ON CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS IN NONLOCAL ELASTICITY

A. Cemal Eringen
PRINCETON UNIVERSITY

Technical Report No. 60
Civil Engng. Res. Rep. No. 84-SM-1

PRINCETON UNIVERSITY
Department of Civil Engineering
ON CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS
IN NONLOCAL ELASTICITY

A. Cemal Eringen
PRINCETON UNIVERSITY

Technical Report No. 60
Civil Engng. Res. Rep. No. 84-SM-1

Research Sponsored by the
OFFICE OF NAVAL RESEARCH
under
Contract N00014-83-K-0126 Mod 4
Task No. NR 064-410

April 1984

Approved for public release:
distribution unlimited

Reproduction in whole or in part is permitted
for any purpose of the United States Government
Continuous Distributions of Dislocations, Nonlocal Theory, Fracture

A linear, nonlocal continuum theory of dislocations is developed. The field equations are given for the dislocation density and the stress fields due to continuous distribution of dislocations. Green's functions are obtained for two- and three-dimensional media & an integral formula is given for line distribution of dislocations generalizing Peach-Koehler formula of the classical...
ON CONTINUOUS DISTRIBUTIONS OF DISLOCATIONS IN NONLOCAL ELASTICITY

A. Cemal Eringen
Princeton University
Princeton, NJ 08544

ABSTRACT

A linear, nonlocal continuum theory of dislocations is developed. The field equations are given for the dislocation density and the stress fields due to continuous distribution of dislocations. Green's functions are obtained for two and three-dimensional media and an integral formula is given for line distribution of dislocations generalizing Peach-Koehler formula of the classical (local) theory. Unlike the classical theory, no stress singularities occur so that self-stress and energies of dislocation loops can be calculated involving no divergences. Exact solutions given for the line and circular distributions of dislocations verify these expectations.
1. INTRODUCTION

In classical (local) theory, the displacement and stress fields due to a continuous distribution of dislocations can be calculated by means of various surface and volume integrals once the Green's functions for the dislocation density is known. To obtain the Greens function one solves partial differential equations for the stress functions. For example, the stress due to a line distribution of dislocation is given by the Peach-Koehler formula. One of the basic difficulties of this theory is that the self-stress and energies of dislocation loops possess mathematical singularities so that calculations will have to be cut-off near the lines of dislocation or the core region.

In several previous papers (cf. 1-4), I have shown that the solutions for the single dislocation involve no stress or energy singularities at the core regions. Moreover, calculated theoretical strengths of solids and dispersion curves for plane waves agree quite well with those known from the atomic lattice dynamics and/or experiments. Therefore, it is expected that the nonlocal theory will eliminate the classical singularities for the self stress and energies of dislocation loops. The raison d'être of the present paper stems from the need to develop a theory of continuous distribution of dislocations based on the nonlocal elasticity, that can hopefully predict the physical phenomena in the microscopic and atomic scales where classical theory fails to apply.

In Section 2, I summarize the basic equations of the nonlocal theory of linear isotropic elastic solids. In Section 3, I develop
the field equations for the continuous distribution of dislocations. Green tensors for the Beltrami and Airy stress functions for solids of infinite extends are obtained in Section 4. The stress fields are given by a volume integral generalizing celebrated Peach-Koehler formula of the classical theory. In Section 5, I derive explicit expressions of the stress fields for the uniform distributions of screw dislocations along a straight line segment and along a circular loop. The exact formulas obtained for these cases contain no singularities, justifying our expectation. Section 6 contains calculations of stress fields and a discussion of the reduction of yield stress with the dislocation pile-up.

The simplicity and the aesthetics of these results, I believe, justifiably indicate the power and the potential of the nonlocal theory in the treatment of the physical phenomena with characteristic lengths in the microscopic and atomic scales.

2. BASIC EQUATIONS

The linear theory of nonlocal elasticity is based on Cauchy's equations of motion

\[ t_{k\ell,k} + \rho(f_{\kappa} - \ddot{u}_{\kappa}) = 0 \]

and the integral constitutive equations

\[ t_{k\ell}(x,t) = \int_{V} c_{k\ell mn}(x'-x) \, e_{mn}(x') \, dv(x') \]

where \( t_{k\ell}, \rho, f_{\kappa}, u_{\kappa} \) and \( e_{k\ell} \) are respectively, the stress tensor, the mass density, the body force density, the displacement vector and the linear strain tensor defined by
(2.3) \[ e_{k\ell} = \frac{1}{2} (u_{k,\ell} + u_{\ell,k}) \]

In (2.2), \( c_{k\ell mn} \) is a function of the vector \( \mathbf{x}'-\mathbf{x} \) and the integral is over the volume of the entire body. Consequently, the stress at a point \( \mathbf{x} \) depends on the strain at all points \( \mathbf{x}' \) of the body.

Throughout this paper, we employ rectangular coordinates \( x_k \), \( k=1,2,3 \), and use the usual summation convention on repeated indices. Also a superposed dot indicates the time rate and an index followed by a comma partial differentiation with respect to \( x_k \), e.g.

\[ \dot{u}_k = \frac{\partial u_k}{\partial t}, \quad u_{k,\ell} = \frac{\partial u_k}{\partial x_{\ell}} \]

The kernel \( c_{k\ell mn} \) possesses certain symmetry regulations and it depends on a length scale. For isotropic solids (2.2) takes the simple form

(2.4) \[ t_{k\ell}(\mathbf{x},t) = \int_V \alpha(|\mathbf{x}'-\mathbf{x}|) \sigma_{k\ell}(\mathbf{x}') \, dv(\mathbf{x}') \]

where \( \sigma_{k\ell} \) is the classical (local) stress tensor given by the Hooke's law

(2.5) \[ \sigma_{k\ell} = \lambda \varepsilon_{\ell k} + 2\mu \varepsilon_{k\ell} \]

and \( \alpha \) is a function of the distance \( |\mathbf{x}'-\mathbf{x}| \). It also depends on a length scale \( \varepsilon \) that may be taken to be proportional to an internal characteristic length \( a \)

(2.6) \[ \varepsilon = e_0 a \]

where \( e_0 \) is a non-dimensional material property which may be determined by one experiment or comparison with calculations based on lattice dynamics\(^3\text{-}^5\). The internal characteristic length \( a \) may be taken as the lattice parameter
for single crystals, granular distance for amorphous materials, and the average distance for fiber composites. As \( \epsilon \to 0 \), \( t_{kl} \to \sigma_{kl} \) and (2.4) reduces to Hooke's law \( t_{kl} = \sigma_{kl} \). Thus, \( \alpha(|x'-x|) \) is a Dirac delta sequence.

