Scaling of the Beam Plasma Discharge for Low Magnetic Fields

by

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A theoretical analysis of the scaling law and the value of the threshold current for beam plasma discharge (BPD) is presented, based on the requirement for an absolute instability near the plasma frequency. It is shown that both the scaling law as well as the numerical values of $I_c$ are consistent with the experimental data, both in the low and high pressure regimes for weak magnetic field experiments ($\omega_e < \omega_L$). The differences in scaling between the regimes are attributed to transition from Bohm to classical diffusion. It is found that the value of the pressure minimum for ignition increases linearly with the ambient magnetic field.
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Abstract

A theoretical analysis of the scaling law and the value of the threshold current for beam plasma discharge (BPD) is presented, based on the requirement for an absolute instability near the plasma frequency. It is shown that both the scaling law as well as the numerical values of $I_c$ are consistent with the experimental data, both in the low and high pressure regimes for weak magnetic field experiments ($\omega_e > \Omega_e$). The differences in scaling between the two regimes are attributed to transition from Bohm to classical diffusion. It is found that the value of the pressure minimum for ignition increases linearly with the ambient magnetic field.
I. Introduction

Laboratory studies of energetic electron beam injection experiments in a neutral gas filled vacuum chamber carried out in the large vacuum facility at the Johnson Space Center (JSC) (Bernstein et al. 1978, 1979, 1980) have provided important informations for the interpretation of data from space based electron beam injection. Perhaps the most important aspect was the determination of an empirical relationship of the form

\[ I_c \sim \frac{E_b^{3/2}}{B^{1/2} L} f(p) \]

for the critical current \( I_c \) required for beam plasma discharge ignition and the values of beam energy \( E_b \), ambient magnetic field \( B \), system length \( L \), and ambient pressure \( p \). The value of \( \lambda = .5-1 \) and the pressure function \( f(p) \) was a function with a minimum at \( p_o = 20 \) mT and varying roughly as \( p^{\pm.5} \) to \( p^{\pm1} \) above and below this pressure (Kellog et al. 1981). The minimum value of \( I_c = 10 \) mA for a system with \( L = 20 \) m, \( E_b = 1 \) keV and \( B = 1 \) G, typical of the experiment. The operating pressure range was between 1-50 mT, with most measurements on the 1-20 mT range. Similar results have been reproduced in a number of other experiments (Konrad et al. 1982, Lyakhov et al. 1982, Bernstein et al. 1983) in which the collisional ionization by the beam is sufficient to bring the plasma density of the system to the point that \( \omega_e > \Omega_e \), where \( \omega_e, \Omega_e \) are the plasma and cyclotron frequency. The observed \( p, B \) scaling is shown graphically in Fig. 1.

Triggering of BPD has been long associated with a beam plasma instability between the electron beam and the beam generated plasma.
(Kharchenko et al. 1962, Getty and Smullin 1963, Galeev et al. 1976, Linson and Papadopoulos 1980, Papadopoulos 1981, Galeev 1983). For \( \omega_e > \Omega_e \) and finite size systems Rowland et al. (1981) and Papadopoulos (1982), have associated the triggering of the BPD with the threshold for an absolute beam plasma instability near \( \omega_e \). As explained in Papadopoulos (1981) for systems such as the JSC tank the system length \( (L \lesssim 20 \text{ m}) \) is not long enough to allow convective modes to grow to sufficient amplitude. The requirement that the waves grow at frequencies near \( \omega_e \), is connected with the fast rate of non-linear energy transfer of the beam energy to ionizing suprathermal electron tails (Papadopoulos and Coffey 1974, Papadopoulos 1975, Papadopoulos and Rowland 1978, Rowland et al. 1980, Galeev 1983). It is the purpose of this paper to develop a model for BPD ignition and the expected scaling laws on the basis of the criterion for an absolute instability near \( \omega_e \). Notice that for systems with \( \omega_e < \Omega_e \) the \( \omega_e \) waves are in the lower hybrid branch (Manickam et al. 1975) which in the absence of internal wave reflections gives always convective amplification. Our analysis therefore applies only to situations where \( \omega_e > \Omega_e \).

