SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES—FULL WAVE SOLUTIONS (U) NEBRASKA UNIV LINCOLN E BAHAR JAN 84 RADC-TR-83-289 F19628-81-K-0025 UNCLASSIFIED
SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES - FULL WAVE SOLUTIONS

University of Nebraska-Lincoln

Ezekiel Bahar

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SCATTERING AND DEPOLARIZATION OF ELECTROMAGNETIC WAVES--FULL WAVE SOLUTIONS

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Traditionally physical optics and perturbation theories have been used to derive the like and cross polarized scattering cross sections for composite random rough surfaces. To this end two scale models have been adopted and the rough surfaces are regarded as small scale surface perturbations that are superimposed on large scale, filtered surfaces. Thus the scattering cross sections are expressed as sums of two cross sections. The first accounts for specular point scattering. It is given by the...
physical optics cross sections for the filtered surface consisting of the large scale spectral components. The second accounts for Bragg Scattering. It is given by the perturbation cross section for the surface consisting of the small scale spectral components that ride on the filtered surface.

On applying the perturbed-physical optics approaches it is necessary to specify the wavenumber $k_d$ where spectral splitting is assumed to occur between the large and small scale spectral components of the rough surface. However, in general, the restrictions on both the large and small scale surfaces cannot be satisfied simultaneously and using the perturbed-physical optics approaches the evaluation of the scattering cross sections critically depend on the specification of $k_d$.

Recently the full wave approach has been used to determine the scattering cross sections for composite random rough surfaces of finite conductivity. Since the full wave solutions account for Bragg scattering and specular point scattering in a self-consistent manner, it is not necessary to decompose the surface into two surfaces with small and large roughness scales. However, when such a decomposition is feasible, the full wave solutions for the scattering cross sections can be expressed in terms of a weighted sum of two cross sections. In an attempt to draw more definite conclusions regarding the choice of $k_d$, it was varied over a wide range of values. Thus, provided that the large scale surface satisfied the radii of curvature criteria and the condition for deep phase modulation, the full wave solutions for the like polarized scattering cross sections based on the two scale model are practically independent of the specified value of $k_d$.

More recently the full wave approach is used to derive a unified formulation for the like and cross polarized cross sections for all angles of incidence. These solutions are compared with earlier solutions based on a two scale model of the random rough surface. Thus, the simplifying assumptions, that are common to all the earlier solutions based on two scale models of the rough surface, are carefully examined. It is shown that while the full wave solutions for the like polarized scattering cross sections based on the two scale model are reasonably good agreement with the unified full wave solutions, the two solutions for the cross polarized cross sections differ very significantly.
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1.0 Overview

1.1 Summary of Research – Statement of Work

(1) Develop generalized field transforms that provide the basis for complete expansions for the electromagnetic fields in rough surfaces. The generalized transforms will be used to reduce Maxwell's electromagnetic equations in conjunction with exact boundary conditions into sets of first order coupled differential equations for forward and backward travelling wave amplitudes.


(2) The iterative solutions for the vertically and horizontally polarized scattered radiation fields when the slope of the rough surface is small will be obtained through the use of the steepest descent method.

The second order iterative solutions for the scattered radiation fields are presented in the Second Annual Report. (26)

(3) Utilize the results of (2) extend the studies to obtain full wave solutions for scattered radiation fields from two dimensionally rough surfaces with arbitrary slopes.

The full wave solutions for the scattered radiation fields from two dimensionally rough surfaces are presented in the Second Annual Report, Section (1.1) (26). This work has also been applied to problems

*The publications cited in this section (1) through (27) are listed in Section 1.2 of this report.
of depolarization of the scattered radiation field by conducting objects of irregular shape above rough land and sea. The full wave approach was also used to determine the scattering and depolarization of radio waves in irregular spheroidal structures.

(4) The full wave solution obtained in (3) will be compared with the perturbational and physical optics solutions.

The comparison of the full wave solutions with the perturbation and physical optics solutions is presented in the Interim Technical Report RADC-TR-81-203 (August 1981). This work has also been used to resolve the discrepancies between different physical optics solutions for rough surface scattering.

(5) The normalized backscatter cross section for an arbitrary rough surface will be obtained from the results of (2), (3), and (4).

The normalized like and cross polarized backscatter cross sections are derived for arbitrary rough surfaces.

(6) The polarization dependence of the backscatter cross section at grazing incidence and incidence at pseudo-Brewster angle with respect to reference plane will be studied.

The polarization dependence of the scattering cross sections are derived for all angles of incidence. These solutions are compared with the perturbed-physical optics technique.