In several previous papers\(^3,5\), I have discussed the properties of \( \alpha(|x'-x|) \) and gave representations which lead to excellent agreement with known atomic calculations on dispersions of waves\(^3,6\) in the entire Brillouin zone and on theoretical strengths of solids.\(^4\) For example, for the two-dimensional case, an appropriate kernel is

\[
(2.7) \quad \alpha(|x|, \epsilon) = \left(2\pi \epsilon^2\right)^{-1} K_0\left(\sqrt{x'^2 + x^2} / \epsilon\right)
\]
which satisfies the equation

\[
(2.8) \quad (1 - \epsilon^2 \nabla^2) \alpha = \delta(|x' - x|)
\]
vanishing at infinity. In fact, for the infinite solid, it can be shown that \( \alpha \) is the Green's function satisfying (2.8) in three-dimensions also.

Using (2.8) in (2.4), we obtain

\[
(2.9) \quad (1 - \epsilon^2 \nabla^2) t_{kl} = \sigma_{kl}
\]

By means of (2.1) and (2.9), we then find that

\[
(2.10) \quad (\lambda + \mu) u_{k,kl} + \mu v_{k,kk} + (1 - \epsilon^2 \nabla^2) (\partial f_{k} - \partial \bar{u}_{k}) = 0
\]
These are the partial differential equations for $u_k$, replacing Navier's equations of classical elasticity. For the static case and vanishing body forces, this reduces classical Navier's equation

$$ (\lambda + \mu) u_{k,kk} + \mu u_{k,kk} = 0 $$

However, note that the stress field is determined by solving (2.9) under appropriate boundary conditions.

3. CONTINUOUS DISTRIBUTION OF DISLOCATIONS

Continuous distribution of dislocations is envisaged as follows:

A small neighborhood $n(x)$ of $x$ in a distorted body of volume $V$, may be relaxed to a small neighborhood $N(X)$ of the image of $X$ of $x$, in an undistorted (or a natural) configuration $V$, by releasing constraints exerted to $n(x)$ by the rest of the body. A line element $dx$ at $x \in n(x)$ can be expressed in terms of its image $dX \in N(X)$ by

$$ dx = A \; dX $$

where $A(X)$ is called the elastic distortion. It is assumed that $A(X)$ is continuously differentiable and possesses unique inverse so that

$$ dX = A^{-1} \; dx $$
Consider a smooth surface $S$ in $V$ bounded by a closed curve $C$. The true Burger's vector $\mathbf{b}$ of the dislocations piercing through $S$ is defined by

$$(3.3) \quad \mathbf{b} = \oint_{C} d\mathbf{x} = \oint_{C} \mathbf{A} \cdot d\mathbf{x} = \iint_{S} \mathbf{n} \cdot ds \quad \text{and}$$

where $\mathbf{n}$ is the unit normal to $S$, the positive sense of $C$ being counter-clockwise when sighting along $\mathbf{n}$. Here $\mathbf{a}$ is called the true dislocation density

$$(3.4) \quad \mathbf{a} = \text{curl} \mathbf{A} \quad \text{or} \quad a_{jk} = \epsilon_{kmn} A_{jm,n}$$

For small distortions, we can write

$$(3.5) \quad A_{k\ell} = \delta_{k\ell} + a_{k\ell} \quad \text{or} \quad A_{k\ell} = \delta_{k\ell} - a_{k\ell}$$

so that

$$(3.6) \quad a_{jk} = \epsilon_{kmn} a_{jm,n}$$

From this, it follows that

$$(3.7) \quad a_{jk,k} = 0$$
The linear strain tensor $e_{k\ell}$ and rotation tensor $r_{k\ell}$ are given by

$$\begin{align*}
(3.8) \quad e_{k\ell} &= \frac{1}{2} (\alpha_{k\ell} + \alpha_{\ell k}), \\
r_{k\ell} &= \frac{1}{2} (\alpha_{k\ell} - \alpha_{\ell k})
\end{align*}$$

The strain incompatibility is expressed by

$$\begin{align*}
(3.9) \quad \epsilon_{ijk} \epsilon_{\ell mn} e_{in,jm} = \eta_{k\ell}
\end{align*}$$

where $\eta_{k\ell}$ is called the incompatibility tensor and is given by

$$\begin{align*}
(3.10) \quad \eta_{k\ell} &= \frac{1}{2} (\epsilon_{kmn} a_{k\ell,m} + \epsilon_{kmn} a_{\ell n,k,m})
\end{align*}$$

All these results are well-known in classical theory (cf. Ref. [7]).

In nonlocal elasticity, the strain tensor can be solved by using (2.9) and (7.5)

$$\begin{align*}
(3.11) \quad e_{k\ell} &= \frac{1}{2\mu} (1 - \epsilon^2 \nu^2) (t_{k\ell} - \frac{\nu}{1+\nu} t_{rr}\delta_{k\ell})
\end{align*}$$

where $\nu = \lambda/2(\lambda+\mu)$ is the Poisson’s ratio. Substituting (3.11) into (3.9), we obtain

$$\begin{align*}
(3.12) \quad (1 - \epsilon^2 \nu^2) [\nu^2 t_{k\ell} + \frac{1}{1+\nu} (t_{rr} k\ell - \nu^2 t_{rr}\delta_{k\ell})] &= 2\mu \eta_{k\ell}
\end{align*}$$

These equations must be solved under the conditions of equilibrium

$$\begin{align*}
(3.13) \quad t_{k\ell,k} &= 0
\end{align*}$$
Following Kröner's approach, modifying the Beltrami solution of (3.13), we take

\[
\frac{t_{k\ell}}{2\mu} = \psi^2 x_{k\ell} + \frac{1}{1-\nu} (x_{rr,k\ell} - \psi^2 x_{rr} \delta_{k\ell})
\]

where the symmetric stress function \( x_{k\ell} \) is subject to

\[
x_{k\ell,k\ell} = 0.
\]

Substituting (3.14) into (3.12), we obtain

\[
(1 - \varepsilon^2 \psi^2) \psi_4 x_{k\ell} = \eta_{k\ell}
\]

Thus, given the dislocation density function \( \alpha_{k\ell} \), through (3.10), we calculate \( \eta_{k\ell} \). The solution of (3.16) gives \( x_{k\ell} \) and (3.14) the stress field.

Equation (3.16) is singularly perturbed and as expected in the limit \( \varepsilon \rightarrow 0 \), (3.16) reduces to the classical equation for \( x_{k\ell} \).

