}{\alpha}
\end{align}

where \( C_{\text{Max}} \) is the maximum velocity. The profile interaction is considered as follows: in the present model the maximum velocity at y = 0, which is given by

\begin{align}
u_{\text{max}} &= C_{\text{Max}}(\alpha^3 - \alpha), \\
V &= \frac{u_{\text{inlet}}}{\alpha}
\end{align}

and then decays outwardly toward zero, becoming negligible beyond some characteristic axial spreading length \( L_{\text{max}} \). Above \( V_{\text{max}} \) the velocity profile is approximately constant and the total composite profile is given by

\begin{align}
\alpha &= \frac{u_{\text{max}}}{u_{\text{inlet}}}, \\
\beta_1 &= \frac{V}{u_{\text{inlet}}}
\end{align}

The reason upstream of the slot and very far downstream where the profile maximum has disappeared can of course be handled by the existing "unknown" version of the present SBLI theory.
where

\[ \beta = \ln(1 + \frac{C_2}{2}) - \ln(1 - \frac{C_2}{2}) \]

is a phase factor insuring the maximum in total velocity at \( y_{\text{max}} \) and

\[ C_x = \frac{(u_{\text{max}}' + AU_{\text{max}}')}{3Au_{\text{max}}'} e^{-y_{\text{max}}'} \]

is a lateral spreading constant (typically \( \approx 0.15 \)) to avoid secondary profile maxima above \( y_{\text{max}}' \).

Below \( y_{\text{max}}' \) on the other hand, we require a functional representation that gives a reasonable monotonic shape and matches smoothly to Eq. (5) at \( y_{\text{max}}' \). Furthermore, we desire some control over the wall-slope in order to represent injection effects on the local skin friction coefficient. The specific constraints on this functional choice are (a) only one maximum in the total composite profile at \( y = y_{\text{max}}' \); (b) a match with the value and slope of the upper (true) function at \( y_{\text{max}}' \), and (c) positive values of the non-dimensional slope

\[ S_w = \frac{\exp(y_{\text{max}}')}{u_{\text{max}}'} \]

leading to physically reasonable skin friction increments

\[ C_f = S_w \left( y_{\text{max}}' - C_3 \exp(C_2 y_{\text{max}}') \right) \]

Now condition (a) so severely restricts the class of monotone functions it admits that no general solution can be generated to accommodate a completely arbitrary combination of conditions (b) and (c); what can be found, however, are functions which allow either an arbitrary choice of all three parameters \( S_w, \Delta u', y_{\text{max}}' \) within a restrictive range or the choice of a wide range of values for the two key parameters \( \Delta u', y_{\text{max}}' \) with \( S_w \) then consequently determined (but still within an interesting range of resulting values). One such function which has proven quite satisfactory for the purposes of this investigation is

\[ \Delta u' = C_1 y_{\text{max}}' - C_3 \left( \exp(C_2 y_{\text{max}}') - 1 \right) y_{\text{max}}' \]

(7A)

where the aforementioned matching conditions are fulfilled if the constants \( C_1, C_2, C_3 \) satisfy the three simultaneous relations

\[ C_1 - C_3 \exp(C_2 - 1) = u_{\text{max}}' \]

\[ C_1 - C_3 \exp(C_2) = y_{\text{max}}' \]

\[ C_1 - C_3 = S_w \]

This trio is readily solved numerically during the implementation of the velocity profile model by using a standard non-linear simultaneous root-finder subroutine.

The aforementioned provides a smooth, piecewise-continuous and physically realistic analytical model of a fully-turbulent boundary layer downstream of a tangential injection slot; it captured the velocity overshoot and negative vorticity region features unique to this kind of flow (i.e., jet) while retaining sufficient basic parameterization to permit sensitivity studies of how the jet-bulge effect influences the SBLI zone. Moreover, it has the advantage of allowing current and later experimental data on turbulent wall-jet boundary layer behavior to be incorporated into the interaction study without tying the present research down to the much more difficult and lengthy effort of such experimental studies. The weak boundary layer compressibility effects on this profile for adiabatic transonic flow are quite satisfactorily accounted for by the reference temperature method.