The plan of the paper is as follows. We discuss next the beam plasma equilibrium expected on the basis of collisional ionization. Section III presents the instability theory for the configuration determined in section II and derives the threshold criteria. Section IV presents a comparison of the model to the BPD ignition values determined in the JSC experiment. The final section summarizes the findings and discusses their applicability to other situations. The numerical values are given in MKS units except for the beam energy (keV), pressure (\( \mu \text{T} \)), magnetic field (G), and density (\( \text{cm}^{-3} \)).
II. Pre-BPD Density Buildup

Before entering the instability analysis it is necessary to establish the equilibrium density profiles for the plasma during the collisional ionization stage. We assume that the beam density profile is given by

\[ n_b(r) = n_b e^{-r^2/a^2} = \frac{I_b}{eV_b a^2} e^{-r^2/a^2} \tag{1} \]

where \( I_b \) and \( V_b \) are the electron beam current and velocity parallel to the magnetic field and \( a \) the beam radius given by

\[ a = \frac{V_b \sin \theta_d}{n_e} \tag{1a} \]

where \( \theta_d \) is the equivalent divergence injection angle (Linson and Papadopoulos 1980). The equation for the ionization at midplane (i.e. ignoring the \( z \) dependence) is

\[ \frac{\partial}{\partial t} n(r) - \frac{1}{r} \frac{\partial}{\partial r} rD \frac{\partial}{\partial r} n(r) + \frac{\alpha n(r)}{L} = \frac{I_b N_o a}{e a^2} e^{-r^2/a^2} \tag{2} \]

where \( D \) is the diffusion coefficient, \( L \) the system length, \( N_o \) the ambient neutral density, and \( \sigma \) the ionization cross section. The term \( \frac{\alpha}{L} n(r) \) describes the axial losses. Notice that if we average (2) over the volume we recover the zero dimensional description, in terms of the confinement time \( \tau \) (Papadopoulos 1982), i.e.

\[ \frac{d}{dt} n = \frac{I_b}{ea^2} N_o \sigma - \frac{n}{\tau} \tag{3} \]
which gives the steady state value of the density as

\[ n = \frac{I_b}{2 \pi a} N_0 \sigma \tau \]  

(4)

The general solution of eq. (2) in terms of the first and second order Bessel functions \( I_0, K_0 \) is

\[ n(r) = A_1 I_0 \left( \frac{r}{b} \right) + A_2 K_0 \left( \frac{r}{b} \right) \]  

(5)

with

\[ b^2 = \frac{LD}{a^2} \]  

(6)

The constants \( A_1, A_2 \) to be found from the boundary conditions. In the thin beam limit \( a \ll b \), in which we have a line source we find

\[ n(r) = \frac{I_b N_0 \sigma}{2 \pi \sigma b} K_0 \left( \frac{r}{b} \right) = n_0 K_0 \left( \frac{r}{b} \right) \]  

(7)

The more general solution gives, for regions inside the source \( (0 \leq r \leq a) \)

\[ n(r) = \frac{I_b N_0 \sigma}{eD \pi a^2} \int_0^a K_0 \left( \frac{r-r'}{b} \right) r'^{-2/2} e^{-r'^2/2} \ dr' \]  

(8)

while outside the source \( (a \leq r \leq b) \)

\[ n(r) = \frac{I_b N_0 \sigma}{eD \pi a^2} \left( \int_0^a I_0 \left( \frac{r'}{b} \right) e^{-r'^2/2} \ dr' \right) K_0 \left( \frac{r}{b} \right) \]  

