Final Report
VECTORIZED SPARSE ELIMINATION

For Period 5/1/80 - 4/30/84
Grant AF-AFOSR 80-0158

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March 1, 1984

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**REPORT DOCUMENTATION PAGE**

1a. REPORT SECURITY CLASSIFICATION  
UNCLASSIFIED

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
AFOSR-TR-84-0367

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
University of Michigan

6b. OFFICE SYMBOL (If applicable)

7a. NAME OF MONITORING ORGANIZATION
Air Force Office of Scientific Research

7b. ADDRESS (City, State and ZIP Code)
Directorate of Mathematical & Information Sciences, Bolling AFB DC 20332

8. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
AFOSR-80-0158.

9. SOURCE OF FUNDING NO.

10. TITLE (Include Security Classification)
VECTORIZED SPARSE ELIMINATION

11. PERSONAL AUTHOR(S)
D.A. Calahan

12. TYPE OF REPORT
Final

13a. TIME COVERED  
FROM 1/5/80 TO 30/4/84

13b. TIME COVERED
1 MAR 1984

14. DATE OF REPORT (Yr., Mo., Day)
15. PAGE COUNT
10

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)

19. ABSTRACT (Continue on reverse if necessary and identify by block number)
This grant concerned the case of vector processors such as the CRAY-1 in the solution of sparse systems of equations. The study produced three major classifications of results. (1) Algorithms and related mathematical software for sparse solution on single processors (uniprocessors). (2) Preliminary projection of vector multiprocessor performance on linear algebra codes. (3) Cooperative work on vector sparse matrix algorithms with AFFDL for CFD and structures codes, and with UC/Berkeley on for circuit simulation.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
UNCLASSIFIED/UNLIMITED ☑ SAME AS RPT. ☐ DTIC USERS ☐

21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL
CPT John P. Thomas, Jr.

22b. TELEPHONE NUMBER (Include Area Code)
(202) 767-5026

22c. OFFICE SYMBOL
NM
I. INTRODUCTION

This grant concerned the case of vector processors such as the CRAY-1 in the solution of sparse systems of equations. The study produced three major classifications of results:

(1) Algorithms and related mathematical software for sparse solution on single processors (uniprocessors).

(2) Preliminary projection of vector multiprocessor performance on linear algebra codes.

(3) Cooperative work on vector sparse matrix algorithms with AFFDL for CFD and structures codes, and with UC/Berkeley on for circuit simulation.
II. TECHNICAL SUMMARY

A. Uniprocessor Studies

Figure 1 indicates how the single topic of general sparse matrix solution using scalar processors may be broken into specialized areas of study when implementation on vector architectures is considered.

First, highly sparse matrices, usually representing ODE/algebraic-modeled systems, are easily decoupled by re-ordering. At a minimum, locally-decoupled equations may be solved in pipelined scalar mode; if the decoupled subsystems can be arranged (a) to have identical sparsity, and (b) to be stored a constant stride apart, then a simultaneous sparse solver may be invoked and a vector solution obtained.

As sparse systems become locally coupled - as occurs in finite element and finite difference problems - then vectors are easily defined within the coupled subsystems. It is worth making a further distinction between

(a) intra-nodal or intra-element coupling, where the dimension of dense submatrices (and hence the vector length) is proportional to the number of unknowns/node or unknowns/finite element, and

(b) inter-nodal or inter-element, where the coupling between grid nodes or finite elements determines the vector length.

Banded and profile matrices result from the latter. The associated vector lengths are the products of the number of unknowns/node (element) and the number of coupled nodes. These
Figure 1
HIERARCHY AND CLASSIFICATION OF SPARSE MATRIX SOFTWARE

GENERAL SPARSE
.25 - 35 MFLOPS

DECOUPLING

PATTERNED;
SIMULTANEOUS SPARSE
SOLVER;
- 70 MFLOPS

UNPATTERNED;
SCHEDULED SCALAR
SOLVER
15 MFLOPS

BLOCKING

RECTANGULAR;
INTRANODAL &
INTRA-ELEMENT
- 141 MFLOPS

DIAGONAL;
INTERNODAL;
BLOCKED PROFILE
- 126 MFLOPS
lengths are therefore always longer than in the former case, so that common bandsolvers potentially offer the highest execution rate (MFLOPS) of any sparse solver applicable to finite element problems.

Algorithms and CRAY-1 mathematical software have been developed on this grant for

(a) general sparse matrices, solved in traditional [15] and reorganized pipelined-scalar mode [7][16];

(b) patterned sparse matrices, in conjunction with a vectorized electronic circuit analysis program [2];

(c) blocked matrices arising from inter-nodal coupling [12];

(d) both symmetric and unsymmetric banded and blocked-profile matrices [13][14];

(e) simultaneous blocked tridiagonal systems, as arising in CFD [17].

