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TITLE: THE MULTISTATE REFLECTOMETER

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SUMMARY

In this report, established multiport reflectometer theory is applied to show that a simple switched reflectometer employing two RF detectors can be calibrated in terms of known standards of voltage reflection coefficient. It considers some design aspects of this multistate reflectometer and distinguishes between its possible application and those of the dual six-port network analyser.
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THE MULTISTATE REFLECTOMETER

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1 INTRODUCTION

1.1 Since Hoer and Engen first described the six-port reflectometer [1,2], and then demonstrated that a network analyser formed of two such reflectometers could precisely measure the voltage reflection coefficient (VRC) of a short-circuit in terms of the characteristic impedance (Zo) defined by a known length of uniform waveguide [3,4], increased activity has been reported on six (and more) port reflectometers. These "multi-port" reflectometers each employ (n-2) detectors each connected to one port of a constant n-port waveguide junction (the two remaining ports of the junction having connected, at port 1, the source and, at port 2, first calibration standards and then the device under test (DUT)). The ratio of the output of each of (n-3) of these detectors to that of the remaining detector is observed and these ratios, together with calibration data, are used to compute the real and imaginary components of the VRC of the DUT. In these multiport instruments, the detector outputs are usually measured in sequence; in this paper we propose a variation whereby only two detectors are used and in which the waveguide junction is sequentially, but repeatably, varied.

1.2 Calibration of this "multistate" reflectometer for VRC measurement appears to require either standards defining at least seven precisely known different VRC, or a minimum of four precisely known lengths of assumed lossless uniform waveguide together with a nominally matched termination and a known short circuit. Because, in practice, the majority of measurements use air-filled waveguide (whose losses are neglected) and do not involve the precise measurement of near-unity VRC, this multistate reflectometer may be a cheaper alternative to the six-port instrument - satisfactory for most purposes. The multistate reflectometer appears suitable for comparing RF power meters if the two detectors each linearly indicate absorbed power. It may also be possible to employ modulation within the instrument, so allowing linear (homodyne) detection - should this be thought worthwhile. Calibration depends on a slightly different application of established multiport theory but, before we explore this topic, we present an example of the multistate reflectometer.
2.1 Although this example somewhat resembles an earlier instrument [5], it allows automated variation of the waveguide junction having ports numbered 1 to 4 in Figure 1. Either highly repeatable, possibly electronic, waveguide switching to different lengths of short-circuited waveguide, or a non-contacting short driven by a stepper motor, appears suitable for producing this variation. The VRC presented by this switched short need not be known, but it must be repeatable. The three-port directional coupler (or power divider) presents to the first detector a sample of (ideally only) the wave incident on the DUT and a four-port junction (such as the second directional coupler in Figure 1), together with the switched short, provides the second detector with samples of both the waves incident on and reflected from the DUT.

2.2 If the switched short assumes k stable states, each causing an output $P_k$ from the second detector and an output $P_R$ from the first detector, then the quantities observed are the ratios $P_k/P_R$. These ratios and calibration data enable an unknown VRC ($r = x + jy$) to be computed. Expressed in the jargon used for multiport instruments, the arrangement described provides $k$ "Q-circles" each centred on or about the unit radius circle in the complex $\Gamma$ plane. Movement of the switched short through an angular length $\theta$ causes the centre of the "Q-circle" to rotate through angle $2\theta$ around the unit radius circle. This would only be exactly true for ideal components, so that operation requires a calibration procedure to take account of the (assumed invariant) properties of the detectors and the (assumed repeatably switched) waveguide junction (with ports numbered 1 to 4).

3 MULTIPORT AND MULTISTATE REFLECTOMETER THEORY

3.1 Established multiport calibration theory relates to a constant n-port waveguide junction having different linear (but not necessarily reciprocal) transmission between its ports with only one mode transmitted at each port (but not necessarily the same mode at every port), there being no evanescent mode present at any port. Denoting the voltages of waves incident on and emergent from port $i$ (where $n > i > 0$) by $a_i$ and $b_i$.
respectively (so that the VRC of the DUT is \( \Gamma = a_2/b_2 \), by definition), then it can be shown [6] that, in general:

\[
b_i = (a_i \Gamma + b_i) b_2 \quad \cdots (3.1)
\]

where \( a_i \) and \( b_i \) are dimensionless coefficients describing transmission and reflection between all ports of the junction.

