SYNTHESIS OF NATURAL TEXTURES ON 3-D SURFACES

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ABSTRACT

This paper presents a new method for the synthesis of textures on 3-D surfaces. To our knowledge, one basic technique has been presented up to now in the literature. In this standard method, textures are synthesized by mapping a rectangular template onto the curved surface. This method is complex, requires substantial computing time, and presents some drawbacks such as the possibility of obtaining aliasing effects and continuity problems along the edges of the curved templates. Procedures to eliminate these problems are available but make this synthesis even more unattractive. The method proposed in this paper does not present the former drawbacks. We do not use a template mapping, which is a drawback in itself. The synthesis is achieved continuously on the surface, so that there are no edge effects and also no aliasing effects. This method is a simple extension of a procedure that we have proposed before in the literature for planar textures. Any kind of texture can be reproduced with a good similarity to the reference texture used. It also has the important advantage that only one set of second order statistics (a small amount of data) needs to be computed on a planar version of the reference texture to synthesize this texture on any surface and at any distance. Some results on simple surfaces are displayed (cylinder, sphere), but the method holds for any surface and is relatively quick and easy.

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1. Introduction

The generation of images by computer is becoming more and more popular nowadays. It has been applied industrially mainly for flight simulators in the past few years, but it is becoming more and more utilized for entertainment. A number of movie films generated by computer have already been presented on television, and even in theaters in the case of the movie TRON.

The techniques available in computer graphics are already very powerful for the representation of 3-D scenes including various illumination effects (shadows, transparency, variations of the reflectance), but their performance is still poor in the case of textured surfaces. The purpose of this work is to find a solution to this problem. We think that the texture synthesis procedure described in this paper should allow the generation of much more realistic images (since we are able to reproduce any kind of texture with good visual similarity) using a rather small amount of data. Only one set of second order statistics per texture is sufficient to synthesize it on any surface having any extent. This method is a generalization of the synthesis procedure we have designed for planar textures [2,3] to the case of 3-D surfaces.

Given a reference feature vector constructed by the concatenation of the second order statistics (second order spatial averages or autocovariance + histogram) of a planar texture field, we construct the projected image of a 3-D textured surface while minimizing the mean square error between the feature vector
of the current 3-D texture and the reference feature vector. The synthesis is achieved point by point, sequentially, thus yielding good continuity properties on the surface so that we have neither aliasing effects nor discontinuity effects, which usually occur with the standard procedure available in the literature [7-13]. The drawbacks of the standard method are inherent to its nature because the synthesis is done by mapping a rectangular planar patch of texture onto the curved surface. The techniques used to eliminate aliasing effects and continuity problems along the boundaries of the patches [11,12] make the method difficult to use in practice. All these reasons favor the new synthesis method proposed here.
2. **Definitions and notation**

Using standard computer graphics terminology, we define **object space** as the three-dimensional space in which an object will ordinarily be described. In order to generate realistic images of objects, we make a perspective transformation of the object from the object space into the **image space**. The image space is also three-dimensional, but objects have undergone a perspective distortion. An orthogonal projection of the image onto, let us say, the x-y plane results in the expected perspective image, which will be called a **projected image**. Since the main purpose of this work is texture synthesis, for simplicity we shall suppose that the viewpoint is at infinity so that object space and image space will be identical (and, from now on, will be called object space), and a projected image will be called simply an image.

A texture is considered to be a realization of a stochastic process \( x \) defined on a domain \( S \). \( S \) is a digital surface in a 3-D space. \( x \) takes its values on a finite set \( L \) of grey levels:

\[
L = \{0, 1, \ldots, L-1\}
\]