To obtain the solution of (3.16), we must find the Green's Tensor \( G_{k\ell,mn}(x,\xi) \) which satisfies

\[
(1 - \varepsilon^2 \psi^2) \psi_4 G_{k\ell,mn} = \delta(x-\xi) \delta_{k\ell} \delta_{mn}.
\]

The solution of (3.16) is then given by

\[
x_{k\ell} = \int V G_{k\ell,mn}(x,\xi) \eta_{mn}(\xi) d\nu(\xi)
\]

subject to supplementary conditions (3.15).
4. GREEN'S TENSORS AND STRESS FIELDS

Here we determine Green's tensors for two and three-dimensional bodies of infinite extends.

(i) Three-Dimensional Infinite Space

The operator $\nabla^2$ is invariant under rotations of coordinates. For the infinite space, we look for a solution of (3.17) which depends on $|x-\xi|$ only, i.e.,

\[(4.1)\quad (1 - \epsilon^2 \nabla^2) \nabla^4 G = \delta(x-\xi)\]

Since the operators $1 - \epsilon^2 \nabla^2$ and $\nabla^4$ are commutative, we set

\[(4.2)\quad (1 - \epsilon^2 \nabla^2) G = H,\quad \nabla^4 H = \delta(|x-\xi|)\]

For the infinite space, $H$ is given by

\[(4.3)\quad H = -\frac{1}{4\pi} \frac{|x-\xi|}{8\pi}\]

In spherical coordinates using

$\nabla^2 = \frac{1}{r^2} (r^2 \frac{d}{dr})$

we obtain for $G$:
Here \( G_0 \) is an arbitrary constant which may be chosen \( G_0 = 1 \) to render \( t_{kk} \) regular at \( x = \xi \). The solution of (3.16) for the infinite solid is given by

\[
(4.6) \quad \chi_{k\ell} = \int \frac{G(|x-\xi|)}{|x-\xi|} \eta_{kk}(\xi) \, dv(\xi)
\]

which satisfies the conditions (3.15) on account of (3.7) and (3.10). If we substitute from (3.10), this gives

\[
(4.7) \quad \chi_{k\ell}(x) = \frac{1}{2} \varepsilon_{ijk} \int \frac{a_{j\ell}(\xi)}{\partial x_i} \frac{\partial G}{\partial x_i} \, dv(\xi)
\]

where we used the Green-Gauss theorem and set a surface term at infinity to zero.

From (4.7), one can obtain various special cases involving surface and line distributions of dislocations. For example, for a line distribution of dislocation along a closed curve \( C \), we obtain
(4.8) \[ x_{kl} = \frac{1}{2} \varepsilon_{kij} b_j \left( \int \frac{\partial G}{\partial x_i} \, dx_k + \frac{1}{2} \varepsilon_{lij} b_j \int \frac{\partial G}{\partial x_i} \, dx_k \right) \]

where \( b_j \) is the Burger's vector per unit length of \( C \) and \( dx_i \) is the element of the arc.

Upon substituting (4.8) into (3.14), we obtain the stress field due to a line distribution of dislocations

(4.9) \[ \frac{t_{kl}}{2\mu} = \frac{1}{2} \varepsilon_{rij} b_j \left[ \int \left( \nabla^2 G_{ij} (\delta_{rl} dx_k + \delta_{rk} dx_l) \right) \right. \]
\[ + \frac{2}{1-\nu} (G_{kl} - \nabla^2 G_{ij} \delta_{kl}) \]

This result is identical to the Peach and Koehler\(^7\) formula with modification that here \( G \) is the nonlocal Green's function (4.4) with \( \varepsilon \neq 0 \).

As we shall see, the most interesting new feature of (4.9) is that, with \( G \) given by (4.4), at a point on the dislocation line \( C \), the stress is finite so that the self-stress and energies of dislocation loops can be calculated, free of infinities.

(ii) Two-Dimensional Infinite Plane

In the case of the plane strain, introducing the Airy's stress function \( \phi(x_1, x_2) \) by

(4.10) \[ t_{11} = \phi_{,22}, \quad t_{22} = \phi_{,11}, \quad t_{12} = -\phi_{,12} \]
we obtain an equation replacing (3.16)

\[(4.11) \quad (1 - \varepsilon^2 \nabla^2) \nabla^4 \phi = 2u \eta \]

where

\[(4.12) \quad \eta = \eta_{33} = a_{23,1} - a_{13,2} \]

\[a_{23} = a_{21,2} - a_{22,1}, \quad a_{13} = a_{11,2} - a_{12,1}\]

depend on \(x_1\) and \(x_2\) only.

Green's function in this case, can be found similar to decomposition (4.2) with \(\nabla^2\) given by

\[\nabla^2 = \frac{1}{r} \frac{d}{dr} (r \frac{d}{dr})\]

Hence,

\[(4.13) \quad G(|x-\xi|) = \frac{G_0}{2\pi} K_0(|x-\xi|/\varepsilon) - \frac{(x-\xi) \cdot (x-\xi)}{8\pi \varepsilon^2} \ln(|x-\xi|/\varepsilon), \; \varepsilon \neq 0\]

\[(4.14) \quad G(|x-\xi|) = -\frac{(x-\xi) \cdot (x-\xi)}{8\pi} \ln(|x-\xi|), \; \varepsilon = 0\]

where \(K_0(z)\) is the modified Bessel's function. Again, we take \(G_0 = 1\) to render \(t_{kk}\) regular at \(x = \xi\).
Airy's stress function is obtained to be

\[ \phi(x) = 2\mu \int_{S} \left[ G_{11} a_{23}(\xi) - G_{12} a_{13}(\xi) \right] ds \]

where we used the Green-Gauss theorem and set a line integral to zero at infinity. For a line distribution of dislocations in the \( x_3 = 0 \) plane, we obtain

\[ \phi(x) = -2\mu \int_{C} \left[ G_{11} b_{2}(\xi) d\xi_1 + G_{12} b_{1}(\xi) d\xi_2 \right] \]

The stress field follows from (4.10)

\[ \tau_{11} = -2\mu \int_{C} (G_{112} b_{2} d\xi_1 + G_{222} b_{1} d\xi_2), \]

\[ \tau_{22} = -2\mu \int_{C} (G_{111} b_{2} d\xi_1 + G_{211} b_{1} d\xi_2), \]

\[ \tau_{12} = 2\mu \int_{C} (G_{112} b_{2} d\xi_1 + G_{212} b_{1} d\xi_2), \]

