THE EFFECT OF TURBULENCE ON THE ATMOSPHERIC TRANSMITTANCE.

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THE EFFECT OF TURBULENCE ON THE ATMOSPHERIC TRANSMITTANCE

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This report describes the effect of turbulence on the atmospheric transmittance. The fluctuation of transmittance due to turbulence is incorporated into the LOWTRAN computer code in terms of two subroutines for plane wave sources and beam wave sources (including spherical wave sources), respectively. These two subroutines calculate the intensity and power scintillation index. The square root of these indices is then used to define the upper and lower bounds of transmittance deviation. The calculations are for point receivers as well as for finite aperture receivers which exhibit the aperture averaging effect.
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I. Introduction

An electromagnetic wave propagating through the earth’s atmosphere is subject to attenuation due to scattering by molecules, aerosols and turbulence (which acts like a source of inhomogeneities) as well as due to absorption of radiation by atmospheric constituents in the atmosphere. In addition to attenuation, scattering by turbulence, in which the refractive index varies through space and time, makes the wave intensity fluctuate, especially for waves with wavelength shorter than millimeters. Because the wavefront is spread by the smaller scales and wandered by the larger scales of the turbulence, the reformatted wavefronts cause the scintillation of the wave intensity.

In this report, we review theoretically the attenuation and fluctuation of the wave propagating through turbulent atmosphere and incorporate the results into the Lowtran computer code in terms of two new subroutines. These added subroutines are used to calculate the normalized intensity (or power) variance, by which we define the upper and lower bounds of atmospheric transmittance. Since the wave attenuation due to turbulence, which will be discussed in section II, is not significant, we do not incorporate it into the calculation of atmospheric transmittance in Lowtran in which the attenuation due to molecular and aerosol’s absorption and scattering is taken into account.

In practical systems, transmitters and receivers with finite apertures are used. A larger receiver aperture not only collects more power, but also reduces wave fluctuation. This is called the receiver aperture averaging effect. Also, a more coherent source gives rise to
smaller scintillations. Hence, the effects of aperture size, source coherence and turbulence on wave scintillation are included in the formulation which will be used to code the new subroutines in Lowtran.

We have chosen pair-correlated field statistics, which act like Gaussian field statistics, to model the coherence properties of sources. It is shown that these statistics yield satisfactory results.

The extended Huygens-Fresnel method has been used to obtain scintillation expressions for partially coherent beam waves as well as for spherical waves. Since the wave structure functions which are used are valid for aperture sizes which are smaller than the Fresnel zone and since the parallel approximation is applied in the Huygens-Fresnel formulation, we cannot extend the beam wave result to the plane wave case simply by letting transmitter size go to infinity. Therefore, we derive the intensity variance for plane wave sources by use of the log-amplitude variance which can be obtained by Rytov's method. Thus, two subroutines, VRANI and SINTL, are coded separately for plane waves and beam waves (including spherical waves), respectively.
II. Transmittance attenuation due to turbulence

The transmittance for a wave propagating through the atmosphere is defined as

\[ \tau = \frac{\langle I \rangle}{\langle I^\prime \rangle} \]  

(1)

where \( I \) and \( I^\prime \) are the received wave intensities in the atmosphere and in vacuum, respectively. \( \langle \rangle \) denotes an ensemble-average.

Excluding the attenuation due to turbulence scattering, the atmospheric transmittance described in LOWTRAN is

\[ T_L = T_{km} \cdot T_{ka} \cdot T_{om} \cdot T_{sa} \]  

(2)

where \( K_m, \sigma_m, K_a, \sigma_a \) are the absorption constant and scattering cross section of the molecules and aerosols in the atmosphere, respectively.

Eq. (2) implies that all scattering energy is thought as a loss. This is a good approximation for a receiver with a very narrow angle of Field of View (FOV) and for scattering by molecules and aerosols that give trivial forward scattering because of their small sizes.

However, the scales of turbulence (1mm ~ 100m) are much larger than optical or infrared wavelengths. A strong forward scattering field due to turbulence is then present. Since the turbulence-induced deviation of the arrival angle is in the order of micro-radians, the scattering energy is not completely lost for a receiver with an angle of FOV larger than micro-radians. Assuming that the scattering by molecules, aerosols and by turbulence are independent, we can then write the atmospheric transmittance \( \tau \) as.
\( \tau = \tau_L \langle \tau_T \rangle \)  \hspace{1cm} (3)

where \( \langle \tau_T \rangle \) is the ensemble-averaged transmittance due to turbulence.

(a) Plane wave

In turbulence, the refractive index variation is small (~ \(10^{-6}\)) and the scales of turbulence are much larger than optical or infrared wavelengths. Therefore, the backscattering due to turbulence is small. For plane waves, neglecting the trivial backscattered fields, almost all incident waves reach the receiver plane though the wavefronts are distorted and the received wave energy is redistributed through the entire receiver plane. Hence, the ensemble-averaged transmittance of a plane wave in turbulence has a value of unity, i.e.,

\[ \langle \tau_T \rangle = 1 \]  \hspace{1cm} (4)

However, the redistributed energy does degrade the coherence of the received wave and induces wave scintillation which we will discuss in section III.

The finite aperture of a receiver with radius \(R\) collects power \(P\):

\[ \langle P \rangle = I_s \tau_{WR}^2 \]  \hspace{1cm} (5a)

\[ = \tau_L I_s \pi R^2 \]  \hspace{1cm} (5b)

where \(I_s\) is the plane wave intensity in the transmitter plane. The transmittance of power can then be defined as

\[ \tau_p = \frac{\langle P \rangle}{\langle P \rangle} = \tau_L \frac{\langle \tau_T \rangle}{\tau_{p,T}} \]  \hspace{1cm} (6)
It is obvious the that \( <P^V> = I_s R^2 \). Substituting \( <P^V> \) into Eq.(6), we find that \( \tau_p = \tau_L \) for a plane wave, i.e. \( <\tau_{p,T}> = 1 \).

(b) Beam waves and spherical waves

The atmospheric transmittance for a beam wave is affected by the aperture size of the transmitter and receiver as well as the coherence of the source. Incorporating these parameters and using the extended Huygens-Fresnel principle, Wang and Plonus\(^8\) derived the intensity and correlation function of the received field for a partially coherent beam wave propagating through turbulence. The intensity transmittance due to turbulence and source incoherence can then be obtained from these derivations, i.e.

\[
<\tau_T> = \frac{1 + \zeta^2 + \xi^2}{1 + \zeta^2 + \xi^2 + \zeta^2} \tag{7}
\]

where \( \zeta = a_s / \rho_s \), \( \zeta = 2a_s / \rho_o \), \( f = k\alpha^2 / L \) is the Fresnel number of the source, \( a_s \) is the source size, \( \rho_s \) is source coherence length, \( \rho_o \) is the coherence length of turbulence, \( L \) is the transmitter-receiver distance and \( k = 2\pi / \lambda \). If a finite aperture receiver of radius \( R \) is used, following the definition of power transmittance \( <\tau_{p,T}> \), Eq.(6), we have

\[
<\tau_{p,T}> = \frac{1 - \exp[-(R/\alpha_c)^2]}{1 - \exp[-(R/\alpha_s c(\xi = 0))^2]} \tag{8}
\]

where

\[
C^2 = (1 - 1/f)^2 + (1 + \xi^2 + \zeta^2)/f^2 \tag{9}
\]
The detailed derivation of the intensity and power transmittance for the various limiting cases has been given in earlier reports. Here, we show some brief results.

(i) For a collimated beam ($F \rightarrow \infty$) and small receiver size ($R \rightarrow 0$) we have

\[ \langle \tau_{p,T} \rangle \approx \frac{C^2(G_0 - 0)}{c^2} = \langle \tau_T \rangle \tag{10} \]

The power transmittance is the same as the intensity transmittance because the received fields in the small receiver area are affected by turbulence uniformly.

(ii) Spherical wave ($\alpha_s \rightarrow 0$)

\[ \langle \tau_p \rangle = \langle \tau_{p,T} \rangle = 1 \tag{11} \]

Like in the case of the plane wave, a spherical wave is affected by turbulence uniformly throughout the entire receiver area. Since no energy is lost due to turbulence scattering, turbulence does not attenuate the atmospheric transmittance (intensity or power) for spherical waves.