With \( k = (n-3) \) detectors, each connected to one of the \( n \) ports and each providing an output \( P_k \) proportional to the RF power it absorbs, plus one more detector arbitrarily designated as the reference and providing output \( P_R \), then it follows from (3.1) that:

\[
Q_k^2 = \left| \frac{d_k \Gamma + e_k}{c \Gamma + 1} \right|^2 \quad (k = 1, 2, 3 \cdots (n-3)) \quad \cdots (3.2)
\]

where \( Q_k = \sqrt{P_k/P_R} \) and \( c, d_k, e_k \) are dimensionless coefficients describing the junction and detectors in terms of the Zo defined by the calibration standards.

We note that it is the \( c \) of the denominator being independent of \( k \) that allows a six-port reflectometer to be reduced algebraically to an equivalent vector-indicating four-port reflectometer [6,7]. Further, it is this reduction algorithm that enables a dual six-port network analyser (DSPNA) to be calibrated with a known length of uniform waveguide defining Zo, plus an unknown but repeatable reflector. Finally, we note that the power flux from the measurement port can be expressed as:

\[
P_L = M \frac{(1-|\Gamma|^2)}{1+|\Gamma|^2} P_R \quad \cdots (3.3)
\]

where \( M \) is a constant for the reflectometer.

Equation (3.3) allows mismatched RF power meters to be compared using a multiport (six-port) reflectometer.

3.2 In our proposed multistate reflectometer, we assume the waveguide junction to be repeatably switched to \( k \) stable states, so that equation (3.1) can be applied to each of those states. With simultaneously observed outputs \( P_R \) from the reference detector and \( P_k \) from the second detector, equations corresponding to (3.2) and (3.3) can be derived for this multistate instrument. They are:

\[
Q_k^2 = \frac{P_k}{P_R} = \left| \frac{d_k \Gamma + e_k}{c_k \Gamma + 1} \right|^2 \quad \cdots (3.4)
\]

\[
P_L = M_k \frac{(1-|\Gamma|^2)}{1+c_k |\Gamma|^2} P_R \quad \cdots (3.5)
\]
After the $3k$ constants ($c_k, d_k, e_k$) of equation (3.4) have been found by calibration, then $\Gamma$ can be found by calculation in a similar way to a multiport instrument. The presence of the suffix $k$ in the denominators of equations (3.4) and (3.5) precludes application of the six-to-four port reduction algorithm, however, and this means that the multistate reflectometer must be calibrated in terms of a number of different precisely known $\Gamma$ or of calculated, assumed lossless, waveguide standards.

3.3 That seven or more standards can be used follows from equation (2.16) of reference [6]; for the multistate reflectometer this becomes:

$$
(Q_k)^2 \left(1 + c_k \left( \frac{x_k^2 + y_k^2}{d_k^2} \right) + 2(c_{Rk} x_k c_{Ik} y_k) \right) = |d_k|^2 \left( x_k^2 + y_k^2 \right) + |e_k|^2 + 2p_k x_k + 2q_k y_k
$$

where $\Gamma_k = x_k + jy_k$ ($k = 1, 2, \ldots$) are different and known and $p_k, q_k$ are dependent on $d_k, e_k$ only, using the notation $c_k = c_{Rk} + jc_{Ik}$.

Hence with $k > 7$ standards, the multistate instrument can be calibrated to find the seven real unknowns $|c_k|^2, c_{Rk}, c_{Ik}, |d_k|^2, |e_k|^2, p_k, q_k$ for each $k$ and then the computation of $\Gamma$ follows the scheme given in reference [6].