(iii) Anti-Plane Strain

In the case of anti-plane strain, equations of equilibrium are satisfied if

\[ \tau_{13} = \frac{\partial \phi}{\partial x_2}, \quad \tau_{23} = -\frac{\partial \phi}{\partial x_1} \]
and we obtain

\[(4.19) \quad (1 - \varepsilon^2 \nabla^2) \nabla^2 \phi = \mu a_{33} \]

where

\[(4.20) \quad a_{33} = a_{31,2} - a_{32,1} \]

Green's function for this case, is obtained to be

\[(4.21) \quad G(|x - \xi|) = -\frac{1}{2\pi} \left[\ln(|x - \xi|/\varepsilon) + K_0(|x - \xi|/\varepsilon)\right], \quad \varepsilon \neq 0 \]

\[(4.22) \quad G(|x - \xi|) = -\frac{1}{2\pi} \ln(|x - \xi|) \quad \varepsilon = 0 \]

The stress field is given by

\[(4.23) \quad \tau_{13} = \mu \int \limits_S G_{,2} b(\xi) \, d\xi_1 \, d\xi_2 \]
\[\tau_{23} = -\mu \int \limits_S G_{,1} b(\xi) \, d\xi_1 \, d\xi_2 \]

For a line distribution of dislocations on the plane \(x_3 = 0\), we have

\[(4.24) \quad \tau_{13} = \mu \int \limits_C G_{,2} b(\xi) \, d\xi, \quad \tau_{23} = -\mu \int \limits_C G_{,1} b(\xi) \, d\xi \]

In plane polar coordinates \((r, \theta)\), the stress field is given by
\( t_{2r} = \mu \int_{C} \frac{1}{r} \frac{\partial G}{\partial \theta} B(\xi) \, d\ell \)

\( t_{2\theta} = -\mu \int_{C} \frac{\partial G}{\partial r} b(\xi) \, d\ell \)

5. UNIFORM DISTRIBUTIONS OF SCREWS

Here we calculate the stress field for two different uniform line distributions of screw dislocations:

(i) **Screw Dislocation Along a Straight Line**

Consider a line distribution of screw dislocations of constant Burger's vector along a straight line segment \( x_2 = 0 \). Green's function is given by

\[
G(|x-\xi|) = \frac{1}{2\pi} \left[ \ln \left( \frac{\rho}{\epsilon} \right) + K_0 (c/\epsilon) \right]
\]

where

\[
\rho = [ (x_1 - \xi)^2 + x_2^2 ]^{1/2}
\]

We can evaluate \( t_{23} \) given by (4.24) immediately since

\[
G, \frac{\partial G}{\partial x_1} = -\frac{\partial G}{\partial \xi}
\]

and we have
\[ t_{23} = \mu b [ G(|x - \xi|) - G(|x + \xi|) ] \]

Explicitly,
\[ t_{23} = -\frac{\mu b}{2\pi} \left\{ \ln \left[ \frac{(x_1 - \xi)^2 + x_2^2}{(x_1 + \xi)^2 + x_2^2} \right]^{\frac{1}{2}} + K_0 \left[ \sqrt{(x_1 - \xi)^2 + x_2^2} \right] \right\} \]
\[ - K_0 \left[ \sqrt{(x_1 + \xi)^2 + x_2^2} \right] \]

Along the line of screws, this gives
\[ t_{23}(x_1, 0) = -\frac{\mu b}{2\pi} \left\{ \ln \left( \frac{|x_1 - \xi|}{|x_1 + \xi|} \right) + K_0(|x_1 - \xi|/\epsilon) - K_0(|x_1 + \xi|/\epsilon) \right\} \]

Calculation of \( t_{13} \) is complicated. However, it is easy to see that
\[ t_{13}(x_1, 0) = 0 \]

Unlike the classical case, \( t_{23}(x_1, 0) \) has no singularity at the end points \( x_1 = \pm \xi \) of the screw line. In fact, we have
\[ t_{23}(\pm \xi, 0) = \pm \frac{\mu b}{2\pi} \left[ \ln(\xi/\epsilon) + K_0(2\xi/\epsilon) \right] \]

for \( \xi \gg 1 \), we have the asymptotic value
\[ t_{23}(\pm \xi, 0) + \pm \frac{\mu b}{2\pi} \left[ \ln(\xi/\epsilon) + (\pi\epsilon/4\xi)^{\frac{1}{2}} \exp(-2\xi/\epsilon) \right] \]
where the second term can also be neglected as compared to the first one for large $\psi/\varepsilon$.

(ii) Uniform Distribution of Screw Dislocations Along a Circle

Suppose that in the plane $x_3 = 0$, there is a uniformly distributed screw dislocations along a circle of radius $R$. In plane polar coordinates, we have

$$
\begin{align*}
(5.9) \quad x_1 &= r \cos \theta, \quad x_2 = r \sin \theta, \\
\xi_1 &= R \cos \phi, \quad \xi_2 = R \sin \phi, \\
|x - \xi| &= \left[ r^2 + R^2 - 2rR \cos (\phi - \theta) \right]^{1/2}
\end{align*}
$$

Greens' function is given by

$$
(5.10) \quad G(|x - \xi|) = -\frac{1}{2\pi} \left[ \ln(|x - \xi|/\varepsilon) + K_0(|x - \xi|/\varepsilon) \right]
$$

To calculate the stress field $t_{zr}$ and $t_{z\psi}$ we must evaluate two integrals

$$
(5.11) \quad I_1 = \int_0^{2\pi + \theta} \ln \left\{ \frac{1}{\varepsilon} \left[ r^2 + R^2 - 2rR \cos (\phi - \theta) \right]^{1/2} \right\} d\phi
$$

$$
(5.12) \quad I_2 = \int_0^{2\pi + \theta} K_0 \left\{ \frac{1}{\varepsilon} \left[ r^2 + R^2 - 2rR \cos (\phi - \theta) \right]^{1/2} \right\} d\phi
$$
To evaluate (5.11), we write

\[ (5.13) \quad \ln \left\{ \frac{1}{\varepsilon} \left[ \frac{r^2 + R^2 - 2rR \cos(\phi - \theta)}{2} \right] \right\} = \ln \left( \frac{|r+R|}{\varepsilon} \right) + \ln \left( 1 - \alpha_0 \cos^2 \frac{\psi}{2} \right) \]

where

\[ (5.14) \quad \alpha_0 = 4 \frac{rR}{(r+R)^2}, \quad \phi - \theta = \psi \]

The second term in (5.13) can be integrated by writing \( x = \cos(\psi/2) \) and consulting Ref. 9, p. 562, No. 38. Consequently

\[ (5.15) \quad I_1 = \begin{cases} 2\pi \ln(r/\varepsilon) & r > R \\ 2\pi \ln(R/\varepsilon) & r < R \end{cases} \]