All points of \( S \) can be indexed by their rank \( i \) (\( i=1,2,\ldots,N \) where \( N \) is the number of points in \( S \)); \( x_i \) will represent the value of \( x \) at location \( i \). We shall deal with **homogeneous** and **ergodic** processes, at least up to the second order, so that probabilistic parameters and estimates of these parameters will be identical. Thus, **first order probability distributions** \( p(\ell) \) can be confused with histogram parameters \( \hat{p}(\ell) \) and **autocovariance**
parameters $M_2(\Delta)$ can be confused with second order spatial moments $\tilde{M}_2(\Delta)$, where

\[
\begin{align*}
\left\{ 
\begin{array}{ll}
p(\ell) &= p(x_m=\ell) & \forall m, \ell \\
\tilde{p}(\ell) &= \frac{1}{N} \sum_{m=1}^{N} \delta(x_m-\ell) & \forall \ell
\end{array}
\right. \\
\end{align*}
\]

\[M_2(\Delta) = E[(x_m-\eta) \cdot (x_{m+\Delta}-\eta)] \quad \forall \Delta \]

(2)

\[M_2(\Delta) = \frac{1}{4\sigma^2} \sum_{m=1}^{I} (x_m-\eta) \cdot (x_{m+\Delta}-\eta) \quad \forall \Delta \]

and

\[
\eta = \frac{1}{N} \sum_{m=1}^{N} x_m
\]

\[
(= E[x_m])
\]

is the texture mean,

\[
\sigma^2 = \frac{1}{N} \sum_{m=1}^{N} (x_m-\eta)^2
\]

\[
(= E[(x_m-E[x_m])^2])
\]

is the texture variance, $E[.]$ denotes the mathematical expectation; $\delta$, the Kronecker function; $\Delta$, a translation in the 3-D space; and $I$ is the number of pairs $m$, $m+\Delta$ belonging to $S$. A more extensive discussion of these homogeneity and ergodicity properties of textures can be found in [1,4].

Psychovisual experiments [1,4] have shown that planar textures are well modelled by second order spatial averages

$\tilde{p}_\Delta(L_1, L_2)$ where $\Delta$ varies in a 2-D space such that $||\Delta|| \leq 9'$ of solid angle (approximately). We have also shown that a simpler model with histogram parameters and autocovariance parameters $\tilde{M}_2(\Delta)$, where $\Delta$ varies in the same range as above, gives acceptable synthesis results for certain natural textures. The synthesis procedure designed in this paper is based upon the latter model, but this is by no means a limitation; we could use a similar
procedure controlling second order spatial averages as well.
The synthesized textures would be of a better visual quality,
but the algorithm would be somewhat more complicated and would
require much longer computation time. An interesting property
of the model we proposed for textures [1,2,4] is that it can
be extended immediately to the case of textures lying on 3-D
surfaces. The fact that this extension is easy reinforces
the former results. The general conjecture becomes:

"We cannot discriminate two textures if \textbf{locally} they possess
the same second order statistics for $\|\Delta\| \leq 9'$ of solid angle"
($\Delta$ belongs to a 3-D space in the case of a 3-D texture).

In practice, the problem we have to solve is to synthesize
a realization of a homogeneous stochastic process defined on
a 3-D surface from a priori given second order statistics (in our
case, histogram + autocovariance function) for $\|\Delta\| \leq 9'$ approxi-
mately. Locally, a texture can be considered as a distorted
version of a planar texture so that we can suppose that our
input data were computed on a plane. $\Delta$ then belongs to a 2-D
space and our model of texture becomes a planar model, which
is easier to work with. But the algorithm will have to take
care of the geometric distortions on the 3-D surface. The im-
portance of the first remark is that \textbf{one} set of second order
statistics (computed on a planar texture field) will allow us
to synthesize this texture on any surface. The model is an
\textbf{intrinsic} one; it does not depend on the type of surface on which
the texture lies. This corresponds well to our intuitive under-
standing of textures.
3. The texture image and its feature vector B

Based upon earlier experiments [1,4], the set of translations \( \Delta \) we should consider should be defined within a circle (since we are dealing with a planar model). For simplicity, we decided to use a set of translations defined by a rectangle \( \text{TR}' \) of \((2Nx+1)*(2Ny+1)\) pixels (see Figure 1). This rectangle will contain the previously considered circle. Since we are dealing with a discrete array, all translations \( \Delta \) are defined by a pair of discrete values \((nx,ny)\). Let us define \( \text{TR}' \) by

\[
\text{TR}' = \left\{ \Delta, \Delta=(n,m) \quad \forall n = -Nx,-Nx+1,...,+Nx, \forall m = -Ny,-Ny+1,...,+Ny \right\}
\]

