Research Report CCS 460

A DEVELOPMENTAL STUDY OF DATA ENVELOPMENT ANALYSIS IN MEASURING THE EFFICIENCY OF MAINTENANCE UNITS IN THE U.S. AIR FORCES

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1. INTRODUCTION.

There are four basic questions related to efficiency and capability which are of particular interest to officials in the military services who are interested in better ways of evaluating military capability and efficiency: (i) What level of military capability can the services achieve with available resources? (ii) What capability is required, and where are the shortfalls? (iii) What resource acquisitions or redistributions are needed to achieve maximum improvement in efficiency and effectiveness? and (iv) How can management systems be changed to improve the identification and correction of factors which limit the readiness and efficiency of our military operations?

The last question, which differs in its emphasis from the other three, provides an opening to the topics that will be addressed in this report. In particular, ~~we shall report the~~ results from a study of DEA (Data Envelopment Analysis) as a method for evaluating the efficiency of Air Force Wings--or, more precisely, their maintenance operations--as elements in Numbered Units in the U. S. Air Force.

By way of background, DEA is an approach to evaluating not-for-profit entities which has emerged from research by A. Charnes, W. W. Cooper and colleagues. In addition to the initial formulations, these persons have been responsible for a series of studies directed to testing DEA for use in measuring the efficiency of a wide variety of not-for-profit institutions ranging from schools and hospitals to army recruitment districts.1/ In such applications, we may note, DEA has advantages like the following: (1) It

1/ See Rhodes [14], Sherman [15] and Charnes, Cooper, Divine, Klopp and Stutz [7] as well as [8] and [10].

deals directly with multiple outputs and multiple inputs and yields an overall measure of relative efficiency for each pertinent decision making unit without requiring either (a) an a priori selection of weights or (b) explicitly hypothesized forms of relations between the various inputs and outputs. (2) Possible sources of inefficiency in each decision making unit are pinpointed for further study. Finally, (3) tradeoff ratios are provided for still further improvement even after efficiency has been attained.

The focus of this report will be on point (1), with points (2) and (3) being dealt with only sketchily. The DMUs (= Decision Making Units) selected for this study are 14 Air Force Wings which are elements of two Numbered Air Forces. Their efficiency will be determine by reference to output and input variables which are commonly used in gauging the performance of aircraft maintenance units in these operations.

Formally, 100% efficiency is attained for any DMU only when

(a) None of its outputs can be increased without either
 (i) Increasing one or more of its inputs or
 (ii) Decreasing some of its other outputs

(b) None of its inputs can be decreased without either
 (i) Decreasing some of its outputs or
 (ii) Increasing some of its other inputs.

Via this definition we avoid the need for assigning a priori measures of relative importance to the different outputs and inputs. Hence output or input inefficiencies, when identified in any DMU, may then be corrected without worsening any other input or output. Moreover, we are not confined to indexes for ranking or ordering of inefficient DMUs. In particular, we will be able to estimate amounts of inefficiency in each such DMU and we shall be able to identify where the inefficiencies are located (i. e., in which inputs or outputs). How these amounts are determined by means of DEA models and methods of analysis and interpretation will be described in the sections that follow.

The above definition is formulated so that efficiency may be determined relative to prior theoretical knowledge or on the basis of some predetermined norm from studies such as are available in physics and engineering, or like disciplines and endeavors. For the kinds of situations which are of interest in this report, however, such prior knowledge will generally not be available. Hence we extend the above definition to one which involves only relative efficiency for use on the kind of data that are likely to be available:

> 100% relative efficiency is attained by any DMU only when comparisons with other relevant DMUs do not provide evidence of inefficiency in the use of any input or output.

Via this characterization the preceding definition is adjusted for immediate application to data of the kind we shall be considering. We should also note, however, that other combinations of the above definitions are also possible so that, in addition, pertinent aspects of any theoretically grounded norms or other types of available knowledge may also be used in common with other data when required.

We now conclude this introductory section by pointing to other studies which have compared DEA to other approaches such as ratio and regression analyses, simulation and similar approaches. For instance, D. Sherman in [15] compared DEA with ratio and regression approaches for use on data generated by Massachusetts hospitals. In this comparison DEA performance was generally favored because it avoided being restricted to considering only 1 or 2 variables at a time--perhaps at aggregated levels--which is an inherent limitation of ratio approaches. Similarly, recourse to DEA made it possible to avoid the assumptions of known functional forms between the inputs and outputs--as is required in regression approaches--along with the difficulties that are often encountered in such approaches when dealing with multiple variables that interact in other than theoretically prescribed manners, and so on. Finally, as shown in [10], the optimizing principles used in regression and DEA are quite different and hence may be expected to yield results which will favor DEA when evaluation of individual DMUs is desired. Regression approaches, for example, tend to optimize by reference to "average" or "representative" behavior--as determined via a least squares analysis--across all observations. DEA, on the other hand, optimizes on each observation and does so not by reference to "average" or "representative" behavior but by reference to subsets of "efficient" DMUs.1/

The manner in which this is accomplished will be described in the sections that follow. Here we are only referencing the fact that such comparisons and evaluations have been made. Finally, therefore, we also note that, in contrast to simulation approaches, DEA does involve optimizations that make it possible to identify its results with available bodies of knowledge in economics, management science and mathematics

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1/ As shown in [1], DEA exhibits similar advantages even relative to regressions adjusted to obtain frontier estimates.

(which are based on similar optimizing principles). Moreover, as will be seen, DEA lends itself to straightforward uses directly on the data without the need for developing numerous intermediate treatments and relations, as in most simulation modeling. Finally, computer codes for effecting such DEA studies are now freely available so that possible trouble from this quarter is also avoided.[1/]

2. THE DEA MODEL

A fuller theoretical development of DEA is provided in the appendix to this report. Here we therefore record only the following version of the Charnes Cooper Rhodes (CCR)[2/] ratio form of the model in order to explain what is being done in the sections that follow.

Objective:

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

Constraints: (1)

Less than Unity Constraints: $$1 \geq \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \; ; \; j = 1,\ldots,14 \quad \text{DMUs(= Air Force Wings)}$$

Positivity Constraints: $0 < u_r \; ; \; r = 1,\ldots,s$
$0 < v_i \; ; \; i = 1,\ldots,m$

Data:

Outputs: y_{rj} = observed amount of r^{th} output for j^{th} Air Force Wing

Inputs: x_{ij} = observed amount of i^{th} input for j^{th} Air Force Wing

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1/ A code as developed by A. Ali and J. Stutz for these purposes is available from the Center for Cybernetic Studies, BEB 454C, The University of Texas at Austin, Austin, Texas 78712

2/ See [8] and [9].