To evaluate \( I_2 \), we employ the integrals 6.684 on p. 741 of Ref. 9, and note that

\[ K_0(z) = \frac{\pi i}{2} \left[ J_0(iz) + iN_0(iz) \right] \]

where \( J_0 \) and \( N_0 \) are zero order Bessel functions. The result is

\[ (5.16) \quad I_2 = \begin{cases} 2\pi I_0(R/\varepsilon) K_0(r/\varepsilon) & r > R \\ 2\pi I_0(r/\varepsilon) K_0(R/\varepsilon) & r < R \end{cases} \]

Consequently,
The stress field is given by

\[ t_{z\alpha} = \mu \frac{bR}{r} \frac{\partial I}{\partial \alpha} = 0 \,, \]

\[ t_{z\theta} = \mu bR \frac{\partial I}{\partial \theta} = \begin{cases} \mu b \frac{R}{\varepsilon} \left[ \frac{\varepsilon}{r} - I_0 \left( \frac{R}{\varepsilon} \right) K_1 \left( \frac{r}{\varepsilon} \right) \right], & r > R \\ \mu b \frac{R}{\varepsilon} I_1 \left( \frac{r}{\varepsilon} \right) K_0 \left( \frac{R}{\varepsilon} \right), & r < R \end{cases} \]

In the special case when \( R \to 0 \) and \( 2\pi R b = b_0 \), we obtain

\[ t_{z\theta} = \frac{\mu b_0}{2\pi r} \left( 1 - \frac{r}{\varepsilon} K_1 \left( \frac{r}{\varepsilon} \right) \right) \]

which is identical to our previous result for a single screw dislocation having Burger's factor \( b_0 \).
6. STRESS DISTRIBUTIONS

Here, I present some numerical results on the stress distributions for the cases discussed in Section 5 and establishes a fracture criteria based on the maximum shear stress.

(i) Single Screw: The shear stress given by (5.19) may be expressed in non-dimensional form:

\[ T_\theta(\rho) = \frac{2\pi \varepsilon / \mu b}{2\rho} \theta^\prime \left[ 1 - \rho K_1(\rho) \right] \]

where

\[ \rho = r / \varepsilon \]

The stress field given by (6.1) is displayed graphically in Fig. 1. It has no singularity at \( \rho = 0 \). In fact, \( T_\theta(\rho) \) vanishes at \( \rho = 0 \) in contradiction to the classical elasticity solution which gives infinite stress at \( \rho = 0 \). The maximum stress occurs at \( \rho = 1.1 \) and is given by

\[ \theta_{\max} = 0.3993 \frac{b}{2\pi \varepsilon} \]

If we write \( h = \varepsilon / 0.3993 \), this agrees with Frenkel's estimate of the theoretical strength of single crystals, based on atomic considerations (cf., Kelly [10], p. 12). In fact, if we use \( \varepsilon = \varepsilon_0 a = 0.39 a \), which is obtained on the basis of matching of the dispersion curve predicted by non-local elasticity and the Born-Kármán lattice model, we find for the single aluminum crystal.
(6.4) \[ t^c/\mu = 0.12 \quad \{\text{Al: } [111]<110>\} \]

This is very close to the theoretical strength \( t_y/\mu = 0.11 \) based on atomic models.

(ii) **Screw Dislocations Along a Straight Line Segment**: Even single crystals contain many dislocations. For a uniform distribution of screws along a line segment \( |x_1| < \xi, \ x_2 = x_3 = 0 \), the shear stress given by (5.5) may be written in non-dimensional form

\[
T^*_2 = T^*_2/d = \xi n \frac{|x+1|}{|x-1|} + K_0(\gamma|x+1|) - K_0(\gamma|x-1|)
\]

where

\[
t_d = \mu b/2n = \mu b_0N/2\pi\xi, \quad x = x_1/\xi, \quad \gamma = \xi/\varepsilon
\]

Here \( b_0 \) is the atomic Burger's vector and \( N \) is the total number of dislocations over a distance \( \xi \).

The distribution of the shear stress (6.5) as a function of \( x \) is shown in Fig. 2 for various values of \( \gamma \). Behavior of \( T^*_2 \) is governed basically by the first term in (6.5) except near \( x = 1 \). At \( x = 1 \), we have

\[
t^*_2(1) = \frac{\mu b_0 N}{2\pi\varepsilon} \frac{n}{\gamma}
\]

The value of \( T^*_2(1) \) is very close to the maximum stress for \( \gamma \geq 3 \)
(cf. Table 1). For a single atomic dislocation according to (6.3), we have the theoretical strength

\[ t^c_y = 0.3993 \frac{ub_0}{2\pi c} \]

Combining (6.7) and (6.8), we obtain

\[ \frac{t^d_y}{t^c_y} = \frac{N}{0.3993} \frac{\ln \gamma}{\gamma} \]

where we set \( t^d_y = t^c_y \) = the yield stress for the distributed dislocations. This gives the shear stress reduction due to the presence of \( 2N \) dislocations distributed uniformly along a straight line segment of length \( 2c \). Since \( t^d_y \leq t^c_y \), the maximum number of dislocations is given by

\[ N_{\text{max}} = 0.3993 \frac{\gamma}{\ln \gamma} \]

For \( \gamma = 4.02 \times 10^4 \), this gives \( N_{\text{max}} = 1514 \), which may be conservative since the distribution is not generally uniform but in an inverse pile-up configuration.

(iii) Uniform Distribution of Screws Along a Circle: In this case, the stress fields given by (5.18) may be expressed in non-dimensional form

\[ T = \frac{t^c_{\omega_0}}{ub} = \begin{cases} \frac{1}{c} - \kappa I_0(\kappa) K_1(\kappa c), & \omega > 1 \\ \kappa K_0(\kappa) I_1(\kappa c), & \omega < 1 \end{cases} \]
where

\( (6.12) \quad \rho = r/R, \quad \kappa = R/\varepsilon \)

\( T \) as a function of \( \rho \), for various values of \( \kappa \), is displayed in Fig. 3. For value of \( \kappa > 50 \), the maxima of \( T \) occurs near \( \rho = 1 \). The locations and values of \( T_{\text{max}} \) are given in Table 2.