(7) The study will emphasize the relationship of scattered fields to the statistics of the height and slope of the rough surface which are given by spectral density of the surface height, the correlation function and the correlation distance. The study will take into account the absorption of electromagnetic energy by the medium.
The scattering cross sections have been computed for rough surfaces that are specified through their surface height spectral density functions \(^{(8)}\) of the surface height autocorrelation function. This work is based on a two-scale model of the composite rough surface. Thus the problem of specifying the wavenumber where spectral splitting occurs is considered in detail.

A uniform full wave approach that does not adopt hybrid two-scale models of rough surfaces has also been developed recently \(^{(13)}\). Details of this formulation are presented in section 2 of this report \(^{(27)}\). In this work, the surface is characterized by a complex dielectric coefficient to account for absorption of electromagnetic energy by the medium.

Scattering by Non-Gaussian rough surfaces has also been presented in the RADC-TR-83-47 Interim Technical report (March 1983). In this work the shadow function and the backscatter cross sections have been computed for Non-Gaussian rough surfaces for which decorrelation implies statistical independence \(^{(10),(11)}\).

In addition to publications in technical and scientific journals \(^{(1)}\) through \(^{(13)}\) and the Interim, Annual and Final Reports \(^{(23)}\) through \(^{(27)}\), the principal investigator has presented invited and contributed papers at several international conferences \(^{(14)}\) through \(^{(22)}\). Ten quarterly reports were also submitted during this period.
1.2 **List of Publications by the Principal Investigator During the Period of the Contract**

1.2.1 **Publications in Technical Journals**


6. "Scattering and Depolarization of Electromagnetic Waves in Irregular Stratified Spheroidal Structures of Finite


1.2.2 Abstracts and Summaries of Papers Presented at International Conferences


18. Joint International IEEE/APS and National Radio Science Meeting at the University of New Mexico, May 24-28, 1982, "Scattering Cross Sections for Composite Surfaces with Large Mean Square Slopes--Full Wave Analysis."


1.2.3 Annual Technical Reports and Interim Technical Reports


2.0 SCATTERING CROSS SECTIONS FOR COMPOSITE ROUGH SURFACES USING THE UNIFIED FULL WAVE APPROACH

2.1 Background

The full wave approach is used to derive a unified formulation for the like and cross polarized scattering cross section of composite rough surfaces for all angles of incidence. Earlier solutions for electromagnetic scattering by composite random rough surfaces are based on two-scale models of the rough surface. Thus on applying a hybrid approach Physical Optics theory is used to account for specular scattering associated with a filtered surface (consisting of the large scale spectral components of the surface) while perturbation theory is used to account for Bragg scattering associated with the surface consisting of the small scale spectral components.

Since the full wave approach accounts for both specular point scattering and Bragg scattering in a self consistent manner the two-scale model of the rough surface is not adopted in this work. These unified full wave solutions are compared with the earlier solutions and the simplifying assumptions that are common to all the earlier solutions are examined. It is shown that while the full wave solutions for the like polarized scattering cross sections based on the two-scale model are in reasonably good agreement with the unified full wave solutions, the two solutions for the cross polarized cross sections differ very significantly.
2.2 Discussion

Traditionally physical optics and perturbation theories have been used to derive the like and cross polarized scattering cross sections for composite random rough surfaces (Beckmann 1963, Rice 1951). To this end two-scale models have been adopted and the rough surfaces are regarded as small scale surface perturbations that are superimposed on large scale, filtered surfaces (Wright 1968, Valenzuela 1968, Barrick and Peake 1968). Thus the scattering cross sections are expressed as sums of two cross sections. The first accounts for specular point scattering. It is given by the physical optics cross section for the filtered surface consisting of the large scale spectral components. The second accounts for Bragg scattering. It is given by the perturbation cross section for the surface consisting of the small scale spectral components that ride on the filtered surface.

On applying the perturbed-physical optics approaches it is necessary to specify the wavenumber \( k_d \) where spectral splitting is assumed to occur between the large and small scale spectral components of the rough surface. Thus Brown (1978) who applied a combination of Burrows' perturbation theory (1967) and physical optics (Beckmann 1968), to obtain the scattering cross sections for perfectly conducting random rough surfaces, specified \( k_d \) on the basis of the characteristics of the small scale surface

\[
\beta = 4k_0^2\langle h_s^2 \rangle = 0.1, \text{ where } k_0 \text{ is the electromagnetic wavenumber and } \langle h_s^2 \rangle \text{ in the mean square height of the small scale surface}.
\]
However, using the approaches of Hagfors (1966) and Tyler (1976) the specification of $k_d$ is assumed to be based on the characteristics of the large scale surface. In general the restrictions on both the large and small scale surfaces cannot be satisfied simultaneously and using the perturbed-physical optics approaches the evaluation of the scattering cross sections critically depend on the specification of $k_d$ (Brown 1978).