Any one of the DMUs represented in the constraints may be selected for such an evaluation. The selected DMU is then represented in the objective as well as in the constraints. Hence we must have max. $h_0 = h_0^* \leq 1$ in order to satisfy the constraints. Within this limitation the optimal $u_i = u_i^*$ and optimal $v_r = v_r^*$ are then chosen so that DMU_0, the DMU being rated, is accorded the highest possible h_0 value.

All of the observed outputs, y_{rj}, and all of the observed inputs, x_{ij}, are assumed to be available as known positive constants. That is, each of the j = 1, 2,..., n DMUs is assumed to have used positive amounts of each pertinent input and to have produced positive amounts of each pertinent output.

Some of the inputs may be varied at the discretion of a manager and some may not. An example of such a "non-discretionary" input is provided by the weather,[1/] e. g., in the form of flying days or days available for flying. Nevertheless, the efficiency rating max $h_0 = h_0^*$ should take this input into account and, moreover, it should do so with the mixes of inputs and outputs (whatever they may be) which are pertinent to the performance of all aircraft wings that utilize these same inputs and produce these same outputs. Hence, both discretionary and non-discretionary inputs were used in the study that will be discussed in this paper, and, moreover, a mix of various types of wings (training and operational) as well as various types of aircraft were used in this study in order to test possible further ranges and limits of DEA.

1/ Further special attention which is needed to treat such non-discretionary (or partially non-discrationary) inputs is not discussed in this report.

As shown in (1), all 14 wings are represented in the less-than-unity constraints. As will later be seen, however, it is the job of DEA to single out a "best" comparison set (or rather subset) for determining the efficiency value of each DMU being rated.

This topic will subsequently be developed in more detail. Here we may note that the "positivity constraints" imply that each output and each input has "some" value with, as already noted, specific optimal values u_r^* and v_i^* assigned to each such output and input. Optimality is determined in each case by reference to the DMU being evaluated, which is represented in the objective as well as the constraints a maximum $h_o = h_o^* \leq 1$ resulting.

These optimal u_r^* and v_i^* have a variety of uses in their own right as when, for instance, they are employed to determine further tradeoff possibilities after efficiency has been attained. They are also called "virtual rates of transformation" which define a "virtual output"

$$y_o = \sum_{r=1}^{s} u_r \, y_{ro} \qquad (2.1)$$

and a "virtual input"

$$x_o = \sum_{i=1}^{m} v_i \, x_{io} \qquad (2.2)$$

so that also

$$h_o = y_o / x_o, \qquad (2.3)$$

with

$$h_o^* = y_o^* / x_o^* \qquad (2.4)$$

when an optimum is achieved. In other words, our definition (1) still retains contact with the classical ratio definitions of efficiency in engineering, physics (and other fields), while also accommodating multiple output and multiple input situations.

3. MAINTENANCE PERFORMANCE STUDY DETAILS

Air Force tactical fighter wings are expected to maintain the readiness of their assigned aircrews, tactical fighter aircraft and ground support. This is a complex problem and the following sections present a small numerical example which can help to provide insight into the complexity of the wing maintenance evaluation proplem. This will then provide a basis for illustrating the possible uses of DEA in such contexts. Although obtained from actual Air Force data bases, the numbers used in this study have been treated and abbreviated in a variety of ways so that the results reported here should only be regarded as illustrative in character.

The input and output measures in this analysis are similar to those used by Air Force commanders and resource managers. Chosen to highlight key objectives and operating characteristics, the inputs and outputs selected for this study reflect, either directly or indirectly, the following peace-time initiatives of wing aircraft maintenance organizations.

Data were obtained from fourteen actual tactical fighter wings, eight of which are organized under one intermediate headquarters (I) and the remaining six under another (II). Both intermediate headquarters report to the Tactical Air Command (TAC) Headquarters. The selected wings (A, B,..., N) fall into one of three mission categories: (1) combat operations, (2) training, or (3) both; and each wing has one assigned aircraft type. The identity of the wings will not be divulged since the results reported here are illustrative. See Table 2 for generic classifications of wing and aircraft types.