If \( b_0 \) is the atomic Burger's vector, then

\( (6.13) \quad 2\pi R b = N b_0 \)

where \( N \) is the number of dislocations on the dislocation circle with radius \( R \). Using (6.8), (6.12) and (6.13), we obtain

\( (6.14) \quad t_c = N T/0.3993\kappa \)

According to Fig. 3, \( 0.324 \leq T_{\text{max}} \leq 1 \). Consequently,

\( (6.15) \quad N/1.2\kappa \leq t_y^d/t_y^c \leq N/0.4\kappa \)

For perfect crystals with \( \varepsilon = 0.39 a \), this gives approximately

\( (6.16) \quad 0.3 Na/R \leq t_y^d/t_y^c \leq Na/R \)

indicating reduction of the yield stress with the presence of large number dislocations uniformly distributed along a circle of radius \( R \).
Table 1: Maximum Shear Stress and its Location

(Line Segment)

\[
\begin{array}{cccccc}
\gamma & = & 1 & 1.5 & 2 & 3 & 5 & 10 \\
\times & = & 1.446 & 1.197 & 1.103 & 1.039 & 1.000 & 1.000 \\
T_{2\text{max}} & = & 0.7478 & 1.0501 & 1.3008 & 1.6851 & 2.3026 & 2.9957 \\
\end{array}
\]

Table 2: Maximum Shear Stress and its Location

(Circle)

\[
\begin{array}{cccccccc}
\kappa & = & 1 & 2 & 3 & 5 & 10 & 50 \\
\sigma & = & 1.8 & 1.5 & 1.4 & 1.3 & 1.2 & 1.1 \\
T_{\text{max}} & = & 0.3243 & 0.4836 & 0.5688 & 0.6630 & 0.7688 & 0.9058 \\
\end{array}
\]
REFERENCES


ACKNOWLEDGEMENT

This work was supported by ONR. The author is indebted to Dr. Basdekas for his encouragement and enthusiasm. My thanks go to Mr. D. Tao for some computer work.
NON-DIMENSIONAL HOOP STRESS FOR SCREW DISLOCATION

FIGURE 1
Part 1 - Government

Administrative and Liaison Activities

Office of Naval Research
Department of the Navy
Arlington, Virginia 22217
Attn: Code 474 (2)
   Code 471
   Code 200

Director
Office of Naval Research
   Eastern/Central Regional Office
   666 Summer Street
   Boston, Massachusetts 02210

Director
Office of Naval Research
   Branch Office
   536 South Clark Street
   Chicago, Illinois 60605

Director
Office of Naval Research
   New York Area Office
   715 Broadway - 5th Floor
   New York, New York 10003

Director
Office of Naval Research
   Western Regional Office
   1030 East Green Street
   Pasadena, California 91106

Naval Research Laboratory (6)
   Code 2627
   Washington, D.C. 20375

Defense Technical Information Center (12)
   Cameron Station
   Alexandria, Virginia 22314

Navy

Undersea Explosion Research Division
Naval Ship Research and Development Center
Norfolk Naval Shipyard
Portsmouth, Virginia 23709
Attn: Dr. E. Palmer, Code 177

Navy (Con't.)

Naval Research Laboratory
Washington, D.C. 20375
Attn: Code 8400
   8410
   8430
   8440
   6300
   6390
   6380

David W. Taylor Naval Ship Research and Development Center
Annapolis, Maryland 21402
Attn: Code 2740
   28
   281

Naval Weapons Center
China Lake, California 93555
Attn: Code 4062
   4520

Commanding Officer
Naval Civil Engineering Laboratory
Code L31
Port Hueneme, California 93041

Naval Surface Weapons Center
White Oak
Silver Spring, Maryland 20910
Attn: Code R-10
   G-402
   K-82

Technical Director
Naval Ocean Systems Center
San Diego, California 92152

Supervisor of Shipbuilding
U.S. Navy
Newport News, Virginia 23607

Navy Underwater Sound Reference Division
Naval Research Laboratory
P.O. Box 8337
Orlando, Florida 32806

Chief of Naval Operations
Department of the Navy
Washington, D.C. 20350
Attn: Code OP-098
Army (Con't.)
Watervliet Arsenal
MAGGS Research Center
Watervliet, New York 12189
Attn: Director of Research

U.S. Army Materials and Mechanics Research Center
Watertown, Massachusetts 02172
Attn: Dr. R. Shea, DREM/R-T

U.S. Army Missile Research and Development Center
Redstone Scientific Information Center
Chief, Document Section
Redstone Arsenal, Alabama 35809

Army Research and Development Center
Fort Belvoir, Virginia 22060

NASA
National Aeronautics and Space Administration
Structures Research Division
Langley Research Center
Langley Station
Hampton, Virginia 23365

National Aeronautics and Space Administration
Associate Administrator for Advanced Research and Technology
Washington, D.C. 20546

Air Force
Wright-Patterson Air Force Base
Dayton, Ohio 45433
Attn: AFFDL (FB)
(FIR) (FEM)
(AFM)

Chief Applied Mechanics Group
U.S. Air Force Institute of Technology
Wright-Patterson Air Force Base
Dayton, Ohio 45433

Air Force (Con't.)
Chief, Civil Engineering Branch
WLRC, Research Division
Kirtland Air Force Base
Albuquerque, New Mexico 87117

Air Force Office of Scientific Research
Rolling Air Force Base
Washington, D.C. 20332
Attn: Mechanics Division

Department of the Air Force
Air University Library
Maxwell Air Force Base
Montgomery, Alabama 36112

Other Government Activities
Commandant
Chief, Testing and Development Division
U.S. Coast Guard
1300 E Street, NW.
Washington, D.C. 20226

Technical Director
Marine Corps Development and Education Command
Quantico, Virginia 22134

Director Defense Research and Engineering
Technical Library
Room 3C128
The Pentagon
Washington, D.C. 20301

Dr. M. Gaus
National Science Foundation
Environmental Research Division
Washington, D.C. 20550

Library of Congress
Science and Technology Division
Washington, D.C. 20540

Director
Defense Nuclear Agency
Washington, D.C. 20303
Attn: SFSS
Other Government Activities (Con't)

Mr. Jerome Persh
Staff Specialist for Materials and Structures
OUSDRAE, The Pentagon
Room 3D1089
Washington, D.C. 20301

Chief, Airframe and Equipment Branch
FS-120
Office of Flight Standards
Federal Aviation Agency
Washington, D.C. 20553

National Academy of Sciences
National Research Council
Ship Hull Research Committee
2101 Constitution Avenue
Washington, D.C. 20418
Attn: Mr. A. R. Lytle

National Science Foundation
Engineering Mechanics Section
Division of Engineering
Washington, D.C. 20550

Picatinny Arsenal
Plastics Technical Evaluation Center
Attn: Technical Information Section
Dover, New Jersey 07801