More recently the full wave approach has been used to determine the scattering cross sections for composite random rough surfaces of finite conductivity (Bahar 1981b, Bahar and Barrick 1983). Since the full wave solutions account for Bragg scattering and specular point scattering in a self-consistent manner, it is not necessary to decompose the surface into two surfaces with small and large roughness scales. However, when such a decomposition is feasible, the full wave solutions for the scattering cross sections can be expressed in terms of a weighted sum of two cross sections (Bahar 1981b, Bahar and Barrick 1983). Thus on adopting a two-scale model, the full wave solution resolves the discrepancies between Valenzuela's (1968) solution (mostly based on physical considerations) and Brown's solution (1978). Furthermore, in an attempt to draw more definite conclusions regarding the choice of $k_d$, it was varied over a wide range of values (Bahar et al 1983). It was shown that while, as expected, the cross sections associated with the individual large and small scale surfaces critically depend upon the choice of $k_d$, the weighted sum of the like polarized cross sections remain practically insensitive to variations in $k_d$ for $\beta > 1.0$. Thus,
provided that the large scale surface satisfies the radii of curvature criteria (associated with the Kirchhoff approximations for the surface fields) and the condition for deep phase modulation, the full wave solutions for the like polarized scattering cross sections based on the two-scale model are practically independent of the specified value of $k_d$. However, on applying the full wave approach to evaluate the like and cross polarized scattering cross sections for two-scale models of composite rough surfaces, several assumptions were made to facilitate the computations. The first assumption was that the large and small scale surfaces were statistically independent (Brown 1978). It would seem reasonable to make such an assumption if the two surfaces are results of independent processes. This would be the case, for example, if the small scale roughness is due to erosion, while the large scale roughness is due to geophysical forces that result in hills and valleys, or as in the case of the sea, where the capillary waves are dependent on surface tension while the large scale rough surface is generated by gravity waves. For the general case, however, one cannot assume statistical independence of the large and small scale surfaces.

The second simplifying assumption that was made was that the mean square slope $\sigma_{FS}^2$ for the total surface was approximately equal to the mean square slope $\sigma_{FS}^2$ for the filtered large scale surface.

The third assumption was that the mean square height of the total rough surface is large compared to a wavelength, and the
surface height characteristic function for the total surface is negligibly small compared to unity.

Finally, the physical optics approximation for the cross polarized backscatter cross section is zero (Brown 1978). As a result, the cross polarized backscatter cross section for the filtered surface is set equal to zero when the two-scale model is used. However, for backscatter, only the specular points on the rough surface do not depolarize the incident wave.

In Section 2.3 the full wave approach is used to derive a unified formulation for the like and cross polarized cross sections for all angles of incidence. These solutions are compared with earlier solutions based on a two-scale model of the random rough surface (Bahar et al 1983). Thus, the simplifying assumptions, that are common to all the earlier solutions based on two-scale models of the rough surface, are carefully examined. It is shown that while the full wave solutions for the like polarized scattering cross sections based on the two-scale model are in reasonably good agreement with the unified full wave solutions, the two solutions for the cross polarized cross sections differ very significantly.

2.3 Application of the Full Wave Solution Without Surface Decomposition

The starting point for this analysis is the full wave expression for the like and cross polarized scattering cross sections of the rough surface \( y = h(x,z) \) (Bahar et al 1983, also reported in Annual Technical Report March 1982-February 1983)
\[ <\sigma^{PQ}> = \frac{k_0^2}{\pi} \left[ \sum_{\gamma}^{\infty} \exp[iv_y(h-h')] \right] \quad \left[ \int \frac{D^{PQ}_{\gamma}(\eta, \eta')}{\eta \cdot \eta'} \right] \left| x(v_y) \right|^2 \]

\[ \cdot \exp[i v_x x + i v_z z] dx \cdot dz \]

(2.1)

In which

\[ \vec{r} = (x-x')\vec{a}_y + (z-z')\vec{a}_z = x_d\vec{a}_x + z_d\vec{a}_z \]

(2.2)

is the radius vector between two points on the reference plane \((x,z)\).

The vector \(\vec{v}\) is

\[ \vec{v} = k_o (\vec{n}^f - \vec{n}^i) = v_x\vec{a}_x + v_y\vec{a}_y + v_z\vec{a}_z \]

(2.3)

where \(k_o\) is the free space wavenumber for the electromagnetic wave and \(\vec{n}^i\) and \(\vec{n}^f\) are unit vectors in the directions of the incident and scattered wave normals respectively. An \(\exp(i\omega t)\) time dependence is assumed in this work. The symbol \(<\>\) denotes the statistical average and

\[ \frac{k_0^2}{\pi} <\sigma^{PQ}_{\gamma}> = \frac{k_0^2}{\pi} \left[ \int \frac{D^{PQ}_{\gamma}(\eta)}{\eta \cdot \eta'} \right]^2 \left| p^2_{\gamma}(\eta^f, \eta^i) \right| p(h_x, h_z) dh_x dh_z \]

\[ \cdot x^2(v_y, -v_y) = \int^{PQ}_{\gamma} (\eta^f, \eta^i) x^2(v_y, -v_y) \]