(9)
Guided by the experimental results we restrict ourselves here to the thin beam limit $a < b$, which implies that $\frac{V_b \sin \theta_d}{\alpha} < \frac{b^{1/2}}{a} \alpha^{1/2}$. In this case the input parameters to the instability analysis are $n_b$, $n_0$, $a$, and $b$ given by eqs. (1), (6) and (7). The values of $n_b$ and $n_0$ in the system units discussed in section I can be found from eqs. (1) and (7) as

$$ n_b = 1.9 \times 10^6 \frac{L_b B^2}{E_b^{3/2}} \frac{1}{\sin^2 \theta_d} $$ (9a)

$$ n_0 = 3.2 \times 10^8 \frac{L_b P}{E_b^{1/2}} \frac{1}{D} $$ (9b)

III. Instability Theory

The homogeneous interaction between the beam and the plasma is described by the dispersion relation

$$ \varepsilon(k, \omega) = 1 + K_p(k, \omega) + K_b(k, \omega) = 0 $$ (10a)

where $K_p$ and $K_b$ are the longitudinal dielectric functions of the plasma and the beam. Both $K_p$ and $K_b$ can be calculated for any type of distribution functions including collisional and finite size geometry effects (Briggs 1964). We choose here models that allow us to emphasize the physics and avoid the mathematical complexity. Consistently with Rowland et al. 1981 and Szuszczewicz et al. 1982 we consider only the synchronous Cerenkov interaction of a slow beam wave with an upper hybrid wave $\omega_0$ of the cold plasma. In this case
\[ K_p(k, \omega) = -\frac{\omega_p^2}{\omega^2} \quad (10b) \]

\[ \omega_p^2 = \frac{1}{2} (\omega_e^2 + \Omega_e^2) + \left[ \frac{1}{4} (\omega_e^2 + \Omega_e^2)^2 - \omega_e^2 \Omega_e^2 \cos^2 \theta \right]^{1/2} \quad (10c) \]

\[ K_B(k, \omega) = -\frac{\omega_b R \cos^2 \theta}{(\omega - k_2 v_b)^2} \quad (10d) \]

\[ \cos^2 \theta = \frac{k_z^2}{k_z^2 + k_\perp^2} \quad (10e) \]

\( \omega_p, \omega_b \) and \( \Omega_e \) are the plasma frequency, beam plasma frequency, and cyclotron frequency respectively, and \( k^2 = k_z^2 + k_\perp^2 \). The potential was assumed to have the form \( J_0(k_z r) \exp[i(k_z z - \omega t)] \) and thus the propagation is axial, with \( k_\perp \) determined by the transverse geometry.

The finite size beam reduction factor \( R \) enters through the boundary conditions at the radii \( a \) and \( b \), and is given in terms of Bessel functions by (Manickam et al. 1975)

\[ R = \frac{\pi}{2} (k_\perp a) \frac{a Y_0(k_\perp b)}{b Y_1(k_\perp b)} \left[ J_0^2(k_\perp a) + J_1^2(k_\perp a) \right] \quad (11a) \]

\[ J_0(k_{\perp b}) = 0 \quad (11b) \]

The value \( \omega_b R^{1/2} \cos \theta \) serves as a reduced effective beam plasma frequency. An important aspect of the dispersion (10) is the absorption of the transverse wavenumber \( k_\perp \) and geometry effects into a single parameter \( R \). Therefore from eq. (11), \( R \) is fixed when the mode number and the ratio \( \frac{b}{a} \) are fixed. Fig. 7 of Manickam et al. (1975) shows the values of \( R \) as a function of \( \frac{b}{a} \) for the fundamental mode. For large \( \frac{b}{a} \gg 1 \), it has a logarithmic dependence approaching the value \( R = 1 \).
In order to determine the conditions for absolute instability in our system we follow the techniques developed by Bers (1972), in the weak coupling approximation. For the beam waves the dispersion relation is

\[ D_b(k,\omega) = 1 - \left( \frac{\omega_b^2}{\omega^2} \right) \left( \frac{\omega}{\omega_b} \right)^2 \cos^2 \theta = 0 \] (12)