TABLE 1

WING AIRCRAFT MAINTENANCE OPERATIONS

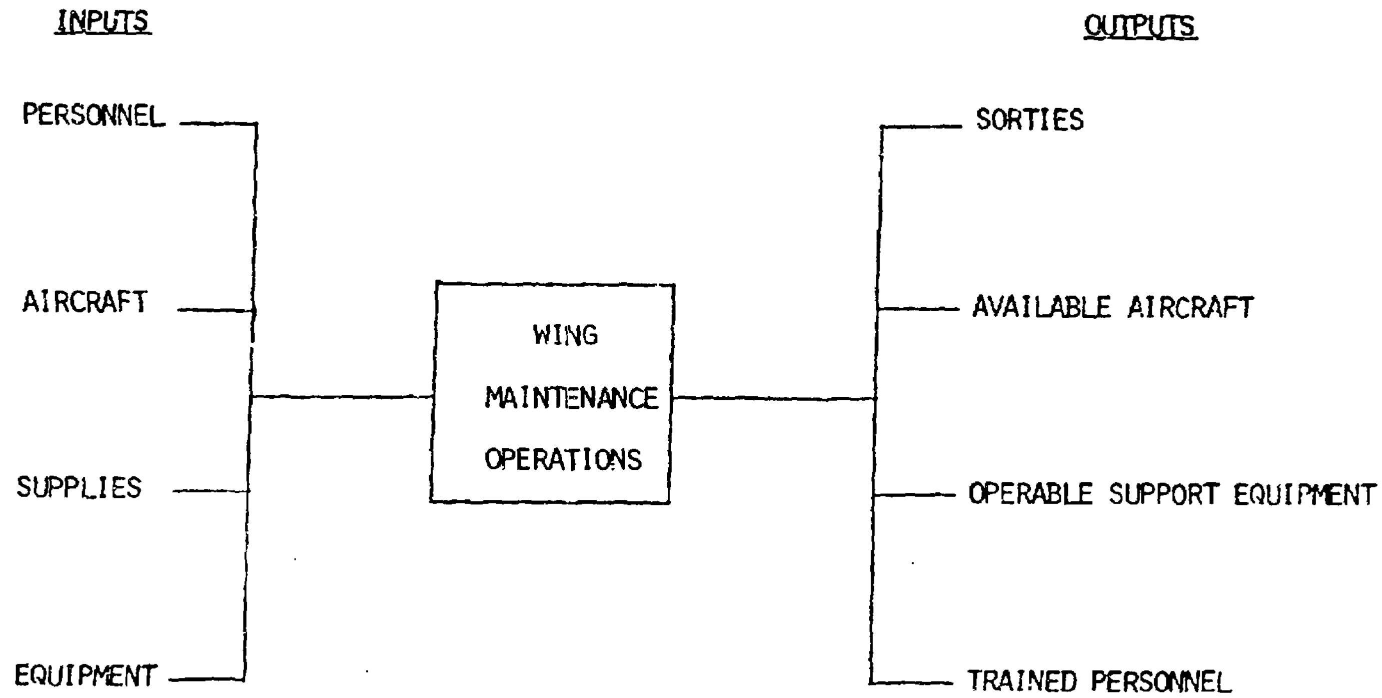

TABLE 2

TACTICAL AIR COMMAND AIRCRAFT MAINTENANCE UNITS IN THE ANALYSIS

NUMBERED AIR FORCES	TACTICAL FIGHTER WINGS (TFWS) REPRESENTED	WING TYPES	AIRCRAFT ASSIGNED AGE	AIRCRAFT ASSIGNED COMPLEXITY
I	A	OPERATIONS	NEW	COMPLEX*
	B	OPERATIONS	NEW	COMPLEX
	C	OPERATIONS	OLD	COMPLEX
	D	OPERATIONS	OLD	COMPLEX
	E	OPERATIONS	NEW	SIMPLE**
	F	TRAINING & OPERATIONS	VERY OLD	COMPLEX
	G	TRAINING	NEW	COMPLEX
	H	TRAINING	NEW	COMPLEX
II	I	OPERATIONS	NEW	COMPLEX
	J	TRAINING & OPERATIONS	OLD	VERY COMPLEX
	K	TRAINING & OPERATIONS	OLD	VERY COMPLEX
	L	TRAINING	NEW	COMPLEX
	M	TRAINING	OLD	COMPLEX
	N	TRAINING	NEW	SIMPLE

*COMPLEX: DIFFICULT TO TROUBLESHOOT AND FIX

**SIMPLE: EASY TO TROUBLESHOOT AND FIX

The four outputs and eight inputs used in this study, with accompanying definitions of the abbreviations employed, are shown in Table 3. Thus, TOTSORTFLO refers to the total number of sorties flown while MICAPHOURS refers to the total number of hours in which the aircraft possessed by each wing were fully or partially mission capable. It should be noted that the first output is measured in units of 10 while the second one is not similarly adjusted. This is done for computational convenience, as may be done with DEA, since the optimum $h_0 = h_0^*$ does not vary with the units of measurement employed. MICAPHRS, on the other hand, is stated in the originally reported units.

TNMCHM, the total number of hours in which aircraft in each wing were <u>not</u> mission capable, is treated in yet another manner. It is measured by subtracting the observed values from 100,000 hours. That is for this variable we use

$$\text{TNMCHM} = 100{,}000 - \overline{\text{TNMCHM}}$$

so that TNMCHM increases as $\overline{\text{TNMCHM}}$, the observed (= reported) value, decreases and <u>vice versa</u>. Any other similarly "large" number could have been used in place of 100,000 hours. The idea is to preserve the isotone property for this output (see Appendix).

This is the approach that was used for this output and other outputs and inputs, as indicated in Table 3, for the monthly evaluations that are reported in Tables 6 and 7. For the cumulative evaluations reported in Tables 4 and 5, however, an alternate approach was used in which these output and input values were converted to reciprocals and the results stated in units of 10^8 hours. This was done to maintain comparability with the magnitudes of other inputs and outputs--and in order to reduce the possibility of efficiency values being affected by computational (roundoff) error--in the data used for the cumulative evaluations reported in Tables 4 and 5.

Such scalings and/or rescalings, we might note, are admissible in a DEA analysis since they do not affect the resulting h_o^* values. This, too, is an advantage not only for purposes of computation but also for avoiding the need for recourse to the elaborate procedures that are sometimes needed to avoid the effects of using different scales in different parts of the same study. In other words, the properties of DEA modeling are such that the different scales used in Tables 4 and 5 and in Tables 6 and 7 do not alter or destroy the comparability of the resulting efficiency values.

Similar remarks are applicable to the inputs with one additional feature that might be observed in AIRMENMEAS. As noted in its definition, this measure is intended to accommodate a comparison between authorized or planned and assigned or actual inputs of airmen. This is accommodated by forming the ratio of the latter to the former.[1/]

1/ Such ratio approaches can cause difficulties that need to be guarded against in uses of DEA. See Sherman [15]. In the present study these ratios were used so that the resulting solutions could also make it possible for higher commands to ascertain the effects of their authorizations on wing efficiencies.

TABLE 3

INPUT/OUTPUT VARIABLE DEFINITION

OUTPUTS	INPUTS
TOTSORTFLO	TOTPOSSAIR
MICAPHOURS	TNMCHS
TNMCHM	OFASSIGNED
FIXRATE	OFMEASURE
	AIRMENASSI
	AIRMENMEAS
	NOCHARGLOS
	CANNRATE

DEFINITIONS OF INPUT AND OUTPUT VARIABLES

TOTSORTFLO	Total number of sorties flown by each wing in a specific month (x 10)
MICAPHOURS	Total number of hours in a specific month in which the possessed aircraft in each wing were fully or partially mission capable
TNMCHM	100,000 minus total number of hours in a specific month in which the possessed aircraft in each wing were not mission capable due to maintenance problems*
FIXRATE	The percentage of code 3 breaks in a wing in a specific month which were fixed within given time intervals
TOTPOSSAIR	The average number of aircraft in the wing in a specific month (x 100)
TNMCHS	100,000 minus total number of hours in a specific month in which the possessed aircraft in each wing were not mission capable due to supply problems*
OFASSIGNED	Number of officers assigned to each wing in a specific month (x 1,000)
OFMEASURE	Assigned officers divided by authorized officers in a wing (x 1,000)
AIRMENASSI	Number of airmen assigned to each wing in a specific month
AIRMENMEAS	Assigned airmen divided by authorized airmen in a wing (x 1,000)
NOCHARGLOS	1,000 minus number of sortie losses due to external reasons (like weather, higher headquarters, air traffic control, etc.) (x 10)
CANNRATE	100 minus total number of cannibalizations in a wing in a specific month divided by the number of sorties flown (x 1,000)

*This was the approach used for the monthly evaluations used in Tables 6 and 7. For the cumulative runs reported in Tables 4 and 5, however, an alternate approach was used in which reciprocals of the reported outputs were used and restated in units of 10^8 hours.

4. PRELIMINARY RESULTS

For purposes of the present study, attention was restricted to already available data for selected months during the period October 1981 through May 1982. Manpower data were obtained from the Headquarters, Airforce Manpower and Personnel Center. The other data were obtained from information supplied by Tactical Air Command Headquarters.

Table 4 presents the results of a cumulative run, i. e., one run, for the entire period, October 1981 through May 1982, using all 4 outputs and 8 inputs defined in Table 3. In Table 5 the grouping is by Type of Mission whereas the grouping in Table 4 is by NAF (= Numbered Air Force) units.