(2.4)

In which

\(\vec{n}(h_x, h_z)\) is the unit vector normal to the rough surface

\[ f(x, y, z) = y - h(x, z) = 0 \]

(2.5)

Thus

\[ \nabla f = \vec{n} |\nabla f| = \nabla (y - h(x, z)) = (-h_x\vec{a}_x + h_y\vec{a}_y - h_z\vec{a}_z) \]

(2.6)

In which the components of the gradient of \(h(x, z)\)

\[ h_x = \partial h/\partial x, \quad h_z = \partial h/\partial z \]

(2.7)
are random variables and \( p(h_x, h_z) \) is the probability density function for the slopes \( h_x \) and \( h_z \). The expression for the scattering cross sections \( \sigma_{PQ} \), (2.1) is valid for all roughness scales. It accounts for shadowing and

\[
P_2(n^f, n^s | \bar{n}) = P_2(n^f, n^i | \bar{n}) S(n^f \cdot \bar{n}) S(n^i \cdot \bar{n})^+ \tag{2.8}
\]

in which \( P_2(n^f, n^i | \bar{n}) \) is the probability that a point on the rough surface is both illuminated and visible given the value of the slopes at the point (Smith 1967, Sancer 1968) and \( P_2(n^f, n^i | \bar{n}) \) is its value at the specular points where the unit vector \( \bar{n} \) is given by

\[
\bar{n} + \bar{n}^s = \frac{n^f - n^i}{|n^f - n^i|} = \frac{\bar{v}}{v} \tag{2.9}
\]

The arguments of the unit step functions \( S(-n^i \cdot \bar{n}) \) and \( S(n^f \cdot \bar{n}) \) vanish at points of the rough surface where the incident and scattered waves are tangent to the rough surface. Thus

\[
S(-n^i \cdot \bar{n}) = 1 \quad \text{and} \quad S(n^f \cdot \bar{n}) = 1.
\]

The characteristic and joint characteristic functions for the surface height \( h \) are respectively (Beckmann and Spizzichino 1963)

\[
\chi(v_y) = \langle \exp(iv_y h) \rangle \tag{2.10}
\]

and

\[
\chi_2(v_y, -v_y) = \langle \exp[iv_y (h-h')] \rangle. \tag{2.11}
\]

It is assumed in this work that the probability density function for the surface height is jointly Gaussian. Thus

\footnote{Sancer's expressions for \( P_2 \) are derived for \( k_0^2<h^2> >> 1. \)}
\[ |\chi(y)|^2 = \exp(-\frac{y^2}{\sigma^2} \langle h^2 \rangle) \]  

and

\[ \chi_2(y_1, y_2) = \exp(-\frac{y_1^2}{\sigma^2} \langle h^2 \rangle + \frac{y_2^2}{\sigma^2} \langle h'h' \rangle) \]

where \( \langle h^2 \rangle \) is the mean square height and \( \langle h'h' \rangle \) is the surface height autocorrelation function. The coefficients \( D^{PQ} \) depend explicitly upon: (1) the polarization of the incident wave (second superscript \( Q=V \) - vertical, \( Q=H \) - horizontal) and (2) the polarization of the scattered wave (first superscript; \( P=V, H \)), (3) the direction of the incident and scattered wave normals \( \vec{n_i} \) and \( \vec{n_f} \) respectively, (4) the unit vector \( \vec{n} \) normal to the rough surface and (5) the complex permeability and permittivity of the medium of propagation respectively (Bahar 1981a). On deriving (2.4), it is assumed that the rough surface is Gaussian and stationary, thus the surface height \( h \) and slopes \( (h_x, h_z) \) are statistically independent (Brown 1978, Longuet-Higgins 1957). It is also assumed that for distances \( |r_d| \) less than the surface height correlation distance, \( L_c, \vec{n}(h_x, h_z) = \vec{n}'(h'_x, h'_z) \). It has been shown that if the principal contributions to the scattered fields come from specular points on the rough surface (\( \vec{n} = \vec{n}_s \), (2.1) reduces to the physical optics solution for the scattering cross section. If, however, the roughness scale of the surface is small compared to the wavelength \( (k_0^2 \langle h^2 \rangle << 1) \) and the surface slopes \( h_x \) and \( h_z \) are very small, (2.1) reduces to the perturbation solution for the scattering cross sections (Rice 1951). Thus, in this case Bragg scattering is
accounted for and the backscatter cross sections for grazing angles are strongly dependent on polarization. Recently a two-scale model was adopted to determine the corresponding full wave solution for the scattering cross sections (Bahar and Barrick 1983). To facilitate the application of the two-scale model it is assumed that the small scale surface $h_s$ and the large scale filtered surface $h_F$ are statistically independent (Valenzuela 1968; Wright 1968; Brown 1978). This assumption is reasonable if the surfaces $h_F$ and $h_s$ are results of independent processes (Brown 1978) as for example, when the small scale roughness is due to erosion while the large scale roughness is due to geophysical forces that result in hills and valleys or as in the case of the sea, where the capillary waves are dependent on surface tension while the large scale surface is generated by gravity waves. In general, however, it cannot be assumed that the large and small scale roughness of the surface are statistically independent. In the general case, if the two-scale model is used to analyze the problem it would be necessary to know the large and small scale surface height joint probability density function for two adjacent points on the rough surface to determine $\chi_2$ (2.11) alone.