From this we find the usual fast and slow waves given by

\[ \omega - k_z v = \pm \omega_b R^{1/2} \cos \theta \] (12a)

The slow wave is a negative energy wave while the fast is positive, i.e.

\[ \omega_b = \pm \frac{2\omega}{\omega_b R^{1/2}} \left( \frac{1}{4} \varepsilon_o \left| \varepsilon_b \right|^2 \right) \] (12b)

where \( \omega_b \) is the wave energy, \( \varepsilon_b \) the beam wave amplitude and \( \varepsilon_o \) the free space dielectric constant. The negative energy wave can couple in synchronous interaction with the backward positive energy upper hybrid wave to produce an instability. Notice that for the upper hybrid wave

\[ D_p(k,\omega) = 1 - \frac{\omega_o^2(k_z)}{\omega^2} \] (13a)

while the wave energy is

\[ \omega_p = \frac{2\omega}{\omega_o(k_z)} \left( \frac{1}{4} \varepsilon_o \left| \varepsilon_p \right|^2 \right) \] (13b)

We examine now the situation where the slow wave of the beam interacts resonantly with the plasma wave \( \omega_o \). We find a set of coupled equations
(\frac{\partial}{\partial t} + v_b \frac{\partial}{\partial z} + v_b) U_b = C_{bp} U_p \tag{14}

(\frac{\partial}{\partial t} + v_p \frac{\partial}{\partial z} + v_p) U_p = C_{pb} U_b

where \( v_b, v_p \) are the group velocities of the beam and plasma waves and
\( v_b, v_p \) are phenomenological damping coefficients of the two waves. \( U_p, b \)
are the usual normalized amplitudes defined as

\[ |U_{p,b}|^2 = \frac{W_{p,b}}{\omega} = \frac{\omega}{4} |e_{p,b}|^2 \tag{15} \]

(Bers 1972, Davidson 1970, Weiland and Wilhelmson 1977) and the coupling
coefficients \( C_{bp}, C_{pb} \) given by

\[
C_{bp} = -\frac{1}{4} \frac{\epsilon_b^* J_{pb}}{W_b} \equiv \frac{P_{bp}}{W_b} \tag{16}
\]

\[
C_{pb} = -\frac{1}{4} \frac{\epsilon_p J_{bp}}{W_p} \equiv \frac{P_{pb}}{W_p}
\]

where \( J_{pb} \) is the perturbed plasma current that interacts with the beam
and \( J_{bp} \) is the perturbed beam current that interacts with the plasma.

For conservative interactions

\[ P_{bp} = -P_{pb} = -P_{pb} \tag{17} \]

From eqs. (12-16) we find the growth rate

\[
\gamma = \frac{|P_{bp}|}{\sqrt{|W_b W_p|}} = \frac{1}{2} \left( \frac{\omega^2 R \cos^2 \theta}{\omega_0^2} \right)^{1/2} \omega_0 \tag{18}
\]
Absolute instability requires (Bers 1972)

\[ v_b v_p < 0 \]  \hspace{1cm} (19a)

\[ \gamma^2 > v_b v_p \]  \hspace{1cm} (19b)

\[ L > \frac{(v_b v_p)^{1/2}}{\gamma} \equiv L_c \]  \hspace{1cm} (19c)

where \( L \) is the system size in the z-direction. The first condition enters through the requirement that the unstable pulse encompasses the origin at all times. The second from the requirement that the pulse growth exceeds the dissipation. The last is equivalent to the breakdown length \( L_c \) of an oscillator and implies that the feedback is stronger than convective losses. Notice that in the absence of wave reflecting boundaries only the upper hybrid branch can be absolutely unstable, since the lower hybrid branch corresponds to a forward wave (i.e. \( v_b v_p > 0 \)). In a plasma with \( \frac{\omega_e}{n_e} < 1 \), waves near the plasma frequency will be convectively unstable.