1.2 Implementation of the Extended Theory

The foregoing approach may be implemented by several straightforward modifications to the existing computer program for the zero-blowing SBLI theory, as follows. To include small-to-moderate wall curvature effects (\( K_6 < 0.1 \)), we add \( K_6 \) as an independent input variable and accordingly modify the input values of \( H_1 \) and \( C_f \) according to Eqs. (1) and (2); furthermore, we eliminate the inviscid curvature singularity, altering the normal shock to a slightly oblique one at the boundary layer edge, by modifying the input effective shock Mach number according to Eq. (3). The influence of tangential injection is accommodated by introducing the two new input parameters \( H_{\text{max}}' \) and \( y_{\text{max}}'/y_{\text{max}}' \) characterizing the magnitude and height, respectively, of the wall jet component effect; in addition, values of the auxiliary parameter \( C_0 \) and \( S_w \) can be set within certain restricted ranges. The program subroutine which evaluates the Walz turbulent boundary layer velocity profile model is modified to add the matched upper and lower wall jet-component increments pertaining to these inputs (Eqs. 4-7), using a Reference Temperature-Method compressibility correction of the appropriate parameters. Figure 7 illustrates some typical boundary layer velocity profiles containing these tangential injection effects. Using the adiabatic temperature-velocity relationship

\[ T = \frac{T_{\text{WAD}}}{T_e - T_{\text{WAD}}} \]

\[ \frac{u^*}{c_f} \]

the associated Mach number profile \( M(y) \) and its derivative \( dM/dy \) (which are both needed in the subsequent SBLI solution routine) are calculated, the corresponding mass flow and momentum defect distributions \( 1 - \frac{U_{\text{se}}}{U_{\text{se}}^*} \) and \( 1 - \frac{U_{\text{se}}}{U_{\text{se}}^*} \) are respectively integrated across the boundary layer to obtain the values of \( \delta^*/6 \) and \( \delta^*/3 \), respectively, associated with the wall jet effect. The resulting values of the displacement thickness and shape factor are shown in Figs. 8a and 8h, to illustrate how the mass and momentum addition to the boundary layer from the wall jet substantially decreases \( \delta^* \) and produces a greater profile "fullness", reflected in a significantly reduced shape factor. Increasing
the height of the jet maximum is seen to have a similar effect, because this enhances the effective strength of the injection effect on the boundary layer profile. Awareness of these overall integral property effects proves helpful in interpreting the predicted interaction properties given below.

Implementation of these wall jet modifications is quite straightforward, except to note that feedback of the aforementioned modified integral properties into the solution sequence must be properly phased: since the wall jet effect on the incoming boundary layer profile shape is already included in the $H(y)$ distribution used in solving the Lighthill interactive pressure equation, the feedback must be done after this pressure disturbance solution is carried out. Subsequent use of the jet-altered values of $A^+$ and $C_f$ then further influences the local interactive displacement thickness and skin friction solution results. To illustrate the importance of this proper feedback of the jet-influenced profile integral properties a typical set of profile shape typified by the increased boundary layer profile fullness and shape factor reduction due to injection causes a significant streamwise contraction of the interactive pressure rider; this is in agreement with experimental observations [see, e.g., Figs. 10a, 11b at Vol. 17]. Accompanying this contraction of the interaction zone, the two thin effects of injection on the ratio $A^+/A^*$ are seen to act in opposite and nearly equal influence: while the profile shape factor effect of injection reduces $A^+$, the corresponding reduction of $A^*$ is approximately of the same magnitude so that the overall change in $A^+/A^*$ is small. This implies that the net injection effect on $A^+$ scales approximately with the corresponding effect on $A^*$. Turning to the interactive skin friction behavior typified in Fig. 9c, it can be seen that the increased $C_f$ level due to the wall jet effect dominates most of the interaction zone both fore and aft of the shock except in the vicinities of the shock foot region, the $C_f$ reduction due to the deepened interactive pressure gradient caused by injection becomes the dominant effect and the local value of $C_f_{min}$ is actually reduced. Stated another way, the SBLI effect adversely counteracts the otherwise favorable $C_f$ increase due to injection.