Mathematical software resulting from these studies have been collected in a library [17].

B. Multiprocessor Studies

In the 1980's, to achieve GIGAFLOP performance with clock times in the range of 10 usec will require multiple vector processors executing cooperative tasks. The ability of extensions of the present CRAY family of processors to execute small tasks in an efficient manner was initially studied by developing a multi-processor extension of the CRAY-1 simulator [18]. Several algorithms were studied for the triangular factorization of small matrices. Figure 2 shows representative
Figure 2. Speedup of Microtasked Solution

![Graph showing speedup of microtasked solution with matrix size on the x-axis and efficiency (speedup/p) on the y-axis.](image-url)
efficiencies* for the cooperative LU factorization of matrices, with the number of processors (p) ranging from 2 to 16. These results were compared with 2-processor CRAY XMP timings, using an experimental in-house multitasking operating system at Cray Research, Inc.

This work will continue under a new AFOSR grant.

C. Air Force-related Applications

1. **Electronics.** A Ph.D. student from UC/Berkeley engaged in electronic circuit simulation was a visitor to our research group during the summer of 1980 to study the CRAY-1 and our sparse matrix research. Under subsequent Bell Laboratories and AFOSR auspices, he produced a series of papers and a vectorized version of the SPICE electronic circuits analysis program which achieves a 5-10:1 speedup on the CRAY-1. The speedup achieved from simultaneous sparse solution of subcircuit matrices—originally proposed in [2]—is the critical feature of this program.

2. **Aerodynamics.** Work was completed in 1980 on FDL-sponsored research on vectorization of computational fluid dynamics (CFD) codes. A four-day seminar on the general topic of vector processing was presented at FDL in 1980.

   Air Force sponsored research on vectorized CFD algorithms has lead to related work currently sponsored by NASA/Ames Research Center.

3. **Structures.** From 1981 - 1983, two related finite

*Efficiency (n)= (uniprocessor time)/(p*(multiprocessor time))
element analysis and optimization codes from FDL were vectorized, under joint AFOSR-FDL sponsorship. By far the greatest speedup (> 2000:1) was due to vectorized banded equation solvers developed under a AFOSR sponsorship [14]. A report, including comparisons with NASTRAN, has been written on these results [20].

III. OTHER COUPLING AND PROFESSIONAL ACTIVITIES

A. Seminars

1. Washington State University (11/17/80)
2. University of California, Berkeley (11/19/80)
3. University of Texas, Austin (10/30/80)
4. 4-day seminar at AFFDL, (6/80)
5. Seminar at LANL (8/80)
6. Review of vector processing research, AFFDL (5/26/81; 7/14/83)
7. Review of the state-of-the-art in scientific computation at AFOSR (5/6/82).

B. Visiting Scientist and Consulting

1. Visiting scientist, AFFDL, to give instruction on algorithms for vector processing, and to study I/O problems associated with Navier-Stokes codes on the CYBER 203/205 (5/1/80 – 9/30/80).
2. Visiting scientist, LANL, on vectorized Monte Carlo (5/1/80 – 9/30/81).
4. Visiting scientist, LANL, on task granularity on vector multiprocessors (10/1/83 – 4/30/84).
5. Industrial consultant, Mobil Research and Development, on the vectorization of 3-D diffusion codes associated with oil reservoir drilling and management (5/1/80 – 1/15/82).

6. Industrial consultant, Chevron Oil Field Research Co., on organization of vectorized sparse matrix algorithms (2/82) and on vector multiprocessors (12/83).

7. Consultant, LLNL, on vectorized Monte Carlo (5/1/82 – 9/30/83).

8. Visiting scientist, AFFDL, on vectorized structural analysis and optimization techniques (5/1/82 – 9/30/83).


C. Related Research

1. Principal investigator, NASA/ARC, on vectorization of computational chemistry codes (8/1/82 – ).


D. Professional

1. A one-week short course on Vector Processing was organized and presented at the University of Michigan during the summers of 1980, 1981, and 1982.

2. As an appointed member of a NASA Technical Review Board, an evaluation was made of proposals from Control Data Corporation and the Burroughs
Corporation for the $100 million Numerical Aerodynamic Simulator (5/1/83 - 7/31/83).

3. Editor, IEEE Transaction on Computers, 8/1/82 - 12/31/83.

Grant-sponsored References

Journal Articles


Conference Publications


Conference Presentations


Reports


Other