(iii) Incoherent sources ($\rho_s + \frac{1}{2\pi}$)

\[ \langle \tau_T \rangle = \langle \tau_{p,T} \rangle = 1 \tag{12} \]

An incoherent beam wave source, acts like a spherical wave source; it radiates waves in all directions, though its field is not coherent. Hence, turbulence scatters wave uniformly and does not give rise
to any attenuation in the atmospheric transmittance for an incoherent source. Also, we have shown that a completely incoherent source ($p_{s} = 0$) does not radiate.

(iv) Coherent source ($p_{s} = \infty$)

$$\langle T_\alpha \rangle = \frac{1 + f^2}{1 + f^2 + \xi^2} \quad (13)$$

It is interesting to note that a completely coherent wave is subject to the most serious attenuation due to turbulence.

Eq. (7) expresses the atmospheric transmittance due to turbulence for a partially coherent source in the turbulent atmosphere. When the field coherence length $p_{0}$ is larger than either the source aperture size $a_{s}$ or the Fresnel zone $\sqrt{\lambda L}$, the transmittance due to turbulence is approximately unity. This is usually true for the weak turbulence case. The behavior of the power transmittance $\langle T_{p}, \alpha \rangle$ is quite similar to the intensity transmittance $\langle T_{\alpha} \rangle$ when a small aperture receiver is used. If the receiver size is large enough to collect all the scattered field due to turbulence, i.e. $R \gg a_{s}$, no attenuation of power occurs. Therefore, the power transmittance approaches the constant unity as $R \to \infty$. (See Eq. (8))

From the above discussion, we conclude that the atmospheric transmittance due to turbulence is definitely unity for plane waves as well as for spherical waves and approaches the value of unity for most cases of beam waves except for a coherent beam wave source in strong turbulence. Hence, we have decided not to incorporate an attenuation factor due to turbulence into the calculation of atmospheric transmittance in Lowtran.
III. Transmittance fluctuation due to turbulence

The most serious effect of turbulence on wave propagation in the turbulent atmosphere is the fluctuation of the received field. We defined the scintillation index $m^2$ as the normalized intensity variance $^2$

$$m^2 = \frac{\sigma^2}{\langle I \rangle^2} = \frac{\langle (I - \langle I \rangle)^2 \rangle}{\langle I \rangle^2} = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1$$

(14)

The power scintillation index is given by

$$m_p^2 = \frac{\sigma^2}{\langle P \rangle^2} = \frac{\langle (P - \langle P \rangle)^2 \rangle}{\langle P \rangle^2} = \frac{\langle P^2 \rangle}{\langle P \rangle^2} - 1$$

(15)

where

$$P = \int_\Sigma I(p) d^2 p$$

(16)

is the wave power received by the receiver aperture $\Sigma$. We can then relate the scintillation index to the deviation of the atmospheric transmittance fluctuation. The detailed derivations of intensity and power transmittance bounds are shown in Appendix 4A and 4B in the Fourth report. From this report we obtain that

$$\tau_u^\tau = \langle \tau \rangle (1 \pm m) = \tau_L^\tau (1 \pm m)$$

(17)

where $\tau_u^\tau$ and $\tau_L^\tau$ are the upper and lower bounds of the transmittance, respectively. And,

$$\tau_p^\tau = \langle \tau_p \rangle (1 \pm m_p)$$

(18)

Note that the upper and lower bounds of transmittance are not the exact bounds as some measured transmittance may be out of the bounds.
However, for many samples, we expect that most measured transmittances will fall inside these bounds. Since we concluded, in section II, that the transmittance attenuation due to turbulence scattering is not significant, the only involvement of turbulence in Eqs. (17) and (18) is in the scintillation index $m^2$ and $m_p^2$. The new subroutines VRANI and SINTL are to calculate $m^2$ and $m_p^2$ for plane waves and beam waves, respectively.

(a) Plane waves

(i) Point receiver

Consider a plane wave $U$ propagating through the turbulent medium represented by

$$U = e^{\chi + iS} \quad (19)$$

where $\chi$ and $S$ present the random log-amplitude and random phase due to turbulence, respectively. Assuming a Gaussian probability distribution for $\chi$, the averaged intensity and variance of intensity can then be stated as

$$\langle I \rangle = \langle U \cdot U^* \rangle = e^{2\langle \chi \rangle + 2\sigma^2_\chi} \quad (20)$$

$$\langle I^2 \rangle = \langle U \cdot U^* \cdot U \cdot U^* \rangle = e^{4\langle \chi \rangle + 8\sigma^2_\chi} \quad (21)$$

Substituting Eqs. (20) and (21) into Eq. (14), the scintillation index for plane waves is

$$m^2 = e^{4\sigma^2_\chi} - 1 \quad (22)$$

The variance of log-amplitude, $\sigma^2_\chi$, has been found by Rytov’s method$^6$. 
However, it is only valid for weak turbulence when applied in Eq. (22). Experimental data indicates that $m^2$ (i.e. $\sigma^2_{IN}$) saturates toward the value of unity. In recent years, theoretical work to prove that the variance of intensity saturates to a constant of unity was performed. Avoiding complex mathematics and hoping to get a model which is sufficiently accurate under weak and strong turbulence conditions, we relate the variance of intensity and log-amplitude by

$$\sigma^2_{IN} = 2 - e^{-x} \quad (23)$$

For small values of $x$, $m = 2\sigma_x$ which agrees with Eq. (22). For large $x$, $m = 1$, which agrees with the saturation condition. Using Ref. (7), the variance of log-amplitude as found by Rytov's method is given by

$$\sigma^2_x = 0.563 k^{7/6} \int_0^L c^2_n(\eta) (L - \eta)^{5/6} d\eta \quad (24)$$

where $k = 2\pi/\lambda$ is the wavenumber.

$c^2_n$ is the structure constant of turbulence.

For a homogeneous medium, $c^2_n$ is constant along the path, and Eq. (24) can be rewritten as

$$\sigma^2_x = 0.31 c^2_n k^{7/6} L^{11/6} \quad (25)$$

A model of $c^2_n$ for the earth's atmosphere is given by Hufnagel, et al. We will show and modify this model to fit in Lowtran later.

(ii) Finite aperture receiver

For a finite size of receiver with radius $R$, the average
received power for the plane wave case can then be obtained from Eq. (16),

\[ \langle P \rangle = \int d^2 \rho \langle i \rangle d^2 P = \langle i \rangle \pi R^2 \]  (26)

and similarly the mean-square received power is

\[ \langle P^2 \rangle = \int d^2 \rho \langle ii \rangle d^2 P_1 d^2 P_2 \]  (27)

Assuming that \( \chi \) is Gaussian, we substitute Eqs. (19), (26) and (27) into Eq. (15) and obtain \( m^2_p \),

\[ m^2_p = \frac{4}{\pi R^2} \int_0^{2R} (e^{4B_\chi (\rho)} - 1) \left[ \cos^{-1} \left( \frac{\rho}{2R} \right) - \frac{\rho}{2R} \sqrt{1 - \frac{\rho^2}{4R^2}} \right] dp \]  (28)

where

\[ B_\chi (\rho) \equiv \langle \chi(\rho)\chi(\rho + \rho) \rangle \]  (29)

The average factor \( G(R) \) is defined as

\[ G(R) = \frac{\pi^2}{m^2} \]  (30)

From Eqs. (22) and (28), we obtain the following expression for \( G(R) \),

\[ G(R) = \frac{4}{\pi R^2} \int_0^{2R} b_1(\rho) \left[ \cos^{-1} \left( \frac{\rho}{2R} \right) - \frac{\rho}{2R} \sqrt{1 - \frac{\rho^2}{4R^2}} \right] dp \]  (31)

where

\[ b_1(\rho) \equiv \frac{e^{4B_\chi (\rho)}}{\sigma^2} - 1 \]  (32)

Using Rytov's Method, the Kolmogorov spectrum and a locally homogeneous medium, the correlation function of log-amplitude can be
found from Ref. (6) under the condition $L \lambda >> L_0$.

$$B_x(\rho) = 0.033\frac{\pi^2}{\Gamma N}(-\frac{5}{6}) L^2 \int_0^L C_n^2(\eta) d\eta J(\rho, h)$$

(33)

where

$$J(\rho, \eta) = \left[ \text{Re} \left( \frac{1}{2} + \frac{1}{2}(\frac{1}{k^2} - \frac{\eta}{\eta}) \right)^{5/6} \right] \Gamma_1(-\frac{5}{6}, 1, -\frac{2}{k^2})$$

(34)

$$+ \left( \frac{1}{k^2} \right)^{5/6} F, (-\frac{5}{6}, 1, x)$$

(35)

$x = k^2 \rho^2 / 4$

$k = 5.92/L_0$

$L_0$ is the inner scale of turbulence

$\Gamma_1(a, b, x)$ is the degenerate hypergeometric function.