The aforementioned tangential injection effect may be readily understood from the overall shape factor and displacement thickness effects shown in Fig. 8: the reduced $H$ and $d$ imply a thinner incoming turbulent boundary layer with a somewhat higher Mach number deep in the layer and a fuller profile shape typical of a favorable upstream pressure gradient history, which in view of the demonstrated sensitivity of SBLI to the shape factor [Fig. 1] have the effect of reducing the streamwise scale and interactive thickness while increasing the corresponding local pressure gradient. Results in a strata of negative vorticity flow above the maximum deep down in the incoming boundary layerprofile (Fig. 6). Now, some earlier basic studies of shock interaction with idealized shear flows (simple velocity discontinuities) suggest that such a strata of vorticity sign reversal might significantly alter the character of the shock transmission and reflection across it, in turn implying possible difficulty with the numerical solution across this strata of the Lighthill interactive pressure disturbance equation in the present SBLI theory (which involves a term $-\mu \partial^2 U/\partial y^2 - \partial U/\partial y$). We therefore examined this point carefully, with the following reassuring conclusion: provided that reasonable care is taken to insure high numerical accuracy with an appropriately smaller step size $\Delta y$, the Lighthill equation solution is quite regular for any smooth albeit rapid variation in sign $(\partial U/\partial y)$ across the strata. Hence the overall interaction solution is modified, but not fundamentally altered, by the presence of the negative vorticity due to the wall jet effect and this is straightforwardly accounted for by our modified velocity profile model in the Lighthill equation and by the associated change of the integral parameters. The underlying reason for this lack of difficulty with rapid local variations in either magnitude or sign of $\partial U/\partial y$ may be found from an analysis of the large scale features of Lighthill's equation, which reveals that its solution essentially depends only on integrals, rather than on local details, of the $\partial U/\partial y$ distribution across the boundary layer.

The presence of a local velocity maximum deep within the boundary layer also raises another possible difficulty, when the wall jet effect is sufficiently large, associated with the existence of a strata of locally supersonic flow astride the velocity maximum (Fig. 10). When this occurs, it is seen that there are two special cases where $\partial U/\partial y$ vanishes at a sonic point within the boundary layer and where a local transonic singularity in the Lighthill pressure equation solution therefore will occur: (a) at a tangential injection rate where $U_{max}$ just goes sonic, and (b) at a slightly higher rate where the local minimum $U$ goes sonic higher up in the boundary layer. In these two isolated cases, there is a local breakdown of the linearization underlying the Lighthill equation and the resulting transonic singularity which causes fundamental difficulties with the numerical solution of this equation that can only be cured by restoring (at least locally) the appropriate non-linear transonic correction term. For all other maximum wall jet velocities (including, interestingly enough, the so-called "overblown" cases where $\gamma_{max} > 1$), the boundary layer contains only one local sonic point that is well removed from $\partial U/\partial y = 0$ (for subsonic $U_{max}$ it lies above $U_{max}$ while for supersonic $U_{max}$ it lies below). In such normal cases, no fundamental difficulties were discerned.

4. Discussion of Parametric Study Results

The present theory has been used to carry out a systematic study of how the key tangential injection parameters influence the essential properties of a subsequent SBLI zone. We now present and discuss the results.

3.3) Imbedded Regions of Negative Vorticity and Supersonic Flow in the Boundary Layer

It has been seen that the wall jet effect...
We note here that the absolute values of the effects of thinning out and delaying separation, as indicated in the trend of this data. The theoretical prediction of a gradual pressure gradient effect is consistent with a large number of corresponding increase in approximate experimental boundary determined by a parametric study of incipient separation conditions. Effects make the boundary layer less resistant to, the strong increase of this slope with wall jet strengths when presented in this ratioed both shock location and/or Mach number effects. Then enhancement effect on the streamwise slope wall jet components-effect, are illustrated in Figure 19, where we see that the effect on injection can significantly reduce the overall shock Mach number. As shown in Fig. 21, this is indeed true for the case when the wall jet effect of separation in any subsequent adverse pressure gradient region it may encounter, and hence diminish the effectiveness of injection in otherwise delaying downstream separation. Regarding the skin friction, these conclusions are summarized in Fig. 23, where there is shown the typical influence of increasing wall jet strength on the post-interactive C$. It is seen that while wall separation at first increases it slightly due to the corresponding increase in &, stronger injection rates have the opposite effect of lowering it (as well as C$) because of the intensified adverse pressure gradient effect.