Appraisal of the units achieving 100% efficiency needs to be held in abeyance until some of the qualifications noted below are attended to. Also, for the present, attention in these tables is confined only to the overall "global" efficiency ratings assigned to the maintenance operations of each of these wings by DEA.

With these provisos we may observe that TFW K (TFW = Tactical Fighter Wing), flying old and very complex aircraft, has the lowest efficiency rating. See No. 11 in Table 4. Turning to Table 5, it is seen that this 0.63 efficiency rating is substantially below the other DMUs which are also engaged in both Operations and Training missions. On the other hand, none of the DMUs which have only Operations missions and none of the DMUs which have only Training missions are flying the kins of aircraft being used by the 3 wings which have both Operations and Training missions. Finally, the 100% efficiency rating assigned to TFW J in NAF II, which also flies old and very complex aircraft, is subject to qualification for reasons that are noted below.1/

1/ See also the Appendix where the possibility of "self evaluation" is discussed along with ways in which its occurrence may be identified.

TABLE 4

CUMULATIVE RUN BY NAF

4 OUTPUTS, 8 INPUTS

NO.	NAF	DMU, MISSION, AIRCRAFT	EFFICIENCY
1	I	TFW A, OPS, New & Complex	1.00000
2	I	TFW B, OPS, New & Complex	.89355
3	I	TFW C, OPS, Old & Complex	.89377
4	I	TFW D, OPS, Old & Complex	.90877
5	I	TFW E, OPS, New & Simple	1.00000
6	I	TFW F, OPS & TNG, Very Old & Complex	.86043
7	I	TFW G, TNG, New & Complex	.99581
8	I	TFW H, TNG, New & Complex	1.00000
9	II	TFW I, OPS, New & Complex	.99731
10	II	TFW J, OPS & TNG, Old & Very Complex	1.00000
11	II	TFW K, OPS & TNG, Old & Very Complex	.62381
12	II	TFW L, TNG, New & Complex	1.00000
13	II	TFW M, TNG, Old & Complex	1.00000
14	II	TFW N, TNG, New & Simple	1.00000

TABLE 5

CUMULATIVE RUN BY TYPE OF MISSION

4 OUTPUTS, 8 INPUTS

NO.	TYPE	DMU, AIRCRAFT	EFFICIENCY
1	OPS	TFW A, New & Complex	1.00000
2	OPS	TFW B, New & Complex	.89355
3	OPS	TFW I, New & Complex	1.00000
4	OPS	TFW C, Old & Complex	.89377
5	OPS	TFW D, Old & Complex	.90877
6	OPS	TFW E, New & Simple	1.00000
7	OPS & TNG	TFW F, Very Old & Complex	.86043
8	OPS & TNG	TFW J, Old & Very Complex	1.00000
9	OPS & TNG	TFW K, Old & Very Complex	.62381
10	TNG	TFW G, New & Complex	.99581
11	TNG	TFW M, Old & Complex	1.00000
12	TNG	TFW H, New & Complex	1.00000
13	TNG	TFW L, New & Complex	1.00000
14	TNG	TFW N, New & Complex	1.00000

6. WINDOW ANALYSES

As noted earlier in this report, the optimization used in DEA differs from the ones used in statistics, e. g., in least squares regression and correlation analyses. The latter are directed to the optimizations across all observations so that the results are applicable to the "class" properties of all of these observations rather than to any one of them in particular. In DEA an opposite orientation is used. The optimizations are undertaken with reference to each observation as, one after another, the different DMUs are positioned in the objective of (1) for evaluation.

In a real sense, then, a DEA analysis should be tested for its results by reference to the behavior of each DMU that may be of interest within the observation set. Nevertheless, any DEA analysis can, and should, be tested in a variety of ways besides the ones admitted by direct observation. The mathematical optimizations and the structural properties of a DEA model can be drawn on for these purposes.

The "window analyses"1/ of Tables 6 and 7 may be used for illustration. The results reported in the preceding Tables 4 and 5 involved 4 inputs and 8 outputs with all values cumulated across the entire period for use in evaluating each of 14 DMUs. This comes close to pressing the limits of the degrees-of-freedom requirements for effecting such efficiency evaluations in DEA--or any other similar evaluation such as, e. g., those of a statistical variety. Hence it is desirable to employ methods which introduce more "degrees of freedom" into the analysis and which here, co-incidentally, can also serve to check other results.

1/ These "window analyses" were first employed in [7] for DEA evaluations of Army recruitment district offices.

Table 6, the Window Analysis for the wings in NAF I, provides an illustration. Each wing is represented as if it were a different DMU for each of 3 successive months. Then a new 3-month set, or "window," is erected in a similar manner starting with the second month and the process then continues for the results portrayed in Table 6.

We illustrate as follows: For the first "window," TFW A is represented in the constraints of (1) as though it were a different DMU in October 81, November 81 and December 81. Therefore when TFW A is to be evaluated for its October 81 efficiency, its own performance data for November 81 and December 81 for TFW A are also included in the constraint sets along with a similar 3-period representation of the other TFWs in NAF I. The efficiency values of 97.89 and 97.31 as obtained in this manner thus apply to TFW A for October 81 and November 81, respectively, with the value of $h_0^* = 98.14$ resulting from a similar insertion of the December 81 behavior into the objective of (1) for evaluation. To recapitulate, the efficiency of TFW A in each of these months is thereby rated relative to its own performance as well as the performance of the other wings in NAF I in each of these three months. This, in effect, provides a 3-fold increase in the number of DMUs considered in these evaluations.

Notice that the successive overlapping windows provide a means of assessing the temporal behavior of the wings. This can be of importance in its own right, as when, e. g., in the case of Army recruitment evaluations it was desirable to examine the behavior of recruitment districts in different quarters of the year to allow for substantial seasonal variations. The overlapping windows also provide a basis for evaluating the stability of the efficiency ratings achieved by the DMUs (= TFWs) when they are obtained from different data sets.