To calculate the power transmittance deviation in subroutine VRANI, we first obtain the $G(R)$ factor and use the modified intensity deviation Eq. (23), that is

$$m_p = m \cdot G(R)$$

(36)

(b) Beam waves and spherical waves

The extended Huygens-Fresnel principle can be used to obtain the receiver field for beam wave or spherical wave sources in the turbulent medium:

$$u(L, p) = \frac{e^{jkL}}{j\lambda L} \int \int d^2x \mu_s(x) \exp\left[ jk \left| \frac{x}{2L} - p \right|^2 + \chi(s, p) \right]$$

(37)

where $\chi(s, p) = \chi + is$ is the random perturbation due to turbulence.
From Eq. (37), we can derive the expressions of the received intensity and intensity-correlation functions that are needed to obtain the intensity and power scintillation index. For a practical system, the source is not necessarily coherent. Thus, there could exist a random part of \( u(s) \) in Eq. (37). However, a slow-response time (narrow bandwidth) receiver can smooth out some fluctuations due to source randomness. Hence, the intensity and power scintillation index are affected, in addition to turbulence, by source incoherence and receiver response time (bandwidth).

The intensity-correlation function, which includes the effects of source incoherence and random medium due to turbulence, has been derived for a partially coherent beam wave source in turbulent medium \(^2\), as

\[
B = \langle I(p_1)I(p_2) \rangle
\]

\[
= \left( \frac{1}{kL} \right)^4 \int d^2s_1d^2s_2d^2s_3d^2s_4 \Phi_4(s_1,s_2,s_3,s_4) \Phi_4(s_1,s_2,s_3,s_4) \exp\left\{ -\frac{4k}{L} (s_1 - s_2) + \frac{4k}{L} (s_3 - s_4) \right\}
\]

\[
\exp\left\{ \frac{\pi k}{2L} (s_1^2 + s_2^2 + s_3^2 + s_4^2) \right\}
\]

(38)

where

\[
\Phi_4(s_1,s_2,s_3,s_4) = \langle u_\alpha(s_1)u_\alpha^*(s_2)u_\alpha^*(s_3)u_\alpha(s_4) \rangle
\]

(39)

is the fourth-order source coherence function and

\[
\Phi_4^*(s_1,s_2,s_3,s_4) = \exp\left\{ \Psi(s_1,p_1) + \Psi^*(s_1,p_1) + \Psi^*(s_3,p_2) + \Psi^*(s_4,p_2) \right\}
\]

\[
+ \Psi^*(s_4,p_2)_m
\]

(40)
is the fourth-order spherical wave coherence function in the turbulent medium. The bracket subscripts s and m denote the ensemble averages over the statistics of source and turbulent medium, respectively.

(1) Source statistics and response time of receiver

For a receiver with response time larger than source coherent time, \( F_s^4 \) in Eq.(39) reduces to a product of second-order spherical wave correlation functions\(^{12,13}\) i.e.

\[
F_s^4 = \langle u_s(s_1) u_s^*(s_2) \rangle \langle u_s(s_1) u_s^*(s_2) \rangle
\]  

Eq.(41) makes the mathematics much simpler and correctly gives zero scintillation in vacuum. On the other hand if the response time is smaller, the fluctuations due to the source are not smoothed out. The following derivation is to model mathematically the source coherence properties and obtain a suitable expansion of Eq.(39).

For a partially (spatially) coherent source, \( u_s(s) \) can be expressed by the product of the deterministic radiation distribution factor \( u_{sd}(s) \) and the random coherence factor \( u_{sr}(s) \). Let \( u_{sd}(s) \) be a source distribution such as that of a fundamental-mode laser:

\[
u_{sd}(s) = A_s \exp\left[-\left(\frac{1}{2\sigma_s^2} + \frac{1}{2}k_s^2 s^2\right)\right]
\]  

and let the random part of the source field be

\[
u_{sr}(s) = e^{i\phi(s)}
\]  

We model the random phase of the source field as\(^{14}\)

\[
\phi(s) = a + b \cdot s
\]
where \( a \) and \( b \) are a random shift and a random tilt vector of the random phase. Assuming the distributions of \( a \) and \( b \) are Gaussian with zero mean, we have two kinds of statistics, namely Gaussian phase statistics and pair-correlated field statistics to apply to \( F_4^s \).

In Ref. (4), we have shown that pair-correlated statistics give the better results. The source coherence function obtained by these statistics can be stated as.

\[
F_4^s(s_1, s_2, s_3, s_4) = A_s^4 \exp \left[ \frac{1}{2} \left( \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2}{2\sigma_s^2} - \frac{1}{2} k (s_1^2 - s_2^2 + s_3^2 - s_4^2) \right) \right] 
\]

\[
\times \left( \exp \left[ \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2}{4\rho_s^2} \right] + \exp \left[ \frac{-s_1^2 + s_2^2 + s_3^2 + s_4^2}{4\rho_s^2} \right] \right)
\]

\[
+ \exp \left[ -\frac{1}{2} \left( \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2}{2\rho_s^2} \right) \right]
\]

\[
- 2\exp \left[ -\frac{1}{4\rho_s^2} \left( \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2}{2\rho_s^2} \right) \right]
\]

where

\[
\rho_s^2 = 1/\langle b^2 \rangle \tag{46}
\]

\( \sigma_a^2 \) is the variance of the random shift \( a \), \( \alpha_s \) is the beam radius and \( F \) is the radius of curvature of beam wavefront. \( \sigma_a^2 \) and \( \rho_s \) are measures of the degree of coherence. As \( \rho_s \to \infty \), \( \sigma_a^2 \to 0 \), we consider the source coherent; if \( \rho_s \to 0 \) or/and \( \sigma_a^2 \to \infty \), the source is incoherent.

(ii) Atmospheric turbulence
In weakly turbulent media, we can assume that the random perturbation \( T \) is Gaussian, i.e. the log-normal field assumption is valid. The fourth-order spherical wave coherence function \( F_4 \) can then be expressed by the structure functions and correlation functions as:

\[
F_4 = \exp \left[ -\frac{1}{2} D(s_1 - s_2, 0) - \frac{1}{2} D(s_1 - s_4, P_d) - \frac{1}{2} D(s_2 - s_3, P_d) - \frac{1}{2} D(s_3 - s_4, 0) \right] 
\]

\[
- \frac{1}{2} D(s_3 - s_4, 0) + \frac{1}{2} D(s_2 - s_4, P_d) + \frac{1}{2} D(s_1 - s_3, P_d) 
\]

\[
+ 2 B_x(s_2 - s_4, P_d) + 2 B_x(s_1 - s_3, P_d) 
\]

\[
+ i D_{\chi S}(s_2 - s_4, P_d) - i D_{\chi S}(s_1 - s_3, P_d) \]

(47)

where \( P_d = P_1 - P_2 \) and \( D, B_x, D_{\chi S} \) are the wave structure function, log-amplitude correlation function and log-amplitude phase structure function, respectively. The two-wave structure functions are known. Hence, for \((\lambda L)^{1/2} \gg |s_d| \gg L_0\) and by use of the quadratic approximation, we can find the wave structure functions,

\[
\frac{1}{2} D(s_d, P_d) = \frac{1}{2} (s_d^2 + s_d \cdot P_d + P_d^2) 
\]

(48)

\[
D_{\chi S}(s_d, P_d) = \frac{1}{2} (s_d^2 + s_d \cdot P_d + P_d^2) 
\]

(49)

where

\[
\frac{1}{\rho_0^2} = 1.575 k^{12/5} L^{-2} \int_0^{\eta(L - \eta)} \left[ \frac{2}{3/2} C_n^2(\eta) \right]^{6/5} \]

\[
(50)
\]

\[
\frac{1}{\rho_{\chi S}^2} = 0.234 k^{13/6} L^{-11/6} \int_0^{\eta(L - \eta)} \frac{C_n^2(\eta)^{11/6}}{[\eta(L - \eta)]^{1/6}} \]