Since the full wave solutions account for both Bragg scatter and specular point scatter in a unified, self consistent manner, in this paper solutions for (2.1) are developed without adopting a two-scale model of the rough surface.
It has been noted in the introduction that the physical optics approximation for the cross polarized backscatter cross section is zero ($\sigma_{PQ}^{P} = 0$ for $P \neq Q$) (Brown 1978). However, even the large scale filtered surface will depolarize the backscattered field at nonspecular points on the surface. Therefore the present analysis should shed more light on the evaluation of the like and cross polarized backscatter cross sections and the suitability of the two-scale model even if it can be assumed that the large and small scale surfaces are statistically independent.

Assuming that $k_{0}^{2} < h_{0}^{2} << 1$ and $|x|^{2} << 1$, the scattering cross section (2.1) can be expressed as follows:

$$\sigma_{PQ} = I_{PQ}^{PQ}(n^{f},n^{i}) \int_{-\infty}^{\infty} [x_{2}(v_{y},-v_{y}) - |x(v_{y})|^{2}]$$

$$\times \exp[i v_{x}x_{d} + i v_{z}z_{d}]dx_{d}dz_{d}$$

$$= I_{PQ}^{PQ}(n^{f},n^{i}) Q(n^{f},n^{i},R) \quad (2.14a)$$

in which $I_{PQ}^{PQ}$ is defined by (2.4) and

$$Q = \int_{-\infty}^{\infty}(x_{2} - |x|^{2})\exp[i v_{x}x_{d} + i v_{z}z_{d}]dx_{d}dz_{d} \quad , \quad (2.14b)$$

is the two dimensional Fourier transform of $(x_{2} - |x|^{2})$ (2.12), (2.13). It therefore depends on the surface height correlation coefficient $R$

$$R = <hh^{'}>/<h^{2}> \quad . \quad (2.15)$$
Using the notation of Rice (1951), the surface height spectral density function $W(v_x,v_z)$ is related to the two dimensional Fourier transform of the surface height autocorrelation function.

$$W(v_x,v_z) = \frac{1}{\pi^2} \int <hh'> \exp[iv_x x_d + iv_z z_d] dx_d dz_d \quad (2.16a)$$

and

$$<hh'> = \int \frac{W(v_x,v_z)}{4} \exp[-iv_x x_d - iv_z z_d] dv_x dv_z \quad (2.16b)$$

Thus assuming that the rough surface is Gaussian and stationary, to compute the scattering cross sections (2.14) it is necessary to prescribe the two dimensional slope probability density function $p(h_x,h_z)$, (2.4) and the surface height autocorrelation function or its Fourier transform (the surface height spectral density function).

Since it is assumed in this work that the surface is isotropic, $<hh'>$ depends only on the distance $r_d = |\bar{r}_d|$ between the two points $(x,h,z)$ and $(x',h',z')$ on the rough surface. Thus

$$<hh'> = 2\pi \int \frac{W(v_{xz})}{4} J_0(v_{xz} r_d) v_{xz} dv_{xz} \quad (2.17)$$

and

$$<h^2> = 2\pi \int \frac{W(v_{xz})}{4} v_{xz} dv_{xz} \quad (2.18)$$

in which $J_0$ is the Bessel function of order zero and

$$v_{xz}^2 = v_x^2 + v_z^2 \quad (2.19)$$

Since

$$J_0''(0) = [d^2 J_0(v_{xz} r_d)/dr_d^2]_{r_d=0} = v_{xz}^2/2 \quad (2.20)$$

it follows that

$$R''(0) = \sigma^2 / 2 <h^2> = -\sigma_x^2 / <h^2> = -\sigma_z^2 / <h^2> \quad (2.21)$$
where
\[ \sigma^2_S = 2\pi \int \frac{W(v_{xz})}{4} v^3_x v^3_z \, dv_x dv_z \]  
(2.22)
is the total mean square slope while \( \sigma^2_x \) and \( \sigma^2_z \) are the mean square slopes in the \( x \) and \( z \) directions. Thus for small values of \( r_d \) the correlation coefficient is given by (Beckmann and Spizzichino 1963, Brown 1978)
\[ R(r_d) = 1 - \frac{r_d^2}{\ell^2_c} = 1 - \frac{\sigma^2_x r_d^2}{2 \langle h^2 \rangle} \]  
(2.23)
where \( \ell_c \) is the correlation distance.

