FINAL REPORT
TO OFFICE OF NAVAL RESEARCH

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"Multiple Time Series Modeling and
Time Series Theoretic Statistical Methods"

Professor Emanuel Parzen, Principal Investigator

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Texas A&M Research Foundation
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1 April 1981 - 31 March 1983
This is a final report on the research project entitled "Multiple Time Series Modeling and Time Series Theoretic Statistical Methods" for the period 1 April 1981 - 31 March 1983.
This final report contains: (1) a list of technical reports which were written on research supported by this project; (2) a list of publications developed from these technical reports, (3) copies of our 1982 and 1983 summary of research program written for the ONR Statistics and Probability Program Summary Report.

Our project on time series analysis has supported collaboration with Professor H. J. Newton (Texas A&M) and Professor Marcello Pagano (Harvard Biostatistics). Its overall goal is to develop algorithms that provide researchers in diverse scientific fields with statistically and computationally efficient techniques for univariate and multivariate time series analysis.

The general ideas and concrete results of the research contributions supported by ONR in this project seem to have had wide impact and applications. Forecasting and spectral analysis (which are among the active problems to which this basic research project has made significant contributions) are statistical procedures of great interest and applicability to many branches of the Defense Department.
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Publication of Technical Reports

1982 Summary of Research Program

Multiple Time Series Modeling and Time Series Theoretic

PROJECT TITLE: Statistical Methods

CONTRACT NO.: N00014-82-MP20001

PRINCIPAL INVESTIGATOR: Distinguished Professor Emanuel Parzen
Institute of Statistics
Texas A&M University
College Station, Texas 77843
(713) 845-3188

LONG RANGE SCIENTIFIC OBJECTIVES: To develop statistically and computationally efficient parameter estimation and model identification techniques for univariate and multivariate time series analysis, and to obtain a unified approach to time series analysis, which serves the needs of each of its diverse fields of application.

PROJECT OBJECTIVES: Time series analysis is regarded as composed of four major problems (model identification, spectral analysis, parameter estimation, and forecasting), which are equivalent in the sense that an answer to any one of these questions generates an answer to the other three questions. These problems must be solved simultaneously, since the 'goodness' of model fit, parameter estimates, spectral estimates, and forecasts are essentially inter-dependent, and must employ simultaneously both the time domain and the frequency domain. Spectral estimation techniques provide new functional inference methods of data analysis.

CURRENT STATUS OF THE PROJECT: It is developing approaches for attaining a goal of time series analysis: from a finite sample, identify models which provide efficient forecasts, and efficient estimators of the spectral density which do not introduce spurious spectral peaks, and resolve close spectral peaks.

The proposed approach first determines the memory type of a time series: long, short, or no (white noise). A short memory time series is modeled by an approximating AR, MA, or ARMA scheme. A long memory time series Y(t) is modeled as: (1) a sum of a long memory signal plus a short memory noise, or (2) as the input of a high-pass filter whose output Y(t) is short memory. Approximating autoregressive schemes play two roles: as spectral estimators, and as diagnostic tools for identification of memory type and ARMA type. Model identification criteria are motivated through concepts of estimating information. Measures of information divergence, entropy, and functional inference are being investigated, yielding a new proof of the maximum entropy character of autoregressive density estimators.

SIGNIFICANT ACCOMPLISHMENTS Our research has introduced new concepts into empirical time series analysis, including: CAT, criterion autoregressive transfer function, provides order determining criteria for the orders of approximating autoregressive schemes, developed for both univariate and multivariate time series [see Newton (1982)], and provides a powerful test for white noise; MEMORY (no, short, and long) provides an initial classification to be assigned to a time series which determines our strategy for modeling it [Parzen (1981)]; ARARMA models for long-memory time series which propose that the transformation from long memory to short memory be a best-lag non-stationary autoregression [Parzen (1982)].

LONG RANGE SCIENTIFIC OBJECTIVES: Our research aims to achieve the basic goals of statistical time series analysis, which is to develop statistically efficient and computationally effective methods for: (1) identification of models for univariate and multivariate time series, both Gaussian and non-Gaussian; (2) estimation of their time domain parameters; (3) spectral analysis; (4) forecasting.

PROJECT OBJECTIVES: This project seeks to develop, both for theoretical elegance and practical guidance, a comprehensive theory of model identification and spectral estimation which clarifies the role of estimators of various types (non-parametric, kernel, maximum entropy, autoregressive, ARMA, ARIMA, bandlimited extrapolation, cepstral, etc.). This project investigates: (1) the proposition that the key ingredient of such a theory is the notion of memory, (2) definitions of memory, (3) various methods for identification of memory type, (4) various estimators of memory index, (5) analogies with recent research on estimation of density-quantile and quantile functions, and (6) roles of statistical computing in time series analysis in two important ways: (a) to rapidly make available to the broader scientific community new algorithms for time series analysis; (b) to make old theoretical ideas of time series analysis practically useful and to stimulate the integration of old and new techniques of time series analysis.

CURRENT STATUS OF THE PROJECT AND RECENT SIGNIFICANT ACCOMPLISHMENTS: To identify models for time series and dynamic systems, information theoretic ideas and approximation theoretic ideas are explored. Models for time series \( Y(t) \) can be formulated as hypotheses concerning the information about \( Y(t) \) given various bases involving past, current, and future values of \( Y(\cdot) \) and related time series \( X(\cdot) \). To determine sets of variables that are sufficient to forecast \( Y(t) \), and especially to determine an ARMA model for \( Y(t) \), an approach is developed which estimates and compares various information increments. We discuss how to non-parametrically estimate the MA(\( \infty \)) representation, and use it to form estimators of the many information numbers that one might compare to identify an ARMA model for a univariate time series. We discuss how to estimate an index of regular variation measuring the long memory behavior of the spectral density at zero frequency. We discuss Toeplitz Gram-Schmidt algorithms for autoregressive modeling.

TRN-32 June 1982 H. Joseph Newton, Marcello Pagano Simultaneous Confidence Bands for Autoregressive Spectra
TRN-33 July 1982 Herbert T. Davis, H. Joseph Newton, Marcello Pagano A Toeplitz Gram-Schmidt Algorithm for Autoregressive Modeling
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