Since the autocovariance function is symmetric \((M_2(\Delta)=M_2(-\Delta))\), we can restrict ourselves to half of the previous domain:

\[
\text{TR} = \left\{ \Delta, \Delta=(n,m), \quad \left((\forall n \in [-Nx,Nx]) \cap (m \in [1,Ny]) \right) \cup \left((\forall n \in [-Nx,-1] \cap (m=0))\right) \right\}
\]

The total number of translations belonging to the set \( \text{TR} \) is then equal to

\[2*Nx*Ny+Nx+Ny.\]

For example, in the case where \( Nx=Ny=1 \), this number is equal to 4.

We denote by \( B \) the vector containing the autocovariance parameters related to the various translations \( \Delta \) belonging to \( \text{TR} \), and by \( H \) the vector containing the histogram coefficients.
4. Synthesis procedure

4.1 Principle

The synthesis is achieved directly on the object surface, but since there is a one to one correspondence between the object and its projected image, we need only to manipulate the points of the image plane (see Figure 3). Suppose, for instance, that the viewpoint is at infinity along the positive z axis as shown on Figure 3. Let us denote by $S'$ the projection of $S$ on the $z=0$ plane. $S'$ is the visible part of $S$ from our viewpoint. If $i$ is a location on the surface $S$, let us denote by $i'$ its projection on the image $S'$ of $S$. For simplicity, we consider that there is no shadow effect and that the reflectance is constant, though such effects could also be treated by our method. If $x_i$ is the luminance of the texture at location $i$, the luminance of the image at location $i'$ is then $x'_i$, where $x'$ can be considered as a stochastic process defined on $S'$. With this hypothesis, we have

$$x'_i = x_i$$

so that it is equivalent to manipulate points of $S'$ or points of $S$. We suppose, of course, that the 3-D surface $S$ is known, so that we know its visible part and the correspondence $i \rightarrow i'$ is one to one. If the surface is known, we can determine the tangent plane $P_i$ to the surface $S$ at each location $i$. We suppose that we also know the local coordinate system $(u_i, v_i)$ associated with $P_i$ for each location. This information is structural information telling us how the texture is positioned locally on the surface $S$. This type of information is a free parameter and is an input to our synthesis procedure. Of course, this information is constrained by the fact that the local coordinate system has to vary smoothly on the surface as a function of $i$. 
The only restrictive hypothesis that we are going to make is that if we consider the neighborhood TR' of location i on the tangent plane P'_i, then all points of TR' also lie on the surface S. This means that the surface S is smooth enough to be approximated by its tangent plane up to a length of \( \max(N_x,N_y) \) around each location i. This hypothesis implies strong differentiability constraints on the surface which may not always be true.

Given this hypothesis, we observe from (1) and (2) that the feature vector related to the surface S corresponds precisely to the set TR of translations. Now it is possible to solve our problem. We have to construct a texture field \( T_x \) on S such that its feature vector \( B^{T_x} \) will be equal to an a priori given vector B. The synthesis is achieved in two steps in a very similar way to that used in our earlier publications [2,3]. First, we create a texture field on S' (or S, equivalently) which is a realization of homogeneous white noise such that its histogram \( H^{T_x} \) is equal to the desired histogram H. Then, at the second step, the texture is modified point by point, sequentially, in a homogeneous way, while minimizing a criterion which is basically the mean square error between the desired feature vector and the current one.
4.2 Inputs to the synthesis procedure

There are three different inputs to the synthesis procedure:

- First, the feature vectors \( H \) and \( B \) containing the histogram and the autocovariance parameters related to the translation set \( TR \) of the desired texture.
- Second, the \( x,y,z \) sampled coordinates of the surface \( S \)
- Third, the local coordinate system \( (u_i,v_i) \) on the tangent plane \( P_i \) of the surface at each location \( i \) (see Figure 3).