2.4 **Illustrative Examples**

For the following illustrative examples the special form of
the surface height spectral density function is chosen (Brown 1978)
\[ W(v_x, v_z) = \left\{ \begin{array}{ll}
\frac{2}{\pi} S(v_x, v_z) = & \left( \frac{2}{\pi} \right) B k^4 / (k^2 + \kappa^2)^4 \quad k < k_c \\
0 & k \geq k_c
\end{array} \right. \]  
(2.24)
where \( W \) is the spectral density function defined by Rice (1951) and
\( S \) is the corresponding quantity used by Brown (1978) (See Fig. 2.1).

For the above isotropic model of the ocean surface
\[ B = 0.0046 \]  
(2.25a)
\[ k^2 = v^2_x + v^2_z \text{ (cm)}^{-2} \, , \, k_c = 12 \text{ (cm)}^{-1} \]  
(2.25b)
\[ \kappa = (335.2V^4)^{-\frac{1}{2}} \text{ (cm)}^{-1} \, , \, V = 4.3 \text{ (m/s)} \]  
(2.25c)
In (2.25c) \( V \) is the surface wind speed. The wavelength for the
electromagnetic wave is
\[ \lambda_o = 2 \text{ (cm)} \, , \, (k_o = 3.1416 \text{ (cm)}^{-1}) \]  
(2.26)
Fig. 2.1 The surface height spectral density function $W(v_x, \omega)(2.24)$. 
Equation (2.18) for the mean square height of the rough surface yields
\[ \langle h^2 \rangle = \frac{B}{2} \left[ \frac{1}{3\kappa^2} - \frac{1}{k_c^2 + \kappa^2} + \frac{2}{(k_c^2 + \kappa^2)^2} + \frac{4}{3(k_c^2 + \kappa^2)^3} \right] \] (2.27)

\[ = \frac{Bk_c^6}{6\kappa^2(k_c^2 + \kappa^2)^3} \]

Thus if the spectral cut-off point \( k_c \) (Brown 1978) is much larger than \( \kappa \) (as for the illustrative example (2.25))
\[ \langle h^2 \rangle = \frac{B}{6\kappa^2} = 87.9 \text{ cm}^2 \] (2.28)

The surface height autocorrelation function \( \langle hh' \rangle \) (2.17) can be expressed in closed form for \( k_c \rightarrow \infty \). Thus the surface height correlation coefficient \( R(r_d) \) (2.15) is given by (Miller et al 1972)
\[ R(r_d) = [1 + \frac{1}{8}(\kappa r_d)^2](\kappa r_d)K_1(\kappa r_d) - (\kappa r_d)^2 K_0(\kappa r_d) \] (2.29)
in which \( K_0 \) and \( K_1 \) are the modified Bessel functions of the second kind and of order zero and one respectively. (See Fig. 2.2)

Since \( k_c \gg \kappa \) and \( k_c > k_o \) the above closed form expression is used for \( R \) in this illustrative example. The total mean square slope of the rough surface is obtained on substituting (2.24) into (2.22).
\[ \sigma_S^2 = B \left[ \frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{11}{12} + \frac{3}{2} \frac{\kappa^2}{k_c^2 + \kappa^2} - \frac{3}{4} \frac{\kappa^4}{(k_c^2 + \kappa^2)^2} + \frac{1}{6} \frac{\kappa^6}{(k_c^2 + \kappa^2)^3} \right] \]
\[ = B \left[ \frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{k_c^2(6\kappa^2 + 15\kappa k_c^2 + 11k_c^4)}{12(\kappa^2 + k_c^2)^3} \right] \] (2.30)
Thus for $k_c \gg \kappa$

$$\sigma^2_S = B \left[ \frac{1}{2} \ln \frac{k_c^2 + \kappa^2}{\kappa^2} - \frac{11}{12} \right] = 0.034 \quad (2.31)$$

For typical sea surfaces the relative complex dielectric coefficient at 15 GHz is given by (Stogryn 1971)

$$\varepsilon_r = 42 - i39$$

The slope probability density function $p(h_x, h_z)$ is assumed to be Gaussian, thus

$$p(h_x, h_z) = \frac{1}{\sigma^2_S} \exp \left[ - \frac{h_x^2 + h_z^2}{2\sigma^2_S} \right] \quad (2.32)$$

