A Collective Interaction Klystron

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It is shown that by just bending the drift region of a two cavity klystron into a circular arc, the current bunching near the output cavity is significantly enhanced. This increase in the bunching results from the "negative mass" behavior of a rotating electron beam. This device thus distinguishes from the conventional klystron as the AC space charges in the latter device adversely affect the grouping of electrons. Operations with both low voltage and high voltage beams are suggested.
A COLLECTIVE INTERACTION KLYSTRON

The klystron may be regarded as the most developed among conventional microwave tubes. It has a wide range of applications, from communication transmission at low power level to high energy particle acceleration where tens of megawatts are required. The simplest (though not necessarily the most practical) klystron configuration consists of two cavities separated by a linear drift region. The input signal is injected into the first cavity to provide a velocity modulation of the electron beam. This velocity modulation, after being carried through the drift region, becomes a density modulation near the second cavity. The density modulation results in an rf current which excites the second (output) cavity. Because of the mutual Coulombic repulsion among the AC space charges, the charge bunching near the output cavity cannot reach the level expected from kinematic (ballistic) considerations. In fact, the efficiency of klystron depends sensitively on the grouping of electrons near the output cavity.

In this paper, we show that by bending the drift tube into a circular arc, the grouping of the space charges can be enhanced. This is possible because of the negative mass effects in rotating electron beams, in which the AC space charges tend to accumulate instead of self-dispersing. In addition to the well-known ballistic effects, a klystron thus constructed makes novel use of a powerful dynamical effect. It therefore distinguishes from the conventional klystron in that the collective motions strengthen rather than weaken the density modulation. Of course, for the electron beam to follow a circular arc in the drift section, a radial force must be externally supplied. This can be achieved via a vertical magnetic field, or preferably in the case of low beam energy, via a radial DC electric field. As we shall see below, the principle outlined here may be applied to electron beam ranging from a few keV to hundreds of keV or higher.

To illustrate the idea, the simple two cavity klystron suffices. In such a configuration [Fig.1], the input and the output cavities, and the drift region may be analyzed separately. Since the novel aspects of the present device lie mainly with the drift region, we shall focus at the charge bunching processes there. Our elementary analysis is analogous to that used in standard textbooks\(^1\), but with emphasis on the negative mass mechanism. However, the result obtained agrees with the classical limit of low current, in which case the bunching is essentially kinematic (ballistic).

In the absence of the rf modulation, the electron beam is assumed to move along the circular arc of the drift tube at linear velocity \( \dot{v}_0 = \dot{\theta} r_0(r) \equiv \theta r_0(r) \), where \( r \) is the radial distance from the center of curvature of the circular arc, and \( \theta \) is the angular variable along the drift tube [Fig.1]. For the time being, we shall leave unspecified the relative strength of the radial electric field \( E_0 \) and the vertical magnetic field \( B_0 \) which are needed to provide the circular motion of the beam. Thus, \( v_0 \) is governed by

\[
\gamma_0 v_0^2/r = - \left( \frac{e/m_0}{\gamma_0} \right) \left( E_0 + v_0 B_0 \right)
\]  

(1)

where \( e \) and \( m_0 \) is respectively the electron charge and rest mass,
\( \gamma_0 = (1 - v_0^2/c^2)^{-1/2} \) is the relativistic mass factor with \( c \) being the speed of light. In writing (1), we have for simplicity ignored the DC self fields of the electrons. The beam is assumed to be monoenergetic, have a small beam cross section, and be located at mean radius \( r = R \) with number density \( N \) per unit arc length along the drift tube.

We shall use a small signal analysis. Associated with the velocity modulation is a longitudinal AC displacement \( \eta \) of an electron from its unperturbed orbit. Similar to plasma oscillation, this displacement leads to
a charge perturbation which then generates a self AC electric field $E_{1\theta}$. In response to this AC electric field $E_{1\theta}$, the angular displacement $\eta$ of an electron obeys the following linearized force law:

$$\ddot{\eta} = rv_o \left( \frac{\partial}{\partial \epsilon} \right) eE_{1\theta}. \tag{2}$$

Here a dot denotes the substantial derivative, $\epsilon$ is the energy of the electron, and $\partial / \partial \epsilon$ is the negative mass factor. For an equilibrium governed by Eq. (1), $\partial \omega_0 / \partial \epsilon$ is given by:

$$\frac{\partial \omega_0}{\partial \epsilon} = -\frac{1}{\gamma_0 \omega_0} \left( \frac{\beta_0^{2+2h}}{1+\gamma_0^2 h^2} \right) \tag{3}$$

where

$$h = -\frac{eE_o}{\gamma_o \omega_o^2 \beta_0^2} \tag{4}$$

and $\beta_0 \equiv v_o / c \equiv \gamma_0 \omega_o / c$.