5. Concluding Remarks

Viewed overall, the present study has shown that the usual favorable tangential injection effects of thinning out and delaying the separation of turbulent boundary layers in subsonic flow can be significantly compromised by transonic
shock boundary layer interaction. Conversely, such injection was seen to appreciably reduce the streamwise extent of an SBLI zone albeit with the added consequence of intensifying the local interactive adverse pressure gradient and onset of shock foot separation. It has further been established that a fundamentally-based transverse theory of SBLI with injection is now available to treat these effects in either external or internal supercritical flow fields. Moreover, this theory has been constructed so as to serve as a locally inseparable interactive module inside the inviscid streamwise extent at an arbitrary streamwise location driven by the attendant local boundary layer properties including an arbitrary non-equilibrium shape factor. Consequently it would be possible to investigate the future interesting problems of allowing in supercritical flow fields, including the use of tangential injection to modify the influence of SBLI upon the viscous streamwise adverse pressure gradient and the the net x / H effects in viscous-inviscid flow field analysis programs for circulation-controlled airfoils and wings flying at supercritical flight speeds.

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Appendix

Because of its convenient analytical form, accurate depiction representation of the combined law of the wall - law of the wake behavior and generality, we have adopted Wall's model for the incoming turbulent boundary layer upstream of the interaction. For the low Mach number small heat transfer conditions appropriate to transonic interactions, it was felt to be sufficiently corrected for compressibility effects in the present reference temperature relation which under these conditions is, in fact, comparable in accuracy to, but far simpler to implement than, the "most perfect" compressibility transformation approach. Let be denote Wall's (incompressible) wake function with .41 Re, 11+ 0.35, 1.176; with .76 and 1.4. For a perfect gas then the compressible form of Wall's composite profile may be written

\[ \frac{U}{U_0} = 1 + \frac{1}{2} \left( \frac{U}{U_0} \right)^2 \frac{R'_{\theta}}{1.23} \left( 1 + \frac{1}{2} \right) \frac{1}{1.23} \]  

\[ \rho \beta \left( \frac{R'_{\theta}}{1.23} \right) = 0.21 \frac{U}{U_0} e \]  

subject to the following condition linking \( \frac{R'_{\theta}}{1.23} \) to \( \rho \beta \) and \( \frac{U}{U_0} \):  

\[ \frac{R'_{\theta}}{1.23} = 0.21 \frac{U}{U_0} e \]  

Eqs. (A-1) and (A-2) have the following desirable properties: (a) for \( \rho \beta \to 0 \) or \( U \to 0 \) it is dominated by a law of the wake behavior which correctly satisfied both the upper limit conditions \( U, U_0 \to 1 \) and \( U, U_0 \to 0 \) for all \( \rho \beta \) values. \( \rho \beta \) assumes a law of the wall behavior consisting of a logarithmic term that is exponentially damped out extremely close to the wall into a linear laminar sublayer profile \( U = R'_{\theta} = 0 \); (b) for very small \( \rho \beta \) values, \( U \) has the form of a wall turbulent boundary layer profile with \( U \to \rho \beta \) or \( U \to 0 \) as \( \rho \beta \to 0 \) and \( \rho \beta \to \infty \) as \( \rho \beta \to \infty \). For \( \rho \beta \) values, the results of Eqs. (A-1) and (A-2) yield the following relationships that link the Wake parameter to the resulting incompressible shape factor \( H_{\text{HI}} = \frac{H_{\text{HI}}}{H_{\text{HI}}} \)

\[ \frac{H_{\text{HI}}}{H_{\text{HI}}} = \frac{R'_{\theta}}{1.23} \left( 1 + \frac{1}{2} \right) \frac{1}{1.23} \]  

Eqs. (A-2) and (A-3) together with the defining relation for \( R'_{\theta} \) enable a rather general and convenient parameterization of the profile and hence the interaction that depends on the in terms of three important physical quantities: the shock strength \( R_{\theta} \), the displacement thickness Reynolds number \( Re_{\theta} \), the wall temperature ratio \( T_w/T_{\infty} \) and the shape factor \( H_{\text{HI}} \) that reflects the prior upstream history of the incoming boundary layer including possible pressure gradients and surface mass transfer effects. With these parameters prescribed, the aforementioned three equations may be solved simultaneously for the attendant skin friction \( C_f \), the value of \( R_{\theta} \) and, if desired, the value appropriate to these flow conditions.