(51)

The use of the quadratic approximation for the structure functions...
does not imply that we are limited to the case of tilt-only medium because in the expansion of terms other than phase-tilt terms are present and which are retained. Fante has introduced a useful log-amplitude correlation function as

\[ B_X(s_d, p_d) = \frac{1}{2} \left( s_d + s_d \cdot p_d + p_d^2 \right) \]

where

\[ \sigma_X^2 = 0.225 k^{7/6} \int_0^L C_n^2(\eta)(L - \eta)^{5/6} \]

is the variance of log-amplitude for spherical waves. To obtain a closed form result for \( m^2 \) and \( \rho_p m^2 \), we should further approximate Eq. (52) as

\[ B_X(s_d, p_d) = \sigma_X^2 \left( 1 - \frac{1}{2} \left( s_d + s_d \cdot p_d + p_d^2 \right) \right) \]

Note that only the structure constant \( C_n^2(\eta) \) contained in Eqs. (50), (51) and (53) characterizes turbulence properties. Therefore, once we know \( C_n^2 \) along the propagation path, we can obtain \( \frac{1}{2} \), \( \frac{1}{2} \) and \( \sigma_X^2 \) for both homogeneous (horizontal path) and inhomogeneous (slant path) turbulent media.

The structure constant \( C_n^2 \) has been measured and modeled for the earth’s atmosphere by Hufnagel, et al. We modify it to fit Lowtran as

\[
C_n^2(h) = \begin{cases} 
4.2 \times 10^{-14} h^{-2/3} \exp(-h/320) & (h > 10 \text{ m}) \\
8.77 \times 10^{-15} & (h < 10 \text{ m}) \\
0 & (h > 100 \text{ Km})
\end{cases}
\]
where \( h \) is the altitude in meters.

(iii) Intensity and power scintillation

A step-function receiver makes the mathematics complicated such that a closed-form of power scintillation cannot be obtained. Therefore, we integrate the given intensity and intensity-correlation function over the receiver aperture weighted by a Gaussian function, which allows us to relax the integration limits to infinity and obtain closed-form results. Using Eqs. (14), (15), (37), (38), (45) and (47), the intensity and power scintillation index for a partially coherent beam wave source in turbulence have been obtained. That is,

\[
m^2 = \frac{4\sigma_s^2}{\alpha_s^4} \left[ \frac{1}{4\alpha_s^2} + \frac{1}{4\rho_s^2} + \frac{1}{\rho_o^2} + \left( \frac{\alpha_A}{\alpha_s} \right)^2 \right] \frac{2k^2}{\left[ \alpha_s^2 + \frac{1}{\rho_s^2} + \frac{1}{\rho_o^2} + \frac{1}{\rho_s^2} \right] \rho_o^2}
\]

\[
+ \frac{\alpha_s^2}{2\pi} \exp \left[ \frac{k^2}{2\lambda z} \right] + \frac{\alpha_s^2}{2\pi} \exp \left[ - \frac{k^2}{2\lambda z} \right]
\]

\[
+ e^{-4\sigma_s^2} \exp \left[ - \frac{k^2}{2\lambda z} \right] - e^{-2\sigma_s^2} \exp \left[ - \frac{k^2}{2\lambda z} \right] - 1
\]

(56)

and

\[
m^2 = \frac{4\sigma_s^2}{\alpha_s^4} \left[ \frac{1}{4\alpha_s^2} + \frac{1}{4\rho_s^2} + \frac{1}{\rho_o^2} + \left( \frac{\alpha_A}{\alpha_s} \right)^2 \right] \frac{k^2}{R^2} \left[ \frac{\alpha_s^2}{1 + \frac{\alpha_s^2}{\rho_s^2} + \frac{\alpha_A^2}{\rho_o^2}} \right]^2
\]

\[
\alpha_s^2 \left[ \frac{1}{\rho_s^2} + \frac{2\alpha_s^2}{\rho_o^2} + \left( \frac{\alpha_A^2}{\rho_s^2} \right)^2 \right].
\]
\begin{equation}
\begin{aligned}
&\frac{\alpha^2}{2BDG(H_1 + \frac{2}{R^2})(F_1 + \frac{1}{2R^2})} + \frac{\alpha^2}{2BKN(H^2 + \frac{2}{R^2})(F^2 + \frac{1}{2R^2})} \\
&- \frac{4\sigma_a^2}{e} + \frac{2\sigma_a^2}{TUDG(H_3 + \frac{2}{R^2})(F_3 + \frac{1}{2R^2})} - 1
\end{aligned}
\end{equation}

where

\begin{align*}
H_1 &= k^2 \\
H_2 &= \frac{k}{BL} \\
H_3 &= \frac{k^2}{UL} \\
H_4 &= \frac{k^2}{YL} \\
F_1 &= -\frac{\sigma_s^2}{\rho_o} - \frac{1}{D} \frac{2\sigma_s^2}{\rho_o^2} + \frac{2\sigma_s^2}{D_p^2} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L} \\
F_2 &= -\frac{\sigma_s^2}{\rho_o^2} + \frac{1}{K} \frac{2\sigma_s^2}{\rho_o^2} + \frac{2\sigma_s^2}{Kp_o} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L} \\
F_3 &= -\frac{\sigma_s^2}{\rho_o^2} + \frac{1}{D} \frac{2\sigma_s^2}{\rho_o^2} + \frac{2\sigma_s^2}{D_p^2} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L} \\
F_4 &= \frac{2\sigma_s^2}{D_p^2} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L} \\
F_5 &= \frac{2\sigma_s^2}{D_p^2} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L} \\
F_6 &= \frac{2\sigma_s^2}{D_p^2} (A - \frac{4}{L}) + \frac{k + \frac{1}{2}}{L}
\end{align*}
\[
F_4 = \frac{4\sigma_0^2}{\rho_0^2} - \frac{1}{2} \left( \frac{\chi_s}{\rho_o} \right)^2 + \left[ -\frac{\chi_s}{2} \left( \frac{A}{2} - \frac{4}{\rho_o^2} + \frac{k}{\rho_o} + \frac{1}{2} \right)^2 \right]
\]

\[
B = \frac{1}{2\alpha_s^2} + \frac{1}{2} \left( \frac{\chi_s}{\rho_o} \right)^2 + \frac{4\sigma_0^2}{\rho_0^2}
\]

\[
D = \frac{1}{2\alpha_s^2} - \frac{\chi_s}{\rho_o^2}
\]

\[
G = D + \frac{1}{2p_s^2} + \frac{\rho_0^{\chi_s}}{4D}
\]

\[
K = D + \frac{1}{2p_s^2}
\]

\[
N = D + \frac{\rho_0^{\chi_s}}{4K}
\]

\[
T = \frac{\alpha_s^2}{\rho_s^2} + \frac{2}{\rho_s^2}
\]

\[
U = \frac{1}{T} + \frac{A^2}{2p_s^2} + \frac{\chi_s}{\rho_o^2} = \frac{A^2}{T} + \frac{1}{2p_s^2} + \frac{1}{2} - \frac{4}{\rho_o^2}
\]

\[
W = G - \frac{1}{1\rho_s^2}
\]

\[
X = \frac{1}{2\alpha_s^2} + \frac{1}{4p_s^2}
\]

\[
Y = X + \frac{A}{4X} + \frac{4}{\rho_o^2}
\]
\[
0 = D + \frac{1}{4\rho_b} \left( A - \frac{1}{2} \right)^2
\]
\[
Z = 0 + \frac{\rho X^2}{4\delta^2}
\]

(58)

Instead of \( n g R_h \sigma \chi^2 \frac{1}{2} \frac{1}{2} D \chi \), we have used Eq. (52) for the log-amplitude correlation function. Therefore, we substitute \( \frac{1}{2} - \frac{2\sigma_x^2}{\rho_o^2} \) by \( \frac{1}{2} - \frac{2\sigma_x^2}{\rho_o^2} \) for all terms in Eq. (59) which can be obtained from Ref. (2). This substitution gives the better results, even though the application range for the approximation is still limited by letting \( \alpha_s \to 0 \), we can obtain the spherical wave results. The intensity and power transmittance deviations for beam waves and spherical waves can then be calculated by the SINTL computer code using Eqs. (56) and (57).
IV. Subroutine VRANI

The subroutine VRANI is designed to calculate, by using Eqs. (23) - (36), the plane wave intensity and power variance.