3.4 If we are prepared to neglect losses in short lengths of air-filled waveguide ("spacers") employed to provide known phase-shift, then the multistate instrument can be calibrated using a procedure similar to that described in reference [5]. At least four spacers are needed (to provide reflected phase changes of up to $2\pi$) and they are used with first a nominally matched termination and then with a short-circuit providing a known VRC. Power ratios $Q_k^2$ are observed and for each $k$ we can rewrite equation (3.4) as:

$$
E_k^2 Q_k^2 = \left| \frac{1 + A_k e^{j\alpha_k} e^{j\phi} }{1 + B_k e^{j\beta_k} e^{j\phi} } \right|^2 \ldots (3.6)
$$

where $E_k = \frac{1}{|e_k|}; A_k = e^{j\gamma_k} = \frac{d_k}{e_k}; B_k = e^{j\gamma_k} = c_k; \Gamma = \alpha e^{j\theta};$ (so that $A, B, F, c, \alpha, \beta, \theta$ are all real).

This gives (suppressing suffix $k$):

$$
E^2 Q^2 \left( 1 + B^2 + \rho^2 + 2B \cos (\beta - \theta) \right) = 1 + A^2 + \rho^2 + 2\alpha \cos (\alpha - \gamma) \ldots (3.7)
$$

First the nominally matched termination could be used with each of the spacers in turn to simulate a perfect $\Gamma = 0$, and this would allow $E$ to be found. Then a minimum of five observations using the known short and the spacers would provide a set of simultaneous non-linear equations (3.7) which could be solved for the unknowns ($A, B, \alpha, \beta$) by a succession of least squares approximations. Having found these calibration constants for each $k$, they and the power ratio $Q_k^2$ observed for a DUT can be
substituted in equation (3.7) which, by substituting $\Gamma = x + jy$ can immediately be recognised as a circle in the $\Gamma$ plane. Thus any unknown $\Gamma$ can be found as the intersection of $k$ known circles; if $k = 3$ then $\Gamma$ can be found unambiguously. We note that if $k > 3$ then the precision of measurement may be increased and application of equation (3.5) for power meter comparison may enhance precision if the $c_k$ are very different from each other (which is unlikely if a high directivity three-port coupler is used in the arrangement of Figure 1).

4 DISCUSSION

4.1 The advantage of the six-port reflectometer for work on RF metrology standards is that, used in a DSPNA, its demonstrated stability and resolution provide the potential for:

(i) comparing conventional waveguide (and coaxial-line) standards

(ii) establishing standards in less conventional uniform cross-section transmission media (e.g., dielectric guide or image guide)

(iii) characterising adaptors and launchers

For all these purposes, calibration of a DSPNA with a single standard defining $Z_0$ is essential and this procedure provides as an incidental the capability of measuring near-unity VRC. Few other applications demand this precision of definition of $Z_0$ so that, provided variations of a waveguide junction can be made sufficiently repeatably, our multistate reflectometer may form an adequate, simple, instrument. The work of Frank Warner on switched coupler travelling standards shows that mechanical waveguide switches are repeatable to better than $\pm 0.001 \, \text{dB}$ [8]. A stepper motor (or even careful manipulation of micrometer) drive can position a non-contacting short-circuit to within $\pm 2 \, \text{micron}$. These factors suggest that the arrangement of Figure 1 could be made sufficiently repeatable for most purposes.

4.2 The arrangement of Figure 1 incorporating a stepper motor drive of the nominal short-circuit should be cheaper than a six-port instrument using only three directional couplers [9]. (A commercial programmable stepper motor drive is about £2.5k, which is less than the approx £4k incurred for an additional coupler plus two thermistor detectors and their associated self-balancing bridges). Although stepper motor drive would be slower and more expensive than an electromechanical multiport waveguide switch plus fixed shorts, it has the advantages of less likelihood of degradation through wear and of providing greater flexibility in positioning the centres of the $Q$-circles. (The ideal may be to use PIN diode switches since these would be faster and possibly more reliable but their temperature dependence is a considerable disadvantage). Irrespective of the method of varying the position of the nominal short in Figure 1, however, the following voltage ratios can be written, assuming ideal matched lossless components (and describing the directional couplers by $|t|^2 = |c|^2 = 1$ with suffixes 1 and 2):

$$\frac{b_1}{a_1} = t_1^2 \frac{t_2^2 \left( \Gamma - \frac{c_2^2}{e_2 e_3^*} \right)}{e_2^*} \quad \ldots \quad (4.1)$$
\[ \frac{b_2}{a_1} = t_1 t_2 \]  

\[ \frac{b_3}{a_1} = j c_2 t_1 t_2 (\Gamma + \Gamma_s) \]  

\[ \frac{b_4}{a_1} = j c_1 \]  