A few words on the negative mass effect are in order. Recall that the negative mass behavior arises if the frequency of rotation $\omega_0$ is a decreasing function of energy ($\partial \omega_0 / \partial \epsilon < 0$). In this case, the angular acceleration is opposite to the applied force $eE_{1\theta}$, as if the inertia of the electron is negative$^{3,4}$. There would then be an intrinsic tendency of beam bunching. If the rotation of the electron is supported solely by a magnetic field, $E_o = 0$ and therefore $h = 0$ by (4). Equations (2) and (3) then yield

$$\ddot{\eta} = -\beta_0^2 eE_{1\theta} / \gamma_o m_o$$

which clearly shows the negative mass nature of a rotating electron, under a uniform magnetic field, as a result of the relativistic mass correction ($c^2 = m_o c^2$). A new class of microwave generation
devices, known as the gyrotron, has been developed based on this principle. Another limiting case is when \( h = 1/\gamma^2 \), i.e., when the rotation of the electron is supported solely by an outward radial electric field [cf. (1,4)]. In this case, \( \partial \omega_o / \partial \epsilon < 0 \) also. This is precisely the equilibrium condition in the microwave generation experiment of Alexeff and Dyer. In fact, one may easily demonstrate from (3) that \( \partial \omega_o / \partial \epsilon \) is maximized with respect to \( h \) when \( h = 1/\gamma^2 \). Thus, among all possible combinations of \( E_o \) and \( B_o \), the negative mass effect is most pronounced when the rotation of the electron is supported just by a radial electric field [cf. (3,4)] for a given beam energy and a given beam radius \( R \). One may compare the negative mass effects for \( h = 0 \) and for \( h = 1/\gamma^2 \) through the relation

\[
\frac{\partial \omega_o}{\partial \epsilon} \bigg|_{h = 1/\gamma^2} = \frac{1}{\beta_o^2} \cdot \frac{\partial \omega_o}{\partial \epsilon} \bigg|_{h = 0}
\]  

which is readily deduced from Eq. (3). Equation (5) clearly suggests the advantage of using only a radial DC electric field (\( h = 1/\gamma^2 \)) at low beam voltage (low \( \beta_o \)). At a high beam voltage, a vertical magnetic field suffices.

A signal of frequency \( \omega \) impressed upon the rotating electron beam yields an angular variation proportional to \( \exp \left(-i\theta \right) \) where \( t = \omega/\omega_o \). Note that \( t \) may be regarded as a propagation constant and that it is not necessarily an integer in the present case. Such an angular variation in the displacement \( \eta \) produces AC space charges whose number density \( N_1 \) per unit arc length is

\[
N_1 = -\frac{N \partial \eta}{\partial \theta} = \frac{i N \eta}{R}.
\]  

This AC line charge yields a self electric field \( E_{1\theta} \) at the beam.
where \( \varepsilon_0 \) is the free space permittivity (MKS units) and \( g \) is essentially the dimensionless impedance experienced by the beam and is given by

\[ g \approx (1 + 2 \ln \frac{2\ln t}{\gamma_0})/\gamma_0^2 \]  

for the dimension specified in Fig. (1).

Upon substituting Eq. (7) into Eq. (2), we obtain

\[ \dot{\eta} - \Gamma^2 \eta = 0 \]  

where

\[ \Gamma^2 = \frac{2c^2}{R^2} \frac{\nu}{\gamma_0} \left( \frac{2\gamma_0^2 + 2\nu}{1 + \gamma_0^2 \nu^2} \right) \]  

with \( \nu = N e^2/4\pi\varepsilon_0 m_o c^2 \) being the dimensionless Budker parameter. Note that \( \Gamma \) is essentially the rate of growth of the negative mass instability in a rotating relativistic electron beam. Equation (9) has its counterpart in the conventional klystron theory in which \( -\Gamma^2 \) is replaced by \( \omega_p^2 \), \( \omega_p \) being the electron plasma frequency.

We may now calculate the fundamental harmonic of the AC current at the output gap. Consider an electron leaving the input gap at \( \theta = 0 \) [Fig. (1)] at time \( t = t_1 \), with a velocity modulation \( \dot{\eta}(t_1) = (a\nu_0/2) \sin \omega t_1 \), where \( a \) is the modulation depth. Then Eq. (9) gives

\[ \eta(t) = (a\nu_0/2\Gamma) \sin \omega t_1 \sinh \Gamma(t-t_1) \]  