For the program, we changed the integration in all formulas to the summation form. From Eq.(55), we find that \( C_n^2 \) varies rapidly when height \( h \) is small. By trading off calculation time and precision of \( C_n^2 \), we choose the subintervals in the summation as,

\[
\begin{align*}
\Delta h_{ij} &= 20 \text{ m} & h_{ij} &< 25 \text{ km} \\
\Delta h_{ij} &= 100 \text{ m} & 25 \text{ km} &< h_{ij} &< 50 \text{ km} \\
\Delta h_{ij} &= 400 \text{ m} & 50 \text{ km} &< h_{ij} &< 100 \text{ km}
\end{align*}
\]

Eq.(24) can then be rewritten as

\[
\sigma^2 = 0.56 k^{7/6} \sum C_n^2(h_{ij})(L - L_{ij})^{5/6} \frac{\Delta L_{ij}}{h_{ij} - h_{i-1}} \Delta h_{ij}
\]

where \( h_{ij} \) is the height which corresponds to the calculated points on the path, "i" is the layer index, "j" is the sub index of each layer, \( L \) is the total path length, \( L_{ij} \) is the path range from the transmitter to the point calculated and \( \Delta L_{ij} \) is the path range for each layer passed.

Since the structure constant \( C_n^2 \) depends on \( h \), the calculation for horizontal and slant paths are different.

(a) Horizontal path

The intensity variance \( m^2 \) can be obtained by Eqs. (23) and (25) after the constant \( C_n^2 \) is specified. \( B_k(\rho) \) can be found with the condition that \( k^2 < \alpha L \) by using Rytov's Method and Kolmogorov spectrum.\(^6,17\)
\[ B_x(\rho) = b_x(\rho) \sigma^2_x \]  \hspace{1cm} (61)

and

\[ 1 - 12.3 \frac{\rho^2}{(\lambda)_{378}^{173}} \frac{\lambda}{\lambda_o} \leq \rho \leq \lambda \leq \sqrt{\lambda L} \]  \hspace{1cm} (62)

where \( \lambda_o \) is the inner scale of turbulence and is assumed to be 3 mm in our subroutine. The power variance can then be obtained by Eqs. (31) and (36).

(b) Slant path

Eq. (60) can be used to calculate the intensity variance for the slant path cases. To obtain results for power variance, Eqs. (33) and (34) have to be approximated so that the integration can be changed to a summation from. The calculation for power variance can then be performed. The approximations are

\[ 1 - 0.8333x - 0.0347x^2 - 0.0045x^3 \quad |x| < 6.5 \]

\[ 1^{5/6}_{-1}(-5, 1, x) = \]

\[ 1.0627 (-x)^{5/6} \quad |x| \geq 6.5 \text{ and } (Re x < 0) \]  \hspace{1cm} (63)

and

\[ (\frac{1-x}{k})^{5/6}(0.259 + 0.805y + 0.009y^2 - 0.0043y^3) \quad y > 6.5, \quad x > 6.5 \]
$J(\eta, \rho) = \begin{cases} 
(\frac{L-\eta}{k})^{5/6}(0.259 + 0.805y + 0.009y^2 - 0.0043y^3) 
- (\frac{1}{K_m})^{5/6}(1 + 0.8333x - 0.0347x^2 + 0.0045x^3) 
\text{where} 
\begin{align*}
x &= -K_m^2 \rho^2/4 
y &= \frac{\rho^2k}{\delta(L-\eta)} 
\end{align*} 
\end{cases}$

The approximations have been checked and the total error is under 10%. Furthermore, we also assume that $\frac{L-\eta}{k} > \frac{1}{K_m^2}$ which means that we neglect a very small part of turbulence near the receiver.

The subroutine VRANI is called by another subroutine TRANS in Lowtran, which calculates the atmospheric transmittance due to absorption and scattering of molecules and aerosol. The program is listed in Appendix B and the definitions of symbols and variables are shown in Appendix A.

For comparison, we used the 1962 U.S. standard atmospheric model the rural aerosal model with visual range of 23km and a 5km propagation distance to plot all the figures. For horizontal paths an altitude of 400m is used for all figures. For an upward path, the altitudes of transmitters and receivers are set to 800m and 400m, respectively. For a downward path, the altitudes of transmitter and receiver are reversed.
Figs. 1 - 3 show the plane wave intensity transmittance for horizontal, downward and upward paths, respectively. Figs. 4 - 6 give the power transmittance for the same condition except that a 10cm receiver is used. The latter figures display a smaller transmittance deviation (scintillation). This is due to the receiver aperture averaging effect.
V. Subroutine SINTL

The subroutine SINTL calculates the intensity and power variance for partially coherent beam wave and spherical wave sources.

(a) Horizontal path

\( C_n^2 \) is constant along the path for the horizontal case. Eqs. (50), (51) and (53) can then be rewritten as

\[
\begin{align*}
\frac{1}{\rho_o^2} &= (0.546 \ c_n^2 \ k^2 \ L)^{6/5} \\
\frac{1}{\rho_{\chi S}^2} &= 0.114 c_n^2 \ k^{13/6} \ L^{5/6} \\
\sigma_{\chi s}^2 &= 0.124 c_n^2 \ k^{7/6} \ L^{11/6}
\end{align*}
\]

After the \( C_n^2 \) constant is calculated from Eq. (55), \( \frac{1}{\rho_o^2} \), \( \frac{1}{\rho_{\chi S}^2} \) and \( \sigma_{\chi s}^2 \) can be obtained immediately from the above equations. Eq. (56) will then give the result of the intensity scintillation for a partially coherent beam wave source.

(b) Slant path

Changing the integrations in Eqs. (50), (51) and (53) into summation form, like that of Eq. (60), we can obtain the results for \( \frac{1}{\rho_o^2} \), \( \frac{1}{\rho_{\chi S}^2} \) and \( \sigma_{\chi s}^2 \). The power scintillation (variance) can then be calculated by the closed-form formula of Eq. (57).

SINTL is also called by the subroutine TRANS when the transmitter size is finite. The program list is shown in Appendix C.

Figs. 7 - 9 show the intensity transmittance for partially
coherent beam wave sources in horizontal, downward and upward paths, respectively. For the finite receiver with radius of 10 cm, Figs. 10 - 12 give the power transmittance. Again, the receiver aperture averaging effect can be found by comparing the intensity and power transmittance deviations. The results of spherical wave cases are shown in Figures 13 - 15 and Figures 16 - 18 for point receivers and finite aperture receivers, respectively.
APPENDIX A

Symbols and Definitions for Subroutines VRANI and SINTL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE</td>
<td>Initial zenith angle in degree</td>
</tr>
<tr>
<td>BI</td>
<td>Covariance of intensity</td>
</tr>
<tr>
<td>BL</td>
<td>Log-amplitude covariance normalized by variance</td>
</tr>
<tr>
<td>BX</td>
<td>Covariance of log-amplitude</td>
</tr>
<tr>
<td>CN2</td>
<td>$C_n^2$ - structure constant of turbulence</td>
</tr>
<tr>
<td>DD</td>
<td>The ratio of the distance from point calculated to receiver over total path length.</td>
</tr>
<tr>
<td>DH</td>
<td>Height interval of slant path integration</td>
</tr>
<tr>
<td>DO</td>
<td>Distance from point calculated to transmitter</td>
</tr>
<tr>
<td>DS</td>
<td>Distance from point calculated to receiver</td>
</tr>
<tr>
<td>DSW</td>
<td>Path length in a layer</td>
</tr>
<tr>
<td>DT</td>
<td>Same as DS, especially used in the downward long path calculation</td>
</tr>
<tr>
<td>DZW</td>
<td>Height for a layer</td>
</tr>
<tr>
<td>FR</td>
<td>Fresnel zone in meter (m)</td>
</tr>
<tr>
<td>GAA</td>
<td>$a^2$, variance of random shift for a partially coherent source. It is assumed to be zero for a temporal coherent source.</td>
</tr>
<tr>
<td>GD</td>
<td>Aperture averaging factor</td>
</tr>
<tr>
<td>HMIN</td>
<td>The minimum height of a downward path</td>
</tr>
<tr>
<td>HW</td>
<td>Height corresponding to the point calculated</td>
</tr>
<tr>
<td>H1</td>
<td>Height of receiver (and transmitter for horizontal path)</td>
</tr>
<tr>
<td>H2</td>
<td>Height of transmitter</td>
</tr>
<tr>
<td>IV</td>
<td>Wavenumber in cm$^{-1}$</td>
</tr>
<tr>
<td>JMIN</td>
<td>The layer index of the minimum height for a downward path</td>
</tr>
</tbody>
</table>
FVR  Power variance
RANGE  Path length in kilometer (km)
RLO  $1/\rho_o^2$, $\rho_o$ is the field coherence length of spherical waves
RIS  Coherence length of source field
RLXS  $1/\rho_s^2$, $\rho_s$ is the structure constant of the log-amplitude and phase structure function
RR  Radius of receiver aperture in meter (m)
SIGM  Variance of log-amplitude for spherical waves
SMI  Intensity scintillation index
SMP  Power scintillation index
TT  Radius of transmitter aperture in meter (m)
VR  Variance of log-amplitude for plane waves
VRI  Intensity deviation
WH2  Height of receiver in meter (m)
WL  Wavelength in meter (m)
WK  $2\pi/\lambda$, wavenumber in m$^{-1}$
WRANGE  Path length in meter (m)
UV  Intensity variance without approximation
XW1  The lowest height of a given path in a given layer
XW2  The highest height of a given path in a given layer
APPENDIX B