Maritime Administration
Office of Maritime Technology
14th and Constitution Avenue, NW.
Washington, D.C. 20230

PART 2 - Contractors and Other Technical Collaborators

Universities

Dr. J. Tinsley Oden
University of Texas at Austin
345 Engineering Science Building
Austin, Texas 78712

Professor Julius Miklowitz
California Institute of Technology
Division of Engineering
and Applied Sciences
Pasadena, California 91109

Universities (Con't)

Dr. Harold Liebowitz, Dean
School of Engineering and Applied Science
George Washington University
Washington, D.C. 20052

Professor Eli Sternberg
California Institute of Technology
Division of Engineering and Applied Sciences
Pasadena, California 91109

Professor Paul M. Maghdi
University of California
Department of Mechanical Engineering
Berkeley, California 94720

Professor A. J. Durelli
Oakland University
School of Engineering
Rochester, Michigan 48063

Professor F. L. DiMaggio
Columbia University
Department of Civil Engineering
New York, New York 10027

Professor Norman Jones
The University of Liverpool
Department of Mechanical Engineering
P. O. Box 167
Brownlow Hill
Liverpool L69 3BX
England

Professor E. J. Skudryzk
Pennsylvania State University
Applied Research Laboratory
Department of Physics
State College, Pennsylvania 16801

Professor J. Klosner
Polytechnic Institute of New York
Department of Mechanical and Aerospace Engineering
333 Jay Street
Brooklyn, New York 11201

Professor R. A. Schapery
Texas A&M University
Department of Civil Engineering
College Station, Texas 77843
Universities

Professor Walter D. Pilkey
University of Virginia
Research Laboratories for the
Engineering Sciences and
Applied Sciences
Charlottesville, Virginia 22901

Professor K. D. Willmert
Clarkson College of Technology
Department of Mechanical Engineering
Potsdam, New York 13676

Dr. Walter E. Maisler
Texas A&M University
Aerospace Engineering Department
College Station, Texas 77843

Dr. Hussein A. Kamel
University of Arizona
Department of Aerospace and
Mechanical Engineering
Tucson, Arizona 85721

Dr. S. J. Fenves
Carnegie-Mellon University
Department of Civil Engineering
Schenley Park
Pittsburgh, Pennsylvania 15213

Dr. Ronald L. Huston
Department of Engineering Analysis
University of Cincinnati
Cincinnati, Ohio 45221

Professor G. C. M. Sih
Lehigh University
Institute of Fracture and
Solid Mechanics
Bethlehem, Pennsylvania 18015

Professor Albert S. Kobayashi
University of Washington
Department of Mechanical Engineering
Seattle, Washington 98105

Professor Daniel Frederick
Virginia Polytechnic Institute and
State University
Department of Engineering Mechanics
Blacksburg, Virginia 24061

Professor A. C. Eringen
Princeton University
Department of Aerospace and
Mechanical Sciences
Princeton, New Jersey 08540

Professor E. H. Lee
Stanford University
Division of Engineering Mechanics
Stanford, California 94305

Professor Albert I. King
Wayne State University
Biomechanics Research Center
Detroit, Michigan 48202

Dr. V. R. Hodgson
Wayne State University
School of Medicine
Detroit, Michigan 48202

Dean B. A. Boley
Northwestern University
Department of Civil Engineering
Evanston, Illinois 60201

Professor P. G. Hodge, Jr.
University of Minnesota
Department of Aerospace Engineering and Mechanics
Minneapolis, Minnesota 55455

Dr. D. C. Drucker
University of Illinois
Dean of Engineering
Urbana, Illinois 61801

Professor N. M. Newmark
University of Illinois
Department of Civil Engineering
Urbana, Illinois 61803

Professor E. Reissner
University of California, San Diego
Department of Applied Mechanics
La Jolla, California 92037

Professor William A. Wash
University of Massachusetts
Department of Mechanics and
Aerospace Engineering
Amherst, Massachusetts 01002
Universities (Con't)

Professor G. Herrmann
Stanford University
Department of Applied Mechanics
Stanford, California 94305

Professor J. D. Achenbach
Northwest University
Department of Civil Engineering
Evanston, Illinois 60201

Professor S. B. Dong
University of California
Department of Mechanics
Los Angeles, California 90024

Professor Burt Paul
University of Pennsylvania
Towne School of Civil and Mechanical Engineering
Philadelphia, Pennsylvania 19104

Professor H. W. Liu
Syracuse University
Department of Chemical Engineering and Metallurgy
Syracuse, New York 13210

Professor S. Bodner
Technion R&D Foundation
Haifa, Israel

Professor Werner Goldsmith
University of California
Department of Mechanical Engineering
Berkeley, California 94720

Professor R. S. Rivlin
Lehigh University
Center for the Application of Mathematics
Bethlehem, Pennsylvania 18015

Professor F. A. Cozzarelli
State University of New York at Buffalo
Division of Interdisciplinary Studies
Karr Parker Engineering Building
Chemistry Road
Buffalo, New York 14214

Universities (Con't)

Professor Joseph L. Rose
Drexel University
Department of Mechanical Engineering and Mechanics
Philadelphia, Pennsylvania 19104

Professor B. K. Donaldson
University of Maryland
Aerospace Engineering Department
College Park, Maryland 20742

Professor Joseph A. Clark
Catholic University of America
Department of Mechanical Engineering
Washington, D.C. 20064

Dr. Samuel B. Batdorf
University of California
School of Engineering and Applied Science
Los Angeles, California 90024

Professor Isaac Fried
Boston University
Department of Mathematics
Boston, Massachusetts 02215

Professor E. Krempl
Rensselaer Polytechnic Institute
Division of Engineering Mechanics
Troy, New York 12181

Dr. Jack R. Vinson
University of Delaware
Department of Mechanical and Aerospace Engineering and the Center for Composite Materials
Newark, Delaware 19711

Dr. J. Duffy
Brown University
Division of Engineering
Providence, Rhode Island 02912

Dr. J. L. Swedlow
Carnegie-Mellon University
Department of Mechanical Engineering
Pittsburgh, Pennsylvania 15213
Universities (Con't)

Dr. V. K. Varadan  
Ohio State University Research Foundation  
Department of Engineering Mechanics  
Columbus, Ohio 43210

Dr. Z. Hashin  
University of Pennsylvania  
Department of Metallurgy and Materials Science  
College of Engineering and Applied Science  
Philadelphia, Pennsylvania 19104

Dr. Jackson C. S. Yang  
University of Maryland  
Department of Mechanical Engineering  
College Park, Maryland 20742

Professor T. T. Chang  
University of Akron  
Department of Civil Engineering  
Akron, Ohio 44325