As mentioned in the introduction we associate the threshold of BPD in the Johnson chamber with an absolute instability near \( \omega_e \). For our parameters the collisional ionization generates a plasma with \( \frac{\omega_e}{n_e} > 2 \), and the collisionality is such that condition (19b) is trivially satisfied. We therefore concentrate on eq. (19c), for a backward wave in the upper hybrid range with \( \frac{\omega_e}{n_e} \gg 1 \). The group velocity of the waves \( v_b, v_p \) are
\[ v_b = V_b \]  
\[ v_p = -2V_b \frac{e \gamma^2}{\omega^2_c} \cos^2 \theta \sin^2 \theta \]  

The value of \( \sin \theta \) can be computed using (10e) and the first root of eq. (11b), i.e. \( k_b = 2.4 \), giving

\[ \sin^2 \theta = \frac{1}{1 + g^2} \]  
\[ g^2 = \frac{b^2}{(2.4)^2} \frac{\omega^2_c}{v_b^2} \]  

From eqs. (18), (19c), (20) and (21) we find the criterion for absolute instability as

\[ \omega_b^2 \geq \frac{2/2}{L} \left( \frac{n_b}{n_0} \right)^{1/2} \frac{n_e V_b}{R^{1/2}} \sin \theta \]  

IV. BPD Ignition Scaling

Eq. (22) is the threshold condition for an absolute instability near the plasma frequency in terms of the plasma parameters of the system. Using eq. (9) we find the current threshold condition as

\[ I \geq I_c = 1.2 \times 10^{-2} \frac{E_b^{3/2}}{p^{1/2}} \frac{d^{1/2}}{L} \frac{\sin \theta \sin \theta_d}{R^{1/2}} \]  

In order to make further process we have to specify the value and scaling of the diffusion coefficient \( D \). In the low pressure regime it
was experimentally determined (Szuszczyewicz et al. 1979) that the diffusion coefficient obeyed the Bohm diffusion law. Therefore

\[ D = D_B = 6.25 \times 10^2 \left( \frac{T_e}{eV} \right) \frac{1}{B} \]  \hspace{1cm} (24)

Using this expression in eq. (23) with \( T_e = 2.5 \text{ eV} \) we find

\[ I_c = \frac{.5 E_b^{3/2}}{p^{1/2}} \left( \frac{1}{B^{1/2}} \right) \left( \frac{1}{L} \right) \left( \frac{1}{R^{1/2}} \right) \sin \theta \sin \theta \]  \hspace{1cm} (25a)

For values of \( g \leq 1 \), the factor in parenthesis is between 2-3, so that to within a factor of two (25a) reads

\[ I_c = \frac{E_b^{3/2}}{p^{1/2}} \left( \frac{1}{B^{1/2}} \right) \left( \frac{1}{L} \right) A \]  \hspace{1cm} (25b)

For the standard parameters \( (E_b = 1 \text{ keV}, B = 1G, L = 20 \text{ m}) \), we find \( I_c = 22 \text{ mA} \). The case analyzed in detail by Kellog et al. (1982), i.e. \( p = 5 \text{ \mu T} \), gives \( I_c = 22 \text{ mA} \). Both values compare favorably with the observed values. While Bohm diffusion is independent of pressure, classical diffusion in pressure dependent, i.e.

\[ D_{cl} = 1.8 \left( \frac{T_e}{eV} \right)^{3/2} \frac{p}{B^2} \]  \hspace{1cm} (26)

We therefore expect that at some value \( D_{cl} \geq D_B \). From eqs. (23) and (26) we find for \( T_e = 2.5 \text{ eV} \)
\[ I_c = 3 \times 10^{-2} \frac{E_b^{3/2}}{B^{1/2}} \frac{p^{1/2}}{B} \frac{1}{L} \left( \frac{\sin \theta \sin \phi}{R^{1/2}} \right) A \]  
(27a)

or

\[ I_c = 6 \times 10^{-2} \frac{E_b^{3/2}}{B^{1/2}} \frac{p^{1/2}}{B} \frac{1}{L} A \]  
(27b)

For the inflection point (i.e. \( p_o = 15 \mu T \)) this will give \( I_c = 14 \) mA.