In Fig. 2.3 the like polarized backscatter cross section $\sigma_{VV}$ is plotted as a function of the angle of incidence $\theta^i_0$ using the expression derived in Section 2.2. These results are compared with the two-scale full wave results (Bahar et al 1983) based on the choice of $k_d$ (the wavenumber where spectral splitting occurs) corresponding to $\beta = 1$. Both results yield the same general dependence of $\sigma_{VV}$ on the angle of incidence. The small difference in level is primarily due to the fact that in (2.3) the mean square slope $\sigma^2_S$ of the total (unfiltered) surface is used, (2.31), while for the solution based on the two-scale model the mean square slope $\sigma^2_{FS}$ for the filtered surface $h_F$ is used (Bahar et al 1983). It should be noted that in deriving the expressions for the scattering cross sections based on the two-scale model, it was assumed that $\sigma^2_{FS} = \sigma^2_S$. Thus the results based on (2.3) are more accurate.
Fig. 2.3 Backscatter cross sections $\langle \sigma_{VV} \rangle$ for rough surfaces characterized by $W(v_x, v_z)$ given by (2.24) with $B = 0.0046$, $V = 4.3$ m/s ($\square$) Two scale model and ($\Delta$) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 42 - 139$, $\lambda_o = 2$ cm.
Furthermore, on deriving the solution based on the two-scale model, the quantity $x(v_y)$ (2.12) is assumed to be negligible compared to $x_2(v_y,-v_y)$ (2.13) for $r_d < c$. Since $4k_0^2 < h^2 = 3468$ for this illustrative example, the resulting approximation is very good except very near grazing angles. In Fig. 2.4, the corresponding results are given for the horizontally polarized backscatter cross sections $\sigma_{HH}$. It is interesting to note that the full wave solution (2.3) yields the proper polarization dependence of the scattering cross sections for all angles of incidence without use of a two-scale model since it accounts for specular point and Bragg scattering in a unified, self-consistent manner. In Fig. 2.5 the cross polarized backscatter cross sections $\sigma_{VH} = \sigma_{HV}$ are plotted as functions of the angle of incidence. Here too, both the solutions based on the two-scale model as well as the solution derived in this section are presented. Unlike the solutions for the like polarized backscatter cross sections $\sigma_{PP}$ (P=V,H), the solutions for the cross polarized backscatter cross sections differ significantly, especially near normal incidence where the difference in level is about 15db. This very significant difference is due to the fact that the physical optics approximations for the cross polarized backscatter cross section is zero (Brown 1978, Bahar et al 1983). For backscatter the surface at the specular points is normal to the incident wave. At these stationary phase points no depolarization occurs. However, since depolarization occurs at the nonspecular points of the filtered surface, the
Fig. 2.4 Backscatter cross sections $<\sigma_{HH}>$ for rough surfaces characterized by $W(x_1,v_2)$ given by (2.24) with $B = 0.0046$, $V = 4.3$ m/s ($\square$) Two scale model and ($\Delta$) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 42 - 139$, $\lambda_0 = 2$ cm.
Fig. 2.5 Backscatter cross sections $\langle \sigma_{vh} \rangle$ for rough surfaces characterized by $W(x_v, \lambda)$ given by (2.24) with $b = 0.0046, V = 4.3 \text{ m/s}$ (D) Two scale model and (\lambda) unified full wave solution (2.4). Relative permittivity is $\varepsilon_r = 4.2, \varepsilon_0 = 2 \text{ cm}.$
physical optics approximations for the cross polarized backscatter cross section is not valid. It is interesting to note that for the two-scale model at normal incidence $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 47 \text{db (P\#Q)}$.

However, using the full wave solution (2.3), $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 31 \text{db (P\#Q)}$.

Consider the additional illustrative examples at X band, 8.91 GHz $\lambda_o = 3.367 \text{ cm, } k_o = 1.87 \text{ (cm)}^{-1}$. The surface height spectral density function $W$ is given by (2.24). The wind velocity is $V = 24 \text{ m/s}$ and the relative dielectric coefficient is $\varepsilon_r = 55 - 137$ (Stogryn 1971).

Two values of $B$ (2.24) are assumed

<table>
<thead>
<tr>
<th>Case (a)</th>
<th>Case (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B = 0.0046$</td>
<td>$B = 0.0092$</td>
</tr>
<tr>
<td>$\langle h^2 \rangle = 0.853 \times 10^5 \text{ cm}^2$</td>
<td>$\langle h^2 \rangle = 0.171 \times 10^6 \text{ cm}^2$</td>
</tr>
<tr>
<td>$\sigma_S^2 = 0.0498$</td>
<td>$\sigma_S^2 = 0.0997$</td>
</tr>
</tbody>
</table>