(11)
if we assume that \( n(t_1) = 0 \). Equation (11) may be used to transform from the Lagrangian to the Eulerian variables at \( \Theta = \Theta_0 \), the angular position of the output cavity [Fig. 1]. From the definition of \( n \), the electron arrives at \( \Theta = \Theta_0 \) at time \( t = T \), where \( \Theta_0 \) and \( T \) are related by \( \Theta(T) = \Theta_0 = \omega_0(T-t_1) + \eta(T) R \); \( n(T)/R = \omega_0(T-t_1) + (\omega_0/2\Gamma) \sin \omega t_1 \sinh \Gamma(T-t_1) \). This relation may be inverted to yield

\[
T = t_1 + \frac{\Theta_0}{\omega_0} - \frac{\Phi}{2\Gamma} \sin \omega t_1 \sinh \frac{\Theta_0}{\omega_0} \tag{12}
\]

in the small signal theory (small \( \alpha \)). The rf current \( I_2(\Theta_0, T) \) at the output gap contains all harmonic frequencies and may be represented in Fourier series as \( I_2(\Theta_0, T) = \sum [a_n(\Theta_0) \cos n\omega T + b_n(\Theta_0) \sin n\omega T] \) where the summation extends from \( n = -\infty \) to \( \infty \). The Fourier coefficient is given by

\[
a_n(\Theta_0) = \frac{\omega I_0}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} dT I_2(\Theta_0, T) \cos n\omega T \]

\[
= \frac{\omega I_0}{2\pi} \int_{-\pi/\omega}^{\pi/\omega} dt_1 \cos n\omega T \tag{13}
\]

where \( I_0 \) is the DC current carried by the beam at the input gap. In writing the last expression, we have used the charge conservation relation \( I_2 dt = I_0 dt_1 \). Upon substituting (12) into (13), we obtain \( a_n(\Theta_0) \) [and a similar expression for \( b_n(\Theta_0) \)]. The total current \( I_2 \) at the output gap is

\[
I_2(\Theta_0, T) = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(X_n) \cos [n\omega(T-\Theta_0/\omega_0)] \tag{14}
\]

where \( X_n = (n\omega/2\Gamma) \sinh (\beta_0/\omega_0) \) and \( J_n \) is the Bessel function of the first kind, of order \( n \).
Equation (14) has a similar structure as the corresponding one for conventional klystron. As a check, we note that in the limit of zero density, \( r \to 0 \) by (10) and the collective effect disappears. In this limit, (14) indeed agrees with the classical result where only ballistic bunching is present. The negative mass effect may be examined by considering just the fundamental harmonic \( 2I_o J_1(X_1) \) in the output current \( I_2 \). The peak value of this quantity is \( 1.16I_o \), occurring at \( X_1 = 1.841 \). Since \( X_1 = (\omega/2\Gamma)\sinh (\Gamma \Theta_o/\omega_o) \), a very small velocity modulation at the input gap may yield the maximum achievable rf current \( 1.16I_o \) at the output gap if \( \Gamma \Theta_o/\omega_o \) is sufficiently large. This is quite different from the conventional klystron, and is due to the enhanced charge bunching resulted from negative mass effect associated with a bent drift tube. In contrast, a weak modulation at the input gap of a conventional two cavity klystron is unable to achieve this peak value of current bunching because of the Coulombic repulsion among the AC space charges\(^1\).

As a proof-of-principle experiment, take \( \Theta_o = 2\pi/3 \), \( R = 4 \) cm, \( \beta_o = 0.1389 \), corresponding to a beam energy of 5 keV. Then \( \omega_o/2\pi = \beta_o c/2\pi R = 0.1658 \) GHz. Let \( \ell = 20 \), say, so that the tube may operate at 3.32 GHz. Then Eq. (10) yields \( \Gamma/\omega_o = 1.86 \) for \( g = 4 \), \( h = 1/\gamma_o \), and a beam current of 0.1 amp. For these parameters, the maximum achievable rf current (according to the present linear theory) may be attained at the output cavity if the modulation factor \( \alpha \) is as low as 0.014. In this example, a radial electric field \( E_o = 2.5 \) keV/cm is used to provide the circular motion in the drift tube.

For high voltage operation, consider, for example, a beam at a voltage of 300 keV and a current of 160 amps. For \( R = 22.36 \) cm, \( \Theta_o = 100^\circ \), \( \phi = 0 \), \( g = 1.59 \), \( \omega/\omega_o = 20 \) (i.e., operation at 3.32 GHz), saturation occurs when \( \alpha = 0.0174 \). Here, only a vertical magnetic field \( B_o = 94 \) G is used to
provide the circular motion in the drift tube. Examples with other beam energy and frequency ranges may similarly be constructed.

In this paper, several well-known principles are synthesized to yield a novel klystron which promises high gain. The device is efficient and compact. It has the simplicity of a conventional two cavity klystron. It also uses the very mechanism which makes gyrotrons efficient, as bunching along the rotational orbits indeed takes place.

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Fig. 1 A schematic drawing of the klystron (left) and the cross sectional view of its drift tube (right).
References