TABLE 6

"WINDOW" ANALYSIS BY MONTHS OF WINGS' EFFICIENCY: NAF I

WING	OCT 81	NOV 81	DEC 81	JAN 82	MAR 82	APR 82	MAY 82
TFW A	97.89	97.31	98.14				
		97.36	97.53	97.04			
			96.21	95.92	94.54		
				95.79	94.63	97.64	
					94.33	97.24	97.74
TFW B	93.90	95.67	96.14				
		96.72	96.42	94.63			
			95.75	94.14	93.26		
				94.54	93.46	96.02	
					93.02	96.02	94.49
TFW C	93.77	91.53	95.26				
		91.77	95.55	94.29			
			93.21	95.04	94.83		
				93.20	93.09	92.21	
					93.59	92.32	92.83
TFW D	99.72	96.15	95.06				
		97.91	95.70	1.000			
			94.79	1.000	94.51		
				99.71	94.39	94.76	
					94.95	94.67	89.37
TFW E	1.000	1.000	1.000				
		1.000	1.000	1.000			
			98.97	99.05	1.000		
				99.37	1.000	1.000	
					1.000	1.000	1.000
TFW F	97.42	93.48	96.07				
		93.00	96.24	93.56			
			94.46	91.75	92.49		
				91.73	92.32	92.35	
					92.68	91.98	99.64
TFW G	90.98	92.80	95.96				
		93.67	96.80	99.52			
			93.34	94.48*	91.73		
				91.94	89.79	95.58	
					89.35	95.14	96.38
TFW H	1.000	1.000	1.000				
		1.000	1.000	1.000			
			1.000	1.000	1.000		
				1.000	1.000	1.000	
					1.000	1.000	1.000

*Extreme mismatch (over 0.05 change)

Consider, for instance, the second line in Table 6 where still another value of $h_0^* = 97.36$ appears for the November 81 efficiency rating of TFW A. This rating emerges from yet another set of 3 x 14 = 52 constraining relations used in the evaluation of TFW A when it is placed in the objective of (1) and related to the data of November 81, December 81 and January 82 for each of the TFW in NAF I. Evidently the value of 97.36 does not differ very much from the 97.31 efficiency value secured from the preceding set for the November 81 behavior of TFW A of NAF I. Indeed, with these "same month" comparisons only 1 pair of values, as show for TFW G in January 82, differs by more than 5% so that, in general, these efficiency ratings exhibit stable behavior.

Turning to Table 7 we are now in a position to say something more about the efficiency evaluations of TFWs J and K which attracted our attention in Table 5. Although never falling to the very low value of $h_0^* = .63$ to which it accumulated in Table 5, the efficiency ratings for wing K are generally low and declining. The one reversal, 93.74, which is starred in January 82 is really very far out of line with the other values in this set and needs to be marked for further investigation as to possible mis-reporting or to some error in the data or the computations.

Turning to the efficiency evaluations for TFW J in Table 7, we find that we have additional reasons to believe that the 100% efficiency rating exhibited for this DMU in Tables 4 and 5 is likely to have resulted from special (and possibly misleading) features in the analyses underlying those Tables. Note that the efficiency ratings of TFW J are persistently below those of all of the other wings except for wing K. In contrast to the latter, however, the efficiency ratings of TFW J do exhibit an upward trend, especially in the later parts of the period and this, too, is information of potential use in evaluating its behavior.

TABLE 7

"WINDOW" ANALYSIS BY MONTHS OF WINGS' EFFICIENCY: NAF II

WING	OCT 81	NOV 81	DEC 81	JAN 82	MAR 82	APR 82	MAY 82
TFW I	99.11	95.94	99.76				
		96.04	1.000	1.000			
			98.16	98.99	94.59		
				98.97	94.62	99.16	
					94.68	98.92	97.28
TFW J	92.85	90.90	91.62				
		91.50	92.12	94.75			
			90.26	93.39	93.83		
				92.92	93.84	95.33	
					94.52	96.07	94.43
TFW K	86.25	84.42	84.03				
		84.98	84.47	93.74*			
			83.37	82.54	80.26		
				82.39	80.14	79.58	
					80.96	78.66	79.75
TFW L	1.000	1.000	1.000				
		1.000	1.000	99.55			
			1.000	99.39	97.39		
				1.000	96.85	1.000	
					96.66	1.000	1.000
TFW M	1.000	1.000	1.000				
		1.000	1.000	1.000			
			1.000	1.000	1.000		
				1.000	1.000	98.75	
					1.000	98.51	99.59
TFW N	1.000	1.000	98.63				
		1.000	1.000	1.000			
			99.45	1.000	1.000		
				1.000	1.000	1.000	
					1.000	1.000	1.000

*Extreme mismatch (over 0.05 change)

Table 8, below, summarizes the results in Tables 6 and 7 in yet another way. Note, for example, that TFW K has not only the lowest mean but it also has the highest variance in its efficiency ratings. Part of the latter may be due to the unusually high rating of 93.74 in the January 82 evaluation of this DMU which, as we previously observed, should be set aside for further examination. On the other hand, low means tend to be accompanied by high variances in Table 8, with the possible exception of TFW D. The latter, which also appears in group D for Table 8, however, does have a marked downturn in its efficiency ratings starting in March 82--see Table 6--and this, too, should be investigated in more detail.

8. EXAMPLE SOLUTION

Table 9, below, will help to interpret Table 5 and it will also provide additional information in its own right. It provides part of a computer printout which includes the $h_o^* = 0.86$ value for DMU 7 = TFW F that was reported in the cumulative evaluation used for Table 5.

Turning first to the outputs we note that the "value if efficient" is obtained by simply adding the "slack" value to the "value measured" (= reported or observed value). E. g., the 8,127 for CUMSORTFLO (= Cumulative Sorties Flown) is obtained by adding the 314, under "slack", to the 7,813 under "value measured."

The other output "values if efficient" recorded in the last column of Table 9 are obtained in the same way with, of course, those having zero slack requiring no such addition to achieve their "value if efficient."

The input "values if efficient" are obtained in a somewhat more elaborate way by first applying the efficiency value of $h_o^* = 0.86$ to the value measured and then subtracting the slack. The basic idea can be illustrated via the NOCHARGLOS (= Number of sortie losses due to external

TABLE 8

MEAN-VARIANCE ANALYSIS ACROSS TIME

WING	MEAN	VARIANCE	GROUP
TFW A	96.62	1.567	B
TFW B	94.95	1.419	B
TFW C	93.50	1.523	B
TFW D	96.11	7.913	D
TFW E	99.89	0.085	A
TFW F	93.98	5.167	C
TFW G	93.87	7.400	D
TFW H	100.00	0.000	A
TFW I	97.75	3.880	C
TFW J	93.22	2.664	C
TFW K	83.04	13.114	D
TFW L	99.32	1.439	B
TFW M	99.79	0.219	A
TFW N	99.96	0.019	A

Groups: A - very low σ^2
B - low σ^2
C - medium size σ^2
D - high σ^2

TABLE 9

SOLUTION FOR DMU 7* = TFW F

EFFICIENCY = 0.86

FACET = 11, 8

OUTPUTS:	VALUE MEASURED	SLACK	VALUE IF EFFICIENT
CUMSORTFLO	7,813	314	8,127
MICAPHOURS	26,225	0	26,225
TNMCHM	1,380	1,275	2,655
FIXRATE	4,786	0	4,786
INPUTS:			
NOCHARGLOS	9,841	2,000	6,463
CUMPOSSAIR	5,945	0	5,113
TNMCHS	4,800	1,915	2,213
OFASSIGNED	1,900	293	1,341
OFMEASURE	1,117	255	705
AIRMENASSI	1,376	279	903
AIRMENMEAS	1,097	223	720
CANRATE	9,382	1,546	6,522

* The DMU number is keyed to Table 5.

reasons like weather) as follows: 0.86 x 9,841 - 2,000 = 6,463. In other words, the value if efficient is obtained by first reducing this input to 86% of its observed value and then subtracting slack. This is to say that TFW F should not have required any more than 86% of the reported input levels. For all inputs except CUMPOSSAIR (= the cumulative value of TOTPOSSAIR), this wing should also have required inputs in the still further reduced amounts indicated in the slack column.