This set of inputs shows that there are many degrees of freedom in the construction of a texture on a 3-D surface.
4.3 **First step of the synthesis procedure**

Given a histogram vector $H$, we compute its cumulative function (integral). Using a random number generator, we can immediately synthesize white noise $T_x$ such that its histogram $H^{T_x}$ is equal to $H$. $T_x$ is defined on $S'$. After this step, the feature vector $B^{T_x}$ of the synthesized texture is equal to zero since $T_x$ is white noise ($M_2(\Delta) = 0, \forall \Delta$). If $B=0$, the procedure is stopped at this point. Otherwise, we have to proceed to the second step.
4.4 Second step

We modify the texture field $T_x$ defined on $S'$ sequentially while minimizing the criterion

$$\text{Err} = \|B - B^{T_x}\|^2 + \alpha \|H - H^{T_x}\|^2$$

where $\alpha$ is a large number. This criterion is roughly equivalent to the minimization of $\|B - B^{T_x}\|^2$ under the constraint $H = H^{T_x}$.

Suppose that we are at location $i$, that $x_i = L_1$, and that $B^{T_x}$ and $H^{T_x}$ are known (they are initialized by $B^{T_x} = 0$ and $H^{T_x} = H$ after the first step). Suppose that we replace $L_1$ by $L_1'$. The idea is to determine the new values of $B^{T_x}$ and $H^{T_x}$.

We have to consider the various neighbors of $x_i$ on the surface $S$ (or equivalently, on the tangent plane $P_i$). The updating of $H^{T_x}$ is given by:

$$T_x(L_1) - L_1 - T_x(L - L_1)$$

For a given translation $\Delta$ belonging to the set $T_R$, $M_2^{T_x}(\Delta)$ is updated by

$$\tilde{M}_2^{T_x}(\Delta) = \tilde{M}_2^{T_x}(\Delta) + \frac{1}{I_{\Delta}}(L_1' - L_1) \ast (x_{i+\Delta} + x_{i-\Delta} - 2\eta),$$

We have neglected the contributions of the variations of $\eta$ and $\sigma$ in the updating of $M_2(\Delta)$ since they are second order terms. Usually $(i+\Delta)$ and $(i-\Delta)$ are not sample points of $S$; in other words, their projections $(i-\Delta)'$ and $(i+\Delta)'$ on the image $S'$ are not sample points (see Figure 4). $x_{i-\Delta}$ and $x_{i+\Delta}$ are computed by bilinear approximation taken from the four nearest neighbors of $(i+\Delta)'$ and $(i-\Delta)'$ on $S'$. 
If we denote by \( \text{Err}(L_1) \) the value of the criterion \( \text{Err} \) before the updating, when \( x_i = L_1 \), it is then possible to compute the new value of the error \( \text{Err}(L'_1) \), if we replace \( L_1 \) by \( L'_1 \). We simply apply (3) and (4) to obtain the new values of \( H^x \) and \( B^x \). Finally, we replace \( L_1 \) by \( L^* \) where

\[
\text{Err}(L^*) \leq \text{Err}(L'_1) \quad \forall L'_1 \in L.
\] (5)

As in our earlier publications [2,3] it is possible to update the error instead of recomputing the error at each time, using

\[
\text{Err}(L'_1) = \text{Err}(L_1) + \sum_{\Delta \in \text{TR}} \left[ \frac{2}{\sigma^2} (\hat{M}_2(\Delta) - \hat{M}_2^x(\Delta)) * (L_1 - L'_1) \right. \\
\times (x_{i+\Delta} + x_{i-\Delta} - 2\eta) + \frac{1}{\sigma^4} (L_1 - L'_1)^2 (x_{i+\Delta} + x_{i-\Delta} - 2\eta)^2 \\
- \frac{2 \hat{\gamma}}{N} (\hat{p}(L'_1) + \hat{p}^x(L_1) - \hat{p}^x(L'_1) - \hat{p}(L_1)) \right]
\]

All points of \( S \), or equivalently all points of \( S' \), are scanned using a random sequence so that the criterion decreases homogeneously all over the surface \( S \) (or \( S' \)). (See [2,3] for a discussion of this phenomenon.) The procedure is such that the criterion decreases monotonically (from (5)) when we update the various points of \( S' \).
5. **Results**

The first experiment was performed to verify that our synthesis procedure gives correct results. For these purposes we chose to achieve the synthesis on a cylinder, which is a ruled (developable) surface, so that it is possible to compare these results with a synthesis achieved starting from a texture synthesized on a plane (which gives correct results [2,3]) and then applied to the cylinder.