31530 SUBROUTINE VRANI(IV)
31540 C
31550 C THIS SUBROUTINE IS TO CALCULATE THE VARIANCE OF INTENSITY
31560 C DUE TO TURBULENCE AND THE CALCULATED STANDARD DEVIATION CAN BE
31570 C USED TO DEFINE HIGH BOUND AND LOW BOUND OF TRANSMITTANCE
31580 C
31590 COMMON /CARD1/ MODEL,IAHAZE,ITYPE,LEN,JP,IM,M1,M2,M3,ML,EMISS,RO
31600 1, BOUND, ISEASN, IVULCN, VIS
31610 COMMON /CARD2/ H1,H2,ANGLE,RANGE,BETA,HMIN,RE,TT,RR
31620 COMMON /CARD3/ V1,V2,DV,AVN,CO,CW,W(15),E(15),CA,PI
31630 COMMON /CNTRL/ LENST,KMAX,M,IJ,JI,J2,JMIN,JEXTRA,IL,IKMAX,NLL,NP1
31640 1,IFIND,NL,IKLO
31650 COMMON /VANG/ K2,DSW(34),DZW(34),XW1(34),XW2(34)
31660 COMMON /VRAN/ VRI
31670 DIMENSION WS(34)
31680 HW=H1*1000.0
31690 WRANGE=RANGE*1000.0
31700 WH2=H2*1000.0
31710 VR=0.0
31720 CN2=0.0
31730 PVR=0.0
31740 WL=0.01/IV
31750 FR=(WL*WRANGE)**0.5
31760 WK=IV*100.*2.*PI
31770 VK=WK**(7./6.)
31780 W0=9.E-6/5.910**2.
31790 IF(ITYPE.NE.1) GO TO 20
31800 C
31810 C VARIANCE CALCULATION FOR HORIZONTAL PATH
31820 C
31830 CN2=4.2E-14*HW**(-2./3.)*EXP(-HW/320.0)
31840 IF(H1.GT.100.0) CN2=0.0
31850 IF(HW.LE.10.0) CN2=8.77E-15
31860 VR=0.31*CN2*WRANGE**(11./6.)
31870 VR=VR*VK
31880 VRI=1.-EXP(-2.*VR**0.5)
31890 IF(RR.LT.0.001) GO TO 91
31900 DO 18 I=1,100
31910 Y=0.01*I
31920 DY=RR**2.*Y
31930 IF(DY.GE.0.003) GO TO 11
31940 BL=1.-12.3*DY**2.0/(FR**(5./3.)*0.003**((1./3.))
31950 GO TO 17
31960 11 XI=WK*DY**2.0/WRANGE
IF(DY.GE.FR) GO TO 12
BL=-1.2.36*XI**(5./6.)+1.71*XI-0.024*XI**2.0
GO TO 17
BL=-0.0242*(XI/4.)**(-7./6.)
CONTINUE
EX=BL*VR
BT=EXP(4.0*BX)-1.
PVR=PVR+BI*(ACOS(Y)-Y*(1.-Y**2.)**0.5)*Y*0.01
CONTINUE
PVR=PVR/PI
W=EXP(4.0*VR)-1.
GD=PVR/W
Q=VR/WD
DO 35 K=1,50
DS=0.1
EX=0.0
Y=0.02*K
DY=EX*Y
RF=DI**2./4.
VR=0.0
Q=RF/WD
DO 34 I=J1,J2
WS(I)=DSW(I)/DZW(I)
IF(XW1(I).LE.25.0) GO TO 21
IF(XW1(I).LE.50.0) GO TO 22
DH=400.0
GO TO 23
21 DH=20.0
GO TO 23
22 DH=100.0
23 WIK=(XW2(I)-XW1(I))*1000.0/DH
IK=WIK
HW=XW1(I)*1000.0
DO 33 J=1,IK
HW=HW+DH
IF(HW.GE.99600.0.AND.ITYPE.EQ.3) XW2(I)=100000.0
IF(HW.GE.99600.0) GO TO 34
CN2=4.2E-14*HW**(-2./3.)*EXP(-HW/320.0)
IF(HW.GT.100000.) CN2=0.0
IF(HW.LE.10.) CN2=8.77E-15
VR=VR+0.56*CN2*DS**(5./6.)*DH*WS(I)
DD=DS/WK
R=RF/DD
IF(DD.LT.WO) GO TO 27
IF(R.GE.6.5 .AND.Q.GE.6.5) GO TO 27
IF(R.GE.6.5 .AND.Q.LT.6.5) GO TO 28
G=DD**((5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)-1.063*RF**((5./6.)))
IF(G.LE.0.0) G=0.
GO TO 31
27 G=0.
GO TO 31
28 G=DD**((5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)-WO**((5./6.))**((1.+0.8333*Q-0.0347*Q**2.+0.0045*Q**3.)
IF(G.LE.0.0) G=0.
GO TO 31
31 BX=BX+CN2*DH*WS(I)
DS=DS+DH*WS(I)
33 CONTINUE
DQ=WRANGE-DS
IF(DQ.LE.0.) DS=DS-DH*WS(I)
VR=VR+0.56*CN2*DS**(5./6.)*(XW2(I)*1000.-HW)*WS(I)
DS=DS+(XW2(I)*1000.0-HW)*WS(I)
36 CONTINUE
IF(RR.LT.0.001) GO TO 36
BX=BX*2.117*WK**2.
BI=EXP(4.*BX)-1.
PVR=PVR+BI*(ACOS(Y)-Y*(1.-Y**2.)**0.5)*Y*0.01
35 CONTINUE
PVR=PVR*16./PI
VR=VR*VK
VRI=1.-EXP(-2.*VR**0.5)
IF(RR.LT.0.001) GO TO 91
WV=EXP(4.*VR)-1.
GD=PVR/WV
VR=VRI*GD**0.5
GO TO 91
37 CONTINUE
C
VARIANCE CALCULATION FOR DOWNWARD PATH
DO 62 MW=1,50
38 DS=0.1
32850  DT=0.1
32860  BX=0.0
32870  Y=0.02*HW
32880  DY=RR*2.*Y
32890  RF=DY**2./4.
32900  VR=0.0
32910  Q=RF/WO
32920  L1=J1
32930  DO 60 L=1,NL
32940  WS(L1)=DSW(L1)/DZW(L1)
32950  IF(XW1(L1).LE.25.0) GO TO 38
32960  IF(XW1(L1).LE.50.0) GO TO 39
32970  DH=400.0
32980  GO TO 40
32990  38 DH=20.00
33000  GO TO 40
33010  39 DH=100.00
33020  40 WIK=(XW1(L1)-XW2(L1))*1000.0/DH
33030  IK=WIK
33040  HW=XW1(L1)*1000.0
33050  DO 57 J=1,IK
33060  HWfrHW-DI
33070  CN2=4.2E-14HW**(-2./3.)*EXP(-HW/320.0)
33080  IF(HW.GT.100000.) CN2=0.0
33090  IF(HW.LE.10.) CN2=8.77E-15
33100  VR=VR+0.56*CN2*DS**(5./6.)*DH*WS(L1)
33110  DD=DS/WK
33120  R=RF/DD
33130  IF(DD.LT.WO) GO TO 51
33140  IF(R.GE.6.5.AND.Q.GE.6.5) GO TO 51
33150  IF(R.LT.6.5.AND.Q.LT.6.5) GO TO 52
33160  G1=DD**((5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)) -1.063*RF**
33170  *(5./6.)
33180  IF(G1.LE.0.) G1=0.
33190  GO TO 53
33200  51 G1=0.0
33210  GO TO 53
33220  52 G1=DD**((5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)) -WO**((5./6.)*
33230  *(1.+0.8333*Q-0.0347*Q**2.+0.0045*Q**3.)
33240  IF(G1.LE.0.0) G1=0.0
33250  53 BX=BX+G1*CN2*DH*WS(L1)
33260  DS=DS+DH*WS(L1)
33270  IF(K2.EQ.0.) GO TO 57
33280  IF(K2.EQ.1.AND.WH2.LE.HW) GO TO 57
VR=VR+0.56*CN2*(WRANGE-DT)**(5./6.)*DH*WS(L1)