Notice, however, that \( I_c \sim p^{1/2} \), while \( I_c \sim \frac{1}{B} \) rather than \( \frac{1}{B^{1/2}} \) found for the low pressure regime.

An interesting scaling results from the above for the dependence of the minimum pressure \( p_o \) for BPD as a function of \( B \). Namely \( p_o \sim B \).

Since \( p_o = 15 \mu T \) for \( B = 1G \), we find \( p_o \) in \( \mu T \) as

\[ p_o = 15 B \]  
(28)

Namely the pressure range of the low pressure scaling increases with \( B \). This is consistent not only with Fig. 1, but also with the small chamber results (Konradi et al. 1983, Bernstein et al. 1983) in which \( B = 38G \) and the low pressure scaling was consistent with eq. 27a. Let me finally note that eqs. (27a,b) are consistent with more recent experimental results (Lyakhov et al. 1982, Kawashima et al. 1982).

An approximate criterion for the ignition of BPD was given before (Rowland et al. 1981, Papadopoulos 1981) and was found consistent with the observations at the J.S.F.C. tank (Szusczewicz et al. 1982) as \( \frac{\omega e}{\Omega e} > 5 \). It is appropriate to comment on its relationship to the present more detailed considerations. Referring to eq. (23) we note that \( I_c \sim \sin \theta \) so that for \( \sin \theta << 1 \) \( I_c \) becomes very small,
independently of other considerations. From eq. (21) \( \sin \theta \ll 1 \)
corresponds to

\[
g = \frac{b}{2.4} \frac{\omega_e}{v_b} \gg 1
\]  

(29)

which is the opposite limit from the one considered before. Taking

\( b = 0(a) \) and using eq. (1a) we recover the condition \( \frac{\omega_e}{n_e} > \frac{2.4}{\sin \theta_d} \approx 5 \)
as an approximate condition at which absolute instability develops.

This criterion is extremely relevant for cases with preionized plasma
such as the ionosphere at daylight conditions or high altitude (i.e. F peak). The \( \frac{\omega_e}{n_e} > 5 \) criterion is a sufficient but not a necessary
condition, and accounts in a natural fashion for the observed hysteresis
during PBD extinction (Bernstein et al. 1979).

V. Summary and Conclusions

We presented a detailed physical analysis of BPD threshold scaling
based on the conjecture proposed by Papadopoulos (1981), that the BPD
threshold is associated with the triggering of an absolute instability
near \( \omega_e \). This conjecture predicts a scaling

\[
I_c \sim \frac{E_b^{3/2}}{p^{1/2} \theta^{1/2} L}
\]

for low pressures (\( p < p_0 \)) and

\[
I_c \sim \frac{E_b^{3/2} p^{1/2}}{B L}
\]
for high pressures \((p > p_0)\). In deriving these scalings we assumed Bohm diffusion for \(p < p_0\) and classical diffusion for \(p > p_0\). The minimum threshold current at \(p = p_0\) is associated with the transition from Bohm to classical diffusion. The predicted scaling, the numerical values of \(I_c\), as well as the dominance of Bohm diffusion for \(p < p_0\) are in good agreement with the data from JSC tank experiment (Bernstein et al. 1979, Szuszczewicz et al. 1979). It is important to restate some of the key assumptions of the theory and their consequences, since many were inspired from the JSC tank parameters and might not be applicable to other situations.