The textures PAPER, WOOL and RATTAN taken from Brodatz [6] were used. Figure 5 shows natural PAPER texture and Figure 6 shows the result of the synthesis of PAPER using the planar technique [3]. Comparison of the two figures shows that the artificial texture is very similar visually to the original one. Figure 7 shows the natural PAPER texture on a cylinder seen at about 30° above the x-axis on the (x,z) plane. We simply applied the texture of Figure 5 onto the cylinder. Figure 8 is a similar construction taken from Figure 6. Figure 9 displays the result of our synthesis procedure achieved directly on the cylinder surface. The input feature vector consisted of 8 histogram parameters and 32 autocovariance parameters (NX=NY=2). The local coordinate systems \((u_i,v_i)\) on the surface of the cylinder were chosen to be consistent with the way we applied the texture plane onto the cylinder in Figure 7. The \(u_i\) vector was tangent to the circle parallel to the base of the cylinder and passing through location \(i\), and \(v_i\) was parallel to the z axis, as shown in Figure 10. Inspection of Figures 7, 8 and 9 shows that our synthesis procedure is correct, so that the hypothesis relating the points of \(P_i\) to the points of \(S\) is correct also.
Figures 11 and 12 present a comparison of natural RATTAN applied onto the cylinder from the plane and the direct synthesis of RATTAN using NX=NY=12. Figures 13 and 14 show similar results for WOOL.

Figures 15 and 16 show the synthesis of PAPER and RATTAN textures on the sphere using the same input data B as before. The sphere is seen from the positive z axis (the visible pole of the sphere is at the center of the circle). On Figures 17 and 18, the sphere is seen from the x positive axis (the poles are not seen). The local coordinate systems chosen on the sphere are such that $u_i$ is tangent to the parallel passing through location $i$, and $v_i$ is tangent to the longitude circle passing through $i$, as shown in Figure 19. The visual aspect of the 3-D textures is better when the poles are not seen, as expected. Another set of local coordinate axes would give another visual aspect of the sphere which would correspond to another way to structure (or deform) the texture onto the sphere or the cylinder.

The reader can verify that it is easy to recognize the various textures (PAPER, WOOL or RATTAN) on the various surfaces, and that we do not have any aliasing effects or any discontinuities on the images. The same 40 parameters are used to synthesize PAPER on the plane, the sphere or the cylinder, and it would be the same for any surface. For RATTAN and WOOL we used 320 coefficients. The visual appearances of the synthesized WOOL and RATTAN are not very good. It would be possible
to design a similar technique controlling the second order spatial averages instead of the autocovariance parameters. This would increase the visual quality of the synthesized textures. In any case, the synthesis procedure is easy to implement and we think that such a technique will have a number of applications in the near future.
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Figure 1: Set TR' of translations Δ considered

Figure 2: Set TR of translations Δ for Nx=Ny=1

Figure 3: A surface S and its image S' (S seen from the z positive axis)

Figure 4: Projections of TR' on S'
Figure 5: Natural PAPER

Figure 6: Artificial PAPER synthesized on the plane

Figure 7: Natural PAPER ruled on a cylinder

Figure 8: Artificial PAPER ruled on a cylinder
Figure 9: 3-D synthesis of PAPER on a cylinder

Figure 10: Local coordinate systems chosen on the cylinder for Figures 9, 12, 14

Figure 11: Natural RATTAN ruled on a cylinder

Figure 12: Synthesis of RATTAN on a cylinder
Figure 13: Natural WOOL ruled on a cylinder

Figure 14: WOOL synthesis on a cylinder

Figure 15: PAPER synthesis on a sphere seen from the $z$ axis

Figure 16: RATTAN synthesis on a sphere seen from the $z$ axis
Figure 17: PAPER synthesis on a sphere seen from the x axis

Figure 18: RATTAN synthesis on a sphere seen from the x axis

Figure 19: Local coordinate systems chosen on the sphere for figures 15 to 18
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