In Figures 2.6, 2.7 and 2.8 the backscatter cross sections are evaluated on the basis of the unified and two-scale models for case (a) and in Figures 2.9, 2.10 and 2.11 the cross sections are evaluated for case (b). Daley et al (1970) published extensive measurements of the like and cross polarized backscatter cross sections at microwave frequencies (Long 1975). From his data at X band (8.91 GHz), with wind speed $V = 24 \text{ m/s}$, one finds that for an angle of incidence $\theta_o = 15^\circ$, $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 20 \text{ db}$. Using the unified full wave approach $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 21.5 \text{ db}$ for case (a) and $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 18 \text{ db}$ for case (b). However, using the two-scale model $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 36 \text{ db}$ for case (a) and $\langle \sigma_{\text{PP}} \rangle / \langle \sigma_{\text{PQ}} \rangle = 34 \text{ db}$ for case (b).
Fig. 2.6 Backscatter cross sections \( \sigma_{VV} \) for rough surfaces characterized by \( W(v_x, v_z) \) given by (2.24) with \( B = 0.0046, V = 24.0 \) m/s (□) Two scale model and (△) unified full wave solution (2.4). Relative complex permittivity is \( \varepsilon_r = 55 - 137, \lambda_0 = 3.367 \) cm.
Fig. 2.7 Backscattered cross sections $\sigma^{HH}_{\text{B}}$ for rough surfaces characterized by $W(v_x, v_z)$ given by (2.24) with $B = 0.0046$, $V = 24.0$ m/s (□). Two scale model and (△) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 55 - j37$, $\lambda_0 = 3.367$ cm.
Fig. 2.8 Backscatter cross sections $<\sigma>_{\text{VH}}$ for rough surfaces characterized by $W(v_x, v_z)$ given by (2.24) with $B = 0.0046$, $V = 24.0$ m/s (□) Two scale model and (Δ) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 55 - 137$, $\lambda_o = 3.367$ cm.
Fig. 2.9 Backscatter cross sections $\sigma^{VV}$ for rough surfaces characterized by $W(x', z')$ given by (2.24) with $b = 0.0092$, $V = 24.0$ m/s (□) Two scale model and (Δ) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 55 - i37$, $\lambda_o = 3.367$ cm.
Fig. 2.10 Backscatter cross sections $\sigma_{HH}$ for rough surfaces characterized by $W(v_x', v_z')$ given by (2.24) with $B = 0.0092$, $V = 24.0$ m/s ($\square$) Two scale model and ($\triangle$) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 55 - i37$, $\lambda_0 = 3.367$ cm.
Fig. 2.11 Backscatter cross sections $<\sigma_{VH}>$ for rough surfaces characterized by $W(v_x, v_z)$
given by (2.24) with $B = 0.0092$, $V = 24.0$ m/s ($\Box$) Two scale model and ($\triangle$) unified full wave solution (2.4). Relative complex permittivity is $\varepsilon_r = 55 - 137$, $\lambda_0 = 3.367$ cm.
Thus the unified full wave results are significantly more in line with published experimental results* (Long 1975).

2.5 Concluding Remarks

It is shown in this section that two-scale models of rough surfaces can be adopted to obtain solutions for the like polarized backscatter cross sections that are in reasonably good agreement with the full wave solutions derived in this section. However, the two-scale model cannot be used to evaluate the cross polarized backscatter cross sections. The significant differences between the solutions derived in this section and those based on the two-scale models are primarily due to the fact that the physical optics approximation for the cross polarized backscatter cross section (associated with the large scale filtered surface) is zero.

For backscatter, the specular points lie on portions of the rough surfaces that are perpendicular to the incident wave normal $\mathbf{n}_i$ ($\mathbf{n}_s = \mathbf{n}_f = -\mathbf{n}_i$). At these specular points, the backscattered waves are not depolarized. However, at nonspecular points of the rough surface, the backscattered waves are depolarized (Bahar 1981b). Thus, it is important to note that even if a surface satisfies the radii of curvature criteria (associated with the Kirchhoff approximations for the surface fields), the physical optics approximations for the scattered fields may not be valid unless for the given incident and scatter angles specular points exist on the surface and significant contributions to the scattered fields come from these stationary phase (specular) points of the surface. This explains

*See also Naval Research Laboratory Report 7142, Daley, J. C., W. T. Davis and N. R. Mills, "Radar Sea Return in High Sea States, September 25, 1970."
why the physical optics approximations for the like polarized backscattered cross sections are not suitable for grazing angles even if the surface meets the radii of curvature criteria associated with the Kirchhoff approximations.

There are additional important reasons for preferring to use the analysis developed in this section over those that are based on two-scale models of rough surfaces. Firstly, if the two-scale model is used, it is necessary to assume that the large and small scale surfaces are statistically independent (Brown 1978). Secondly, even if the assumption of statistical independence is acceptable, when the two-scale model is used, it is still necessary to judiciously specify $k_d$ (where spectral splitting is assumed to occur). These problems do not arise when the unified full wave formulation is used to evaluate the scattering cross sections.

2.6 References


Acknowledgments. This paper was sponsored by the U. S. Air Force contract F19628-81-K-0025. The manuscript was prepared by Mrs. E. Everett.
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