Here we have brought the variable NOCHARGLOS up front in order to observe that some of these values are in need of qualified interpretations and possibly further treatment as well. In all cases these computer printouts such as the one in Table 9 will need to be buttressed by competent judgment aided, perhaps, by additional inquiries directed to each individual DMU. In the portrayal used here, moreover, each input is treated as though it can be varied at management discretion. This is not true for weather dependent variables like NOCHARGLOS, however, and so further treatment is needed for this input as being a "non-discretionary resource," at least in part.

It is not proposed to undertake a discussion here of the details on how such adjustments can be made for non-discretionary inputs. The point does need to be made explicitly that such non-discretionary inputs--e. g., favorable flying weather[1/]--can be important and when this is the case they should not be omitted. Judged from the standpoint of output attainment, the evaluation should take account of these inputs even when non-discretionary. In a DEA analysis, this is done by reference to relative

---

1/ Examples of non-discretionary inputs in these studies have included unemployment rates in Army recruitment districts and parental and neighborhood backgrounds as inputs to educational attainment by school children. See [7] and [14].

performances achieved by other units such as those identified as facet members for DMU 7 at the top of Table 9 which also utilize these kinds of non-discretionary inputs.

Before turning to a discussion of these facet members, we should emphasize that full efficiency is achieved in a DEA analysis only when all of the indicated adjustments can be accomplished. That is, all output and input adjustments, as in the final column of Table 9, are needed to achieve full efficiency. Thus, to rate as 100% efficient, TFW F would have had to increase its outputs and reduce its inputs and this, in DEA terminology, would have projected it on to its efficient facet.

9. FACET MEMBERS

The efficiency rating is only relative, as was noted at the outset of this paper. Here the rating is determined relative to DMUs 11 and 8 which, as can be seen in Table 5, are both rated as 100% efficient via the cumulative runs which form the basis of the evaluation in Table 9.

All evaluations in a DEA analysis are effected by reference to subsets of DMUs which are rated as 100% efficient. Among such efficient subsets of wings, the optimization in (1) has picked DMUs 11 and 8 as the "best" reference set for DMU 7 = TFW F in this case.

Formally, no other reference set can yield a higher h_o^* value for TFW F, since otherwise the 0.86 efficiency value would not be maximal. Finally, but less formally, we may say that each DEA optimization tends to pick the reference sets for these evaluations from among the available (efficient) DMUs which are most like the unit being evaluated. Thus wing F which has the mission of both training and operations, while flying very old and complex aircraft, is evaluated by reference to DMU 11, which is TFW M with training as its mission while flying old and complex aircraft, and by reference to DMU 8, which is TFW J with the mission of both training and operations while flying old and very complex aircraft.

One reason for displaying the DMUs in the facet set, as is done in Table 9, should now be apparent since this information supplies further insight into the standpoint from which the evaluation was effected. Hence, this information, too, can be taken into account in any managerial follow-ups that might be effected on the basis of such DEA-initiated inquiries. In particular this facet evaluation information suggests that these members of the "efficient" DMUs should provide information for study in the evaluation of the inefficient DMUs which they help to rate.

There is a great deal of additional information for managerial use that is included in the computer printouts that are availa le from such DEA studies. Now, however, we want to turn attention to Table 10 which records the number of appearances of each wing as a member of a facet used to evaluate <u>other</u> wings. Here the reference is to the efficiency evaluations reported in the window analyses of Tables 6 and 7.

Note, first, that TFW G does not appear in any of the facets used to evaluate <u>other</u> wings. This provides further evidence that this wing is not really efficient but is being rated so--perhaps justifiably--by virtue of its specialized features. In contrast TFW E appears 189 times as a member of an efficient facet--i.e., as a member of the efficient reference group--used to evaluate other DMUs, and this is corroborating evidence of its <u>relative</u> efficiency. Notice, for instance, that this performance exceeds event that of TFW H despite the fact that the latter achieved a rating of 100% efficiency in every one of the evaluations reported in Table 7.

The important point to make here is that such corroboration is available and can be effected in a variety of other ways in DEA analyses. Finally, and equally important, this kind of information can be used in a variety of additional ways that can have managerial value. For instance, Table 10 can be used to guide analyses directed to ascertaining the effect on <u>other</u> DMUs which might accrue from altering the behavior of any of the particular DMUs in that Table. Notice, for instance, that a reorganization which eliminated the TFW E could affect the efficiency evaluations of many other DMUs whereas a similar reorganization of the TFW G would not be likely to have any such further consequences.[1/]

10. CONCLUSION

The above discussion does not exhaust the uses of DEA. We have not touched on the duality relations, for instance, and how the information that is generated from these aspects of the optimizations can be put to managerial use. All that needs to be said, at this point, is that the situation is analogous to the ones found in other parts of mathematical programming where these duality relations have added major features to the "policy analyses" that have accompanied mathematical programming applications in many different types of management planning problems.[2/] Here the values of these dual variables provide important information on possible substitutions and tradeoffs that can be used to further <u>other</u> managerial objectives <u>after</u> efficiency has been achieved by each DMU.

1/ See [2] for further discussion in the context of reorganizations proposed for the San Antonio Community College.

2/ See [5] or [3] for detailed examples and discussions.

TABLE 10

NO. OF APPEARANCES OF EACH WING IN THE FACET OF ANOTHER WING*

WING \ WINGS IN FACET	D	G	H	E	I	L	M	N
A	1		13	19	3	13	9	5
B			14	17	2	7	15	4
C	5		1	16		2	15	13
D	2		5	17		2	19	9
G		1	2	24		5	2	8
H			24	11		1	9	7
F	4		3	16		3	17	7
E			8	20		5	5	5
I			6	18	4	7	15	9
L			10	7		21		7
M	1		6	8		1	22	6
J	3			16			17	12
K	3		4	17		1	15	7
N			2	3		1		18
TOTAL NO. OF APPEARANCES IN OTHER FACETS	17	0	74	189	5	48	138	99

*Maximum possible = 45 facet appearances = 5 windows x 3 DMU repetitions per window x 3 possible appearances in a facet.