DD=(WRANGE-DT)/WRANGE

R=RF/DD

IF(DD.LT.WO) GO TO 54

IF(R.GE.6.5.AND.Q.GE.6.5) GO TO 54

IF(R.LT.6.5.AND.Q.LT.6.5) GO TO 55

G2=DD**(5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)-1.063*RF**

*(5./6.)

IF(G2.LE.0.) G2=0.0

GO TO 56

54 G2=0.0

GO TO 56

55 G2=DD**(5./6.)*(0.259+0.805*R+0.009*R**2.-0.0043*R**3.)-WO**(5./6.)*

*(1.+0.8333*Q-.0347*Q**2.+0.0045*Q**3.)

IF(G2.LE.0.0) G2=0.0

GO TO 56

56 BX=BX+G2*CN2*DH*WS(L1)

5450 DT=DT+DH*WS(L1)

5460 57 CONTINUE

5470 DQ=WRANGE-DS

5480 IF(DQ.LE.0.) DS=DS-DH*WS(L1)

5490 VR=VR+0.56*CN2*DS**(5./6.)*(HW-XW2(L1)*1000.)*WS(L1)

5500 BX=BX+G1*CN2*(HW-XW2(L1)*1000.)*WS(L1)

5510 DS=DS+(HW-XW2(L1)*1000.0)

5520 IF(K2.EQ.0) GO TO 58

5530 IF(K2.EQ.1.AND.WH2.LE.HW) GO TO 58

5540 VR=VR+0.56*CN2*(WRANGE-DT)**(5./6.)*(HW-XW2(L1)*1000.)*WS(L1)

5550 BX=BX+G2*CN2*(HW-XW2(L1)*1000.)*WS(L1)

5560 DT=DT+(HW-XW2(L1)*1000.0)*WS(L1)

5570 58 CONTINUE

5580 L1=L1-1

5590 IF(K2.EQ.0.AND.L1.LE.J2) GO TO 61

5600 IF(L1.LE.JMIN.AND.K2.EQ.1) GO TO 61

5610 60 CONTINUE

5620 61 CONTINUE

5630 IF(RR.LT.0.001) GO TO 90

5640 BX=BX*2.117*WK**2.

5650 BI=EXP(4.*BX)-1.

5660 PVR=PVR+BI*(ACOS(Y)-Y*(1.-Y**2.)**0.5)*Y*0.01

5670 62 CONTINUE

5680 PVR=PVR*16./PI

5690 90 VR=VR*VK

5700 VR=1.-EXP(-2.*VR**0.5)

5710 IF(RR.LT.0.001) GO TO 91

5720 WV=EXP(4.*VR)-1.
GD = PVR/WV
VRI = VRI * GD ** 0.5
91 CONTINUE
RETURN
END
APPENDIX C

SUBROUTINE SINTL(IV)

C THIS SUBROUTINE IS TO CALCULATE THE VARIANCE OF INTENSITY OR
C POWER DUE TO TURBULENCE FOR SPHERICAL WAVE SOURCE OR BEAM WAVE
C WITH PARTIALLY COHERENT SOURCE
C THE CALCULATED STANDARD DEVIATION CAN BE USED TO DEFINE
C HIGH BOUND AND LOW BOUND OF TRANSMITTANCE

COMMON /CARD1/ MODEL,IAZE,ITYPE,LEN,JP,IM,M1,M2,M3,ML,LEMISS,RO
           1,TBOUND,ISEASN,IVULCN,VIS
COMMON /CARD2/ H1,H2,ANGLE,RANGE,BETA,HMIN,RE,TT,RR,RLS
COMMON /CARD3/ V1,V2,DV,AVM,CO,CW,W(15),E(15),CA,PI
COMMON /CNTRL/ LENST,KMAX,M,IJ,JI,J2,JMIN,JEXTRA,IL,IKMAX,NLL,NPL
COMMON /WANG/ K2,DSW(34),DZW(34),XWI(34),XW2(34)
COMMON /VRAN/ VRI
DIMENSION WS(34)
HW=H1*1000.0
WRANGE=RANGE*1000.0
WH2=H2*1000.0
GAA=0.
TLX=0.

TLG=0.
CN2=0.0
WL=0.01/IV
FR=(WL*WRANGE)**0.5
WK=IV*100.*2.*PI
VK=WK**(7./6.)
DC=TT/RLS
WK2=WK**2.
WR2=WRANGE**2.
AM=WK/WRANGE
SA=AM**2.
IF(RLS.LT.0.0001) RLS=0.0001
RW=1./RLS**2.
TT=TT**2.
IF(ITYPE.NE.1) GO TO 20