Professor Charles W. Bert  
University of Oklahoma  
School of Aerospace, Mechanical, and Nuclear Engineering  
Norman, Oklahoma 73019

Professor Satyam N. Atluri  
Georgia Institute of Technology  
School of Engineering and Mechanics  
Atlanta, Georgia 30332

Professor Graham F. Carey  
University of Texas at Austin  
Department of Aerospace Engineering and Engineering Mechanics  
Austin, Texas 78712

Dr. S. S. Wang  
University of Illinois  
Department of Theoretical and Applied Mechanics  
Urbana, Illinois 61801

Professor J. P. Abel  
Cornell University  
Department of Theoretical and Applied Mechanics  
Ithaca, New York 14853

Universities (Con't)

Professor V. H. Neubert  
Pennsylvania State University  
Department of Engineering Science and Mechanics  
University Park, Pennsylvania 16802

Professor A. W. Leissa  
Ohio State University  
Department of Engineering Mechanics  
Columbus, Ohio 43212

Professor C. A. Brebbia  
University of California, Irvine  
Department of Civil Engineering  
Irvine, California 92717

Dr. George T. Rahn  
Vanderbilt University  
Mechanical Engineering and Materials Science  
Nashville, Tennessee 37235

Dean Richard B. Gallagher  
University of Arizona  
College of Engineering  
Tucson, Arizona 85721

Professor E. F. Rybicki  
The University of Tulsa  
Department of Mechanical Engineering  
Tulsa, Oklahoma 74104

Dr. R. Raftka  
Illinois Institute of Technology  
Department of Mechanics and Mechanical and Aerospace Engineering  
Chicago, Illinois 60616

Professor J. G. de Oliveira  
Massachusetts Institute of Technology  
Department of Ocean Engineering  
77 Massachusetts Avenue  
Cambridge, Massachusetts 02139

Dr. Bernard W. Shaffer  
Polytechnic Institute of New York  
Route 110  
Farmingdale, New York 11735
Industry and Research Institutes

Dr. Norman Hobbs
Kaman AviDyne
Division of Kaman
Sciences Corporation
Burlington, Massachusetts 01803

Argonne National Laboratory
Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. H. C. Junger
Cambridge Acoustical Associates
54 Rindge Avenue Extension
Cambridge, Massachusetts 02140

Dr. R. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215

Newport News Shipbuilding and
Dry Dock Company
Library
Newport News, Virginia 23607

Dr. W. F. Bozich
McDonnell Douglas Corporation
3301 Bolsa Avenue
Huntington Beach, California 92647

Dr. H. A. Abramson
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeRart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Industry and Research Institutes (Con't)

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacificas Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. W. F. Dozich
McDonnell Douglas Corporation
5301 Balsa Avenue
Huntington Beach, California 92647

Dr. H. N. Abramson
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. I. D. Mindlin
Tribological Associates
89 Deer Hill Drive
Ridgefield, Connecticut 06877
Industry and Research Institutes

Dr. Norman Robb
Kaman AviDyne
Division of Kaman
Sciences Corporation
Burlington, Massachusetts 01803

Argonne National Laboratory
Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedaln Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Dr. B. D. Eibbit
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. H. D. Mibbit
Mibbit & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877

Newport News Shipbuilding and Dry Dock Company
Library
Newport News, Virginia 23607

Dr. W. F. Bozich
McDonnell Douglas Corporation
5301 Bolsa Avenue
Huntington Beach, California 92647

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. M. C. Junger
Cambridge Acoustics Associates
94 Ridge Avenue Extension
Cambridge, Massachusetts 02140

Mr. J. R. Torrance
General Dynamics Corporation
Electric Boat Division
Groton, Connecticut 06340

Dr. J. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215

Dr. Robert T. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedaln Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Dr. B. D. Eibbit
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. M. C. Junger
Cambridge Acoustics Associates
94 Ridge Avenue Extension
Cambridge, Massachusetts 02140

Mr. J. R. Torrance
General Dynamics Corporation
Electric Boat Division
Groton, Connecticut 06340

Dr. J. E. Greenspon
J. G. Engineering Research Associates
3831 Menlo Drive
Baltimore, Maryland 21215

Dr. W. F. Bozich
McDonnell Douglas Corporation
5301 Bolsa Avenue
Huntington Beach, California 92647

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. B. D. Eibbit
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. B. D. Eibbit
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Library Services Department
9700 South Cass Avenue
Argonne, Illinois 60440

Dr. B. D. Eibbit
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. R. C. DeHart
Southwest Research Institute
8500 Culebra Road
San Antonio, Texas 78284

Dr. M. L. Baron
Weidlinger Associates
110 East 59th Street
New York, New York 10022

Industry and Research Institutes

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedaln Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. E. D. Hambitt
Hambitt & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877

Industry and Research Institutes

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedaln Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. H. D. Hambitt
Hambitt & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877

Industry and Research Institutes

Dr. T. L. Geers
Lockheed Missiles and Space Company
3251 Hanover Street
Palo Alto, California 94304

Mr. William Caywood
Applied Physics Laboratory
Johns Hopkins Road
Laurel, Maryland 20810

Dr. Robert E. Dunham
Pacifica Technology
P.O. Box 148
Del Mar, California 92014

Dr. M. F. Kanninen
Battelle Columbus Laboratories
505 King Avenue
Columbus, Ohio 43201

Dr. A. A. Hochrein
Daedaln Associates, Inc.
Springlake Research Road
15110 Frederick Road
Woodbine, Maryland 21797

Dr. James W. Jones
Swanson Service Corporation
P.O. Box 5415
Huntington Beach, California 92646

Dr. Robert E. Nickell
Applied Science and Technology
3344 North Torrey Pines Court
Suite 220
La Jolla, California 92037

Dr. Kevin Thomas
Westinghouse Electric Corp.
Advanced Reactors Division
P. O. Box 158
Madison, Pennsylvania 15663

Dr. H. D. Hambitt
Hambitt & Karlsson, Inc.
132 George M. Cohan Boulevard
Providence, Rhode Island 02903

Dr. R. D. Mindlin
89 Deer Hill Drive
Ridgefield, Connecticut 06877
Industry and Research Institutes (Con't)

Dr. Richard E. Dame
Mega Engineering
11961 Tech Road
Silver Spring, Maryland 20904

Mr. G. M. Stanley
Lockheed Palo Alto Research Laboratory
3251 Hanover Street
Palo Alto, California 94304

Mr. R. L. Cloud
Robert L. Cloud Associates, Inc.
2972 Adeline Street
Berkeley, California 94703