"Tradeoffs," as the name itself suggests, are not required until after efficiency has been achieved by each DMU. See the definitions of efficiency contained in section 1, p. 2, above. In other words, the tradeoff concept is related to the notion of an efficiency frontier within which improvements in any input or output can be achieved only at the expense of other inputs and outputs. To guide decisions where some outputs are to be improved at the expense of others, it is important to know the tradeoff (or substitution) ratios as well as the ranges of input and output alterations within which these ratios are valid. Much of this is supplied automatically in the computer printouts. Further research under way is directed toward providing an even more complete picture.

Via routes like these one is able to move from "control" and evaluation of each DMU to other (related) problems such as "planning"1/ for still more effective performance by resource allocations across DMUs by methods such as "goal programming," etc., which the authors of this report have developed for similar uses in other military planning contexts.2/

In this report we have confined ourselves to straightforward uses of what is already available from DEA. The preceding examples are only illustrative--for use in examining DEA--rather than directed toward actual managerial evaluation of performance of the air force wings included in these illustrative examples.

Within this context we have also tried to highlight differences between DEA and other approaches to the topics examined here. This has included the comments we offered in section 1 on possible uses of ratio

1/ The distinctions between planning and control uses of mathematical programming are elaborated in further detail in Chapter 1 of [5].

2/ See, e.g., [4] and [13].

analysis, regression and simulation techniques which also have been used in past and present efficiency evaluations. A use of DEA does not, however, preclude the use of these other techniques. On the contrary, DEA is best regarded as an additional method of analysis which can be used in conjunction with any or all of these other presently used alternatives. This should therefore be born in mind when considering the following recapitulation.

In contrast with the usual ratio approaches, DEA deals simultaneously with all inputs and outputs that are considered to be pertinent, and in whatever mixes are present, in the performance reports of all DMUs when arriving at its efficiency evaluation ratios. See expression (2.1) to (2.4) and the accompanying discussions on page 7 in section 2, above.

In contrast with the usual regression approaches--e. g., as in standard versions of least squares-correlation analyses--DEA optimizes on the observations for each DMU. Moreover, instead of being directed toward averages, the focus on DEA is directed to frontier location and estimation. These twin attributes then lend themselves to the kinds of inefficiency estimates and characterizations for each DMU that were discussed in connection with Table 10 at the end of the preceding section.

In contrast with the usual simulation modeling approaches, DEA does involve optimizations and via this route it provides access to bodies of theory that are available both for immediate use and further extensions. Access to this prior body of knowledge was used, for example, in the identification of the facet members that were discussed as providing the basis used for the efficient facet evaluation of the (cumulative run) performance of the TFW F in Table 10. It also provided guidance for interpreting the results of the window analysis by reference to the number of facet appearances in evaluating <u>other</u> wings. In this manner we were able to identify facets which had achieved 100% efficiency ratings by means of "self" comparisons--e. g., because of one or more special features of the operations of these particular DMUs--as well as others who had achieved 100% <u>relative</u> efficiency by virtue of comparisons with a great many other wings.

In conclusion we may therefore point to the window analysis exhibited in Tables 6 and 7. This window analysis is a new feature of DEA which has emerged from recent applications to military problems.[1/] It seems likely that still other such developments will emerge from similar research on DEA for use on Air Force problems which, we hope, has been demonstrated as having potential value for use in controlling and managing Air Force resources.

1/ Its first appearance was in [7].

APPENDIX

In this section we present the rigorous mathematical form for the Data Envelopment Analysis of the CCR ratio form for relative efficiency and exhibit the type of mathematical properties, duality, etc. that the mathematical system has by means of a proof of the efficiency of the projection of an inefficient DMU onto the facet of its associated efficient DMUs.

Of course this is not the only type of change in inputs and outputs which could bring the DMU to a relative efficiency of 1, ceteris paribus. For example, if the DMU has an efficiency of h_0, then raising all its outputs by a factor $(1/h_0)$ would result in an efficiency of unity.

Other mathematical properties of the Data Envelopment Analysis systems useful for analysis and evaluation of desirable directions for improvement of particular desired outputs are to be found in CCS 459, "Pareto-Optimality, Efficiency Analysis and Empirical Production Functions," by A. Charnes, W.W. Cooper, B. Golany, L. Seiford, J. Stutz, May 1983.

We now move from the individual variable form in the text to vector-matrix notation. The CCR ratio problem then is expressed as:

$$\text{Max} \quad \frac{\eta^T Y_0}{\xi^T X_0}$$

$$\text{subject to} \quad \frac{\eta^T Y_j}{\xi^T X_j} \leq 1 \qquad , j = 1, \ldots, n$$

$$\text{(A)} \qquad -\frac{\eta^T}{\xi^T X_0} \leq -\varepsilon e^T$$

$$-\frac{\xi^T}{\xi^T X_0} \leq -\varepsilon e^T$$

with $\epsilon > 0$ as a non-Archimedean infinitesimal.

Here the input vector X_j and output vector Y_j are assumed to have positive components. The quantities η^T and ξ^T are called "virtual multipliers." The scalar products $\eta^T Y_j$, $\xi^T X_j$ are called "virtual outputs" and "virtual inputs" respectively. The inequalities

$$\left(\eta^T Y_j\right) \Big/ \left(\xi^T X_j\right) \leqslant 1 \quad , \quad j = 1, \ldots, n$$

are called the "efficiency technology." The further conditions on η^T , ξ^T are called the "multiplier positivity" conditions since they assure (i) that only positive values are secured for these and (ii) that there is algebraic closure to the inequality system.

By means of the Charnes-Cooper transformation of linear fractional programming $\mu^T = \xi^T/\xi^T X_o$, $\nu^T = \xi^T/\xi^T X_o$, we obtain a linear programming problem and its dual as shown in (A.2).

The dual LP problems may be characterized respectively as:

(1) Maximize the virtual output of DMU_o at unit virtual input subject to the efficiency technology and positivity conditions.

(2) Minimize the intensity of the input vector with the input and output vectors "enveloped" respectively from below and above--i.e., "Data Envelopment Analysis."

<u>Projection on Efficiency Facets</u>

The dual non-Archimedean linear programs for evaluating the relative efficiency of the DMU with input-output vectors (X_o, Y_o) are:

$$
\begin{array}{ll|l}
 & \max \quad \mu^T Y_o & \min \theta \quad - \epsilon e^T s^+ - \epsilon e^T s^+ \\
\text{subject to} & \nu^T X_o = 1 & Y\lambda - s^+ = Y_o \\
\text{(A.2)} & \mu^T Y - \nu^T X \leqslant 0 & \theta X_o - X\lambda - s^- = 0 \\
 & -\mu^T \leqslant -\epsilon e^T & \lambda, s^+, s^- \geqslant 0 \\
 & -\nu^T \leqslant -\epsilon e^T &
\end{array}
$$

where $X = [X_1, \ldots, X_n]$, $Y = [Y_1, \ldots, Y_n]$, ϵ is the non-Archimedean infinitesimal.