(a) Weak magnetic field in the sense that at prebreakdown \(\frac{\omega_e}{n_e} > 2\). For situations where \(\frac{\omega_e}{n_e} < 1\), the \(\omega_e\) waves lie in the lower hybrid branch of the dispersion curve which corresponds to forward waves and therefore produces convective instability. This seems to be the situation in Boswell and Kellog (1983) and the expected scaling should be derived from different considerations. The case \(\Omega_e < \omega_e < 2\Omega_e\) requires special consideration due to the presence of strong cyclotron damping which was neglected here. Note also that end plate reflections can produce an absolute instability even in the lower hybrid branch.

(b) Thin beam in the sense \(a \ll b\). For situations where \(a = b\), axial losses could be dominant and the more general eqs. (8) and (9) should be used instead of (7).

(c) Weak coupling limit eqs. (14) are valid in the limit where the parameter \(\left(\frac{\omega_p}{\omega_e}\right)^{1/2} = \left(\frac{n_b}{n_o}\right)^{1/2} \ll 1\). Otherwise the saddle
point method (Le Queau et al. 1980) should be used to
determine the threshold condition given by eq. (22).

(d) In associating the threshold for absolute instability with
the BPD ignition we implicitly assumed that the energy
deposition from the beam to the plasma will produce ambient
electron fluxes whose ionization rate exceeds the beam
ionization rate. This seems to be clearly satisfied in the
low pressure regime. However as noted in Papadopoulos et al.
(1983) by increasing the pressure there is a limit at which
the energy deposition by the beam plasma instability is not
sufficient to overcome line emission so that the electron
energy stays below the ionization energy. This case will
have many of the BPD signatures (i.e. broadening of the
radiated region) but the electron plasma density will be
controlled by beam ionization only (i.e. BPD without D). The
small chamber experiment (Bernstein et al. 1983) is possibly
indicative of such behavior.

Before closing we should comment on the applicability of the above
concepts to electron beam injection in space. There are two fundamental
differences between the laboratory and the ionospheric beam injection:

(i) The existence of ambient plasma with long density gradient
scales (i.e. $L \rightarrow \infty$) in the ionosphere.

(ii) The laboratory experiments are steady state, while the
vehicle motion across the magnetic field line can be thought
as producing beam pulses with time length $\tau = \frac{U}{a}$ where $U$ is
the cross-field motion of the vehicle (i.e. $1-2 \frac{\text{km}}{\text{sec}}$ for
rockets, $4-8 \frac{\text{km}}{\text{sec}}$ for the shuttle).
In assessing BPD for the ionospheric case we have to ask the following questions:

(i) Is the plasma density produced by the collisional ionization due to the beam during $t \ll \tau$, larger than the ambient ionospheric density.

(ii) Is there a beam instability at $\omega_e$ based on the ambient ionospheric plasma density.

(iii) Is the ionization time due to the hot electrons produced by the instability shorter than the injection time $\tau$ (i.e. $v_{\text{ion}} \tau >> 1$).

If the answer to question (i) is yes and to question (ii) no, the threshold condition is similar to eq. (23), with the length $L$ given by the plasma density gradient due to collisional ionization by the beam. In view of the short injection time, however, eq. (23) is sufficient only for the beam plasma instability (BPI) but not BPD. In order to have BPD we need in addition to eq. (23) a positive answer to question (iii). The condition $v_{\text{ion}} \tau >> 1$ is equivalent to the Townsend condition (Galeev et al. 1976, Papadopoulos 1981). If the answer to question (i) is no and to question (ii) is yes, the BPI criterion will be given by eq. (22) with $n_0$ the value of the ambient plasma density. BPD requires in addition $v_{\text{ion}} \tau > 1$. The above comments should be only taken as guidelines. More precise considerations require non-linear BPI computations for the evaluation of $v_{\text{ion}}$ as a function of the beam parameters and will be reported in the future.
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Figure 1  Current threshold for BPD ignition as a function of pressure for three magnetic field values ($E = 1.5$ kV and $L = 20$ m).