References

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NOWAL SHOO

INCOMING PROFILE

\( M_{s+1} \)

LINEARIZED SUPERSONIC

LINEARIZED SUBSONIC

DISPLACED EDGE

ROTATIONAL INVISCID FLOW WITH FROZEN TOTAL SHEAR STRESS

\( \tau_c(y) = \tau_{w_0} \)

THIN SHEAR-DISTURBANCE SUBLAYER

\( \alpha, \beta \)

Fig. 1. Shape and Factor Effect on Interaction Results

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Fig. 2. Experiment

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Fig. 3. Interaction Theory with Hugoniot Shock Jump Relations

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Fig. 4. Interaction Theory with Shock Jump Relations for Maximum Stream Deflection
Fig. 5
Schematic of Turbulent Boundary Layer Development Downstream of a Wall Jet

Fig. 6
Model for the Wall Jet Effect on the Turbulent Boundary Layer Velocity Profile

Fig. 7
Typical Turbulent Boundary Layer Profiles with Injection

\[ M_i = 1.20, \quad Re_{i} = 3.5 \times 10^5 \]
\[ H_i = 1.40 \text{ (unblown)} \]
Fig. 8  Blowing Effect on Internal Properties of the Boundary Layer

$M = 1.20$
$R_{e_0} = 3.5 \times 10^4$
$H_{i_{,n, n, b, low}} = 1.40$

$\frac{\delta_0^*}{\delta_0}$
$\Delta U_{\text{max}}/U_e$

Fig. 9  Typical Blowing Effect on Interactive Property Distributions

$\frac{\Delta U_{\text{max}}}{U_e} = 0.5$
$y_{\max}/\delta_0 = 0.05$
$\Delta C_{p, b, low} = 6.5 \times 10^{-6}$
ALL BLOWING EFFECTS, INCLUDING $\Delta C_f = 4.5 \times 10^{-2}$

Fig. 9 (continued)

FIG. 10
Sonic and Supersonic
Regions within a Blown
Boundary Layer (Schematic)

$M_1 = 1.25$

$R_{\infty} = 1.5 \times 10^{4}$

$M_{l,1} = 1.40$ (UNRESOLVED)

FIG. 11
Parametric Study of Wall
Jet-Effect on Interaction
Pressure Distribution
Fig. 12 Blowing Effect on
Downstream Influence
Distance

\[ \frac{L_{up}}{d_0} \]

\[ \frac{L_{down}}{d_0} \]

Re, \( 3.5 \times 10^4 \)

M_1 = 1.20

M_1 = 1.40

(Non-Blown)

Fig. 13 Blowing Effect on
Upstream Influence Distance

Re, \( 3.5 \times 10^4 \)

M_1 = 1.40

(Non-Blown)

Re, \( 3.5 \times 10^5 \)

M_1 = 1.40

(Non-Blown)

Fig. 14 Reynolds and Mach Number Effects on Blown
Upstream Influence Distance

Fig. 15 Reynolds and Mach Number Effects on Blown
Downstream Influence Distance

13
Fig. 15: Shape factor effect on blown downstream influence

\( \frac{l_{\text{down}}}{\delta_0} \)

Fig. 17: Shape factor effect on blown downstream influence

Fig. 16: Parametric study of wall jet effect on interactive displacement thickness distribution

Fig. 19: Injection effect on viscous wedge angle
Fig. 3a. Mach Number Requirement for Separation at Shock Foot

\[ \frac{C_{T_{\text{mix}}}}{C_{T_0}} \]

\[ \Delta U_{\text{mix}} = 0 \]

\[ \frac{U_{\text{mix}}}{U_e} = 0.10 \]

\[ \frac{U_{\text{mix}}}{U_e} = 0.25 \]

\[ \frac{U_{\text{mix}}}{U_e} = 0.50 \]

\[ \frac{U_{\text{mix}}}{U_e} = 0.70 \]

\[ U_{\text{mix}} = 0.5 \delta_0 \]

\[ \text{Ray}^+ = 3.5 \times 10^5 \]

\[ H_{\text{mix}} = 1.40 \]

Fig. 2b. Mach Number Requirement Separation at Shock Foot
STRONG INTERACTION REGIONS

SENSITIVE TO INCOMING BOUNDARY LAYER

SHOCK-DISTORTED DOWNSTREAM BOUNDARY LAYER

Fig. 22 Schematic of Global Viscous-Inviscid Interaction Problem on Supercritical Airfoils

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Fig. 23 Blowing Effects on Downstream Skin Friction Level

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$\frac{C_f \times 10^3}{\text{max} \ 1.0}$

$\text{Blowing Effect of Increased Interactive Inlet Flow}$

$\text{Re} = 1.0 \times 10^6$