If $\theta^\star$, $\lambda^\star_B$, $s^{\star +}_B$, $s^{\star -}_B$ is an optimal basic solution (with dual evaluators $\mu^\star_B$, $\nu^\star_B$) designating the coefficient vectors or matrices

$$
\text{(A.3)} \qquad \begin{bmatrix} 0 \\ X_o \end{bmatrix} , \begin{bmatrix} Y_B \\ -X_B \end{bmatrix} , \begin{bmatrix} -I^+_B \\ 0 \end{bmatrix} , \begin{bmatrix} 0 \\ -I^-_B \end{bmatrix}
$$

as the basis of this optimal solution, they satisfy the reduced system of equations:

$$
\text{(A.4)} \qquad
\begin{array}{rl}
Y_B \lambda^\star_B - s^{\star -}_B & = Y_o \\
\theta^\star X_o - X_B \lambda^\star_B - s^{\star -}_B & = 0
\end{array}
$$

If one replaces (X_o, Y_o) by $X'_o = \theta^\star X_o - s^{\star -}_B$ and $Y'_o = Y_o + s^{\star +}_B$, then $\theta = 1$, $\lambda = \lambda^\star_B$, $s^-_B = 0$, $s^+_B = 0$ is a feasible basic solution for (X'_o, Y'_o) with this same basis.

Now μ_B^{*T} , ν_B^{*T} are still the dual evaluators for the basis and

(A.5)
$$\mu_B^{*T} Y_B \lambda_B^* \;=\; \mu_B^{*T}(Y_o + s_B^{*-}) \;=\; \mu_B^{*T} Y_o'$$
$$\nu_B^{*T} X_B \;{}_B^* \;=\; \nu_B^{*T}(\theta^* X_B - s_B^{*-}) \;=\; \nu_B^{*T} X_o'$$
$$=\; \nu_B^{*T}(\theta^* X_o') \text{ , where } \theta = 1$$

But $\mu_B^{*T} Y_B - \nu_B^{*T} X_B = 0$ for the dual inequalities designated by λ_B^* .
Hence

(A.6) $$\mu_B^{*T} Y_o' \;=\; \mu_B^{*T} Y_B \lambda_B^* \;=\; \nu_B^{*T} X_B \lambda_B^* \;=\; \nu^{*T} X_o'$$

i.e., the new inequality replacing $\mu^T Y_o - \nu^T X_o \leqslant 0$ is also satisfied by μ_B^{*T} , ν_B^{*T} . Further

(A.7) $$\left(\mu_B^{*T} Y_o'\right) \Big/ \left(\nu^{*T} X_o'\right) \;=\; 1$$

Thus $\tilde{\mu}_B^T = \mu_B^{*T} \Big/ \left(\nu^{*T} X_o'\right)$, $\tilde{\nu}_B^T = \nu_B^{*T} \Big/ \left(\nu^{*T} X_o'\right)$ is a feasible (basic) solution to the (X_o', Y_o') problem with functional value $\tilde{\mu}_B^T Y_o' = 1$ equal to the dual problem functional value $\theta = 1$. Thus the "projection" $X_o \to X_o' = \theta^* X_o - s^{*-}$, $Y_o \to Y_o' = Y_o + s^{*+}$ yields an efficient DMU.

REFERENCES

[1] Banker, R.D., A. Charnes, W.W. Cooper and A. Maindaratta, "A Comparison of DEA and Translog Estimates of Production Frontiers Using Simulated Observations from a Known Technology," Research Report: Pittsburgh, PA, Carnegie-Mellon University, School of Urban and Public Affairs, Oct., 1982.

[2] Bessent, A., W. Bessent, A. Charnes, W.W. Cooper and N. Thorogood, "Evaluation of Educational Proposals by Means of DEA," CCS 426 (Austin, Texas: The University of Texas at Austin, Center for Cybernetic Studies, Feb., 1982).

[3] Bradley, S.P., A.C. Hax and T.L. Magnanti, Applied Mathematical Programming (Reading, Mass., Addison-Wesley, 1977).

[4] Charnes, A. and W.W. Cooper, "Goal Programming and Multiple Objective Optimizations," European Journal of Operational Research I, No. 1, 1977, pp. 39-54.

[5] __________ and __________, Management Models and Industrial Applications of Linear Programming (New York: John Wiley & Sons, Dec., 1961).

[6] __________ and __________, "Programming with Linear Fractional Functionals," Naval Research Logistics Quarterly 9, 1962, pp. 181-186.

[7] __________, __________, D. Divine, G.A. Klopp and J. Stutz, "An Application of Data Envelopment Analysis Recruitment Districts," CCS 436 (Austin, Texas: The University of Texas at Austin, Center for Cybernetic Studies, 1982).

[8] __________, __________ and E. Rhodes, "Data Envelopment Analysis as an Approach for Evaluating Program and Managerial Efficiency--with an Illustrative Application to the Program Follow Through Experiment in U.S. Public School Education," Management Science 27, No. 6, June, 1981, pp. 668-697.

[9] __________, __________, and __________, "Measuring the Efficiency of Decision Making Units," European Journal of Operational Research 2, No. 6, Nov., 1978, pp. 429-444.

[10] __________, __________, and H.D. Sherman, "A Comparative Study of Data Envelopment Analysis and Other Approaches to Efficiency Evaluation and Estimation," Operations Research (submitted).

[11] Jagannathan, R. and S.Schaible, "Duality in Generalized Fractional Programming via Farkas' Lemma," Working Paper 82-8, University of Iowa, 1982.

[12] Lewin, A.Y. and R.C. Morey, "Evaluating the Administrative Efficiency of Courts," Omega (forthcoming).

[13] Niehaus, R., Computer Assisted Human Resources Planning (New York: John Wiley & Sons, Dec., 1979).

[14] Rhodes, E., Data Envelopment Analysis and Related Approaches for Measuring the Efficiency of Decision Making Units with an Application to Program Follow Through in U.S. Public School Education, Ph.D. Thesis (Pittsburgh: Carnegie-Mellon University, School of Urban and Public Affairs, 1978).

[15] Sherman, H.D., Measurement of Hospital Technical Efficiency: A Comparative Evaluation of Data Envelopment Analysis and Other Approaches for Locating Inefficiency in Health Care Organizations, DBA Thesis (Cambridge: Harvard University, Graduate School of Business, 1981).