VARIANCE CALCULATION FOR HORIZONTAL PATH

CN2=4.2E-14*EXP(-HW/320.)/HW**0.667
IF(H1.GT.100.0) CN2=0.0
IF(HW.LE.10.0) CN2=8.77E-15
34220 SIGM=0.124*CN2*WK*WRANGE**1.833
34230 RLO=(0.546*CN2*WK*WRANGE)**1.2
34240 RLX=0.425*CN2*WK**2.167*WRANGE**0.833
34250 RLXS=0.114*RLX/0.425
34260 GO TO 80
34270 20 IF(ANGLE.GT.90.0) GO TO 37
34280 C
34290 C VARIANCE CALCULATION FOR UPWARD PATH
34300 C
34310 DS=0.1
34320 DO 34 I=J1,J2
34330 WS(I)=DSW(I)/DZW(I)
34340 IF(XW1(I).LE.25.0) GO TO 21
34350 IF(XW1(I).LE.50.0) GO TO 22
34360 DH=400.0
34370 GO TO 23
34380 21 DH=20.0
34390 GO TO 23
34400 22 DH=100.0
34410 23 WIK=(XW2(I)-XW1(I))*100.0/DH
34420 IK=WIK
34430 HW=XW1(I)*1000.0
34440 DO 33 J=1,IK
34450 HW=HW+DH
34460 IF(HW.GE.99600.0.AND.ISTYPE.EQ.3) XW2(I)=100000.0
34470 IF(HW.GE.99600.0) GO TO 34
34480 CN2=4.2E-14*EXP(-HW/320.)/HW**0.667
34490 IF(HW.LE.10.0) CN2=8.77E-15
34500 DQ=WRANGE-DS
34510 TLG=TLG+CN2*DS**0.833*DH*WS(I)
34520 TLO=TLO+CN2*DS**1.667*DH*WS(I)
34530 TLX=TLX+CN2*DQ**1.833*DH*WS(I)/DS**0.167
34540 DS=DS+DH*WS(I)
34550 33 CONTINUE
34560 DQ=WRANGE-DS
34570 IF(DQ.LE.0.) DS=DS-DH*WS(I)
34580 IF(DQ.LE.0.) DQ=DQ+DH*WS(I)
34590 XY=(XW2(I)*1000.-HW)*WS(I)
34600 IF(XY.LE.0.) GO TO 34
34610 TLG=TLG+CN2*DS**0.833*XY
34620 TLO=TLO+CN2*DS**1.667*XY
34630 TLX=TLX+CN2*DQ**1.833*XY/DS**0.167
34640 DS=DS+XY
34650 34 CONTINUE
C  VARIANCE CALCULATION FOR DOWNWARD PATH
C
DS=0.1
DT=0.1
L1=J1
DO 60 L=1,NL
WS(L1)=DSW(L1)/DZW(L1)
IF(XW1(L1).LE.25.0) GO TO 38
IF(XW1(L1).LE.50.0) GO TO 39
DH=400.0
GO TO 40
GO TO 40
38 DH=20.00
GO TO 40
39 DH=100.00
40 WIK=(XW1(L1)-XW2(L1))*1000.0/DH
IK=WIK
HW=XW1(L1)*1000.0
DO 57 J=1,IK
HW=HW-DH
CN2=4.2E-14*EXP(-HW/320.)/HW**0.667
IF(HW.LE.10.0) CN2=8.77E-15
DQ=WRANGE-DS
TLG=TLG+CN2*DS**0.833*DS*WS(L1)
TLO=TLO+CN2*DS**1.667*DS*WS(L1)
TLX=TLX+CN2*DQ**1.833*DS*WS(L1)/DS**0.167
DS=DS+DS*WS(L1)
IF(K2.EQ.0) GO TO 57
IF(K2.EQ.1.AND.WH2.LE.HW) GO TO 57
TLG=TLG+CN2*(WRANGE-DT)**0.833*DS*WS(L1)
TLO=TLO+CN2*(WRANGE-DT)**1.667*DS*WS(L1)
TLX=TLX+CN2*DQ**1.833*DS*WS(L1)/(WRANGE-DT)**0.167
DT=DT+DS*WS(L1)
57 CONTINUE
DQ=WRANGE-DS
IF(DQ.LT.0.0) DQ=DQ+DS*WS(L1)
IF(DQ.LT.0.0) DS=DS*WS(L1)
XY=(HW-XW2(L1)*1000.)*WS(L1)
IF(XY.LE.0.0) GO TO 61
TLG=TLG+CN2*WS**0.833*XY
TLO=TLO+CN2*WS**1.667*XY
TLX=TLX+CN2*DQ**1.833*XY/WS**0.167
35100 DS=DS+XY
35110 IF(K2.EQ.0) GO TO 58
35120 IF(K2.EQ.1.AND.WH2.LE.HW) GO TO 58
35130 TLG=TLG+CN2*(WRANGE-DT)**0.833*XY
35140 TLO=TLO+CN2*(WRANGE-DT)**1.667*XY
35150 TLX=TLX+CN2*DT**1.833*XY/(WRANGE-DT)**0.167
35160 DT=DT+XY
35170 58 CONTINUE
35180 LL=LL-1
35190 IF(K2.EQ.0.AND.LL.LE.J2) GO TO 61
35200 IF(LI.LE.JMIN.AND.K2.EQ.1) GO TO 61
35210 60 CONTINUE
35220 61 CONTINUE
35230 75 CONTINUE
35240 SIGM=0.225*VK*TLG
35250 RLO=1.575*WK*2.4*TLO*1.2/WRANGE**2.
35260 RLXS=0.235*WK*VK*TLX/WRANGE**1.833
35270 80 CONTINUE
35280 F4=EXP(4.*SIGM)
35290 C CALCULATION FOR SPHERICAL WAVES
35300 C
35310 C
35320 IF(TT.GE.0.001) GO TO 81
35330 F5=1.+RR**2.*SIGM*RLO*8.
35340 SMF=F4/F5*(2.+EXP(-4.*GAA)-2.*EXP(-2.*GAA))-1.
35350 GO TO 89
35360 81 CONTINUE
35370 C CALCULATION FOR BEAM WAVES
35380 C
35390 C
35400 RDI=1./STT
35410 F1=4.*RDI**2.
35420 TM=2.*RDI+2.*RW
35430 XM=0.5*RDI+0.25*RW
35440 HX=RLO*SIGM*2.
35450 AML=AM-RLXS*4.
35460 AMLS=AML**2.
35470 BM=0.25*TM+STT*SA/2.+4.*RLO
35480 DM=0.5*RDI+HX*2.
35490 EK=DM+0.5*RW
35500 GM=EK+AMLS/(4.*DM)
35510 EN=DM+AMLS/(4.*EK)
35520 QM=DM+0.25*RW
35530 UM=SA/ST+0.25*TM+4.*RLO
35540  WM=GM-0.5*RW
35550  YM=XH+SA/(4.*XM)+.5*RLO
35560  ZM=QM+AMLS/(4.*QM)
35570  HA=SA/BM
35580  HB=SA/UM
35590  HC=SA/YM
35600  F2=(TM/8.0+RLO+STT*SA/4.)**2.
35610  FA=0.5*STT/(BM*DI1*GM)
35620  FB=0.5*STT/(BM*EK*EN)
35630  FC=EXP(-4.*GAA)/(TM*UM*DN*WM)
35640  FD=EXP(-2.*GAA)/(2.*XM*YM*QM*ZM)
35650  F3=FA+FB+FC-FD
35660  SMI=F1+F2+F3+F4-1.
35670  IF(SMI.LE.0.) SMI=0.
35680  VRI=SMI**0.5
35690  IF(RR.LT.0.001) GO TO 90
35700  SF1=HX**2.-HX**2./DM+(AML*HX/DM+AM+RLXS*2.)**2./(GM*4.)
35710  SF2=HX**2.-HX**2./EK+(AML*HX/EK+AM+RLXS*2.)**2./(EN*4.)
35720  SF3=HX**2.-HX**2./DM+(AML*HX/DM+AM+RLXS*2.)**2./(WM*4.)
35730  SF4=HX**2.-HX**2./QM+(AML*HX/QM+AM+RLXS*2.)**2./(ZM*4.)
35740  Q1=FI*F2*F4
35750  Q2=STT*SA/(1.+DC**2.+STT**2.*SA+STT*4.*RLO)
35760  RR2=1./RR**2.
35770  Q3=FA/((SF1+RR2*0.5)*(HA+RR2*2.))
35780  Q4=FB/((SF2+RR2*0.5)*(HA+RR2*2.))
35790  Q5=FC/((SF3+RR2*0.5)*(HB+RR2*2.))
35800  Q6=FD/((SF4+RR2*0.5)*(HC+RR2*2.))
35810  SMP=Q1*(Q2+RR2)**2.*(Q3+Q4+Q5+Q6)-1.
35820  89 IF(SMP.LE.0.) SMP=0.
35830  VRI=SMP**0.5
35840  90 CONTINUE
35850  RETURN
35860  END
References


LOWTRAN V PREDICTIONS WITH VARIANCES

Fig. 1. Plane wave sources; point receivers; horizontal paths.
Fig. 2. Plane wave sources; point receivers; upward paths.
Fig. 3. Plane wave sources; point receivers; downward paths.
Fig. 4. Plane wave sources; receivers radius R = 10 cm; horizontal paths.
LOWTRAN V PREDICTIONS WITH VARIANCES

Fig. 5. Plane wave sources; receivers radius $R = 10$cm; upward paths.
Fig. 6. Plane wave sources; receivers radius $R = 10\text{cm}$; downward paths.
Fig. 7. Beam wave sources, $\alpha = 2$ cm; point receivers; horizontal paths.
Fig. 8. Beam wave sources, $\alpha_s = 2\text{cm}$; point receivers; upward paths.
Fig. 9. Beam wave sources, $a_s = 2\text{cm}$; point receivers; downward paths.
Fig. 10. Beam wave sources, $a_s = 2\text{cm}$; receiver radius $R = 10\text{cm}$; horizontal paths.
Fig. 11. Beam wave sources, $\alpha_s = 2\text{cm}$; receiver radius $R = 10\text{cm}$; upward paths.
Fig. 12. Beam wave sources, \( \alpha_s = 2\text{cm} \); receiver radius \( R = 10\text{cm} \); downward paths.
Fig. 13. Spherical wave sources; point receivers; horizontal paths.
Fig. 14. Spherical wave sources; point receivers; upward paths.
Fig. 15. Spherical wave sources; point receivers; downward paths.
Fig. 16. Spherical wave sources; receivers radius $R = 10$ cm; horizontal paths.
Fig. 17. Spherical wave sources; receivers radius $R = 10 \text{cm}$; upward paths.
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