But \( \Gamma_s = -\rho e^{-j2\theta} \) where \(|\rho|a_1\), the magnitude of the VRC of the nominal short.

Hence \( Q_k^2 = \frac{P_k}{P_R} = |\Gamma - \rho \cos 2\theta_k + j\rho \sin 2\theta_k| \frac{2 \left( \frac{C_1}{C_2} \right)^2}{t_1^2 t_2^2} \)  

Equation (4.5) describes a circle in the \( \Gamma = x + jy \) plane centred at \(-\Gamma_s\), i.e. (very nearly) on the unit radius circle. A reference \( \theta_k = 0 \) can be established by connecting a short to the measurement port and adjusting \( \theta \) to obtain a minimum indication of \( P_k \); thereafter the angle \( \theta \) can be increased to values appropriate to any frequency of interest by operating the stepper motor. Figure 2 illustrates that we have achieved an analogue of the multiport reflectometer with \((k + 3)\) ports, without the attendant problems of balancing the phase of interconnecting waveguides between couplers, or of designing near perfect waveguide junctions to prevent \( Q \) circle centres approaching unfavourable positions in the \( \Gamma \) plane. It is probable that values of \( \theta_k \) of approximately 0, 60°, 120° at the longest wavelength of a waveguide operating range would suffice (since these would become 0, 120°, 240° at the shortest). However, the calibration theory only requires a three-port junction (e.g., a power divider) and a variable four-port junction (possibly a switched short and a resistive bridge) and by a suitable choice of stepper motor drive settings, allowing the \( Q \)-circle centre to rotate more than one revolution, a greater
than octave frequency range could be covered. If necessary, the redundancy introduced by using more than $k = 3$ short circuit settings could be used to enhance the precision of measurement either by averaging or by discarding small $Q_k^2$ values.

4.3 The equations (4.1) to (4.5) lead to the following estimates of input mismatch loss (for all $|\Gamma| \leq 1$) and power relations (in terms of $|a_1|^2$, the power incident on the junction) for the arrangement of Figure 1:

| Coupling factor dB | Max mismatch loss at input for ($|\Gamma| = 1$) = $10 \log_{10} \left| \frac{1 - |b_1|^2}{|a_1|^2} \right|$ dB | Max power to matched load $= \frac{b_2}{|a_1|^2}$ for all $|\Gamma| = 1$ | Max $P_k^2$ for all $|\Gamma| = 1$ | $P_R^2 = \frac{|b_4}{|a_1|^2}$ |
|-------------------|-------------------------------------------------|-----------------------|--------------------------|---------------------|
| Coupler 1 | Coupler 2 | $-1.2$ | $0.25$ | $0.5$ | $0.5$ |
| 3 | 3 | $-3.5$ | $0.375$ | $0.75$ | $0.25$ |

These suggest that the use of a 6dB three-port coupler together with a 3dB four-port coupler would be preferable for comparing equal range RF power meters and that two 3dB couplers might provide slightly better resolution in measuring all VRC of $|\Gamma| \leq 1$.

4.4 Looser coupling would enable measurements of VRC to be made at high power levels, for those applications in which this is necessary. We also note that, although much has been made by some theorists on measurement of $|\Gamma| > 1$, this is possible with both multistate and multiport instruments, provided that extrapolation of $Q_k^2$ indications is permissible. If the VRC of an active device be written as $\Gamma_A^t$, so that $1 < 1$, then substitution of $\frac{1}{\Gamma_A}$ in equation (3.4) leads to

$$Q_k^2 = \left| \frac{d_k + e_k \Gamma_A^t}{c_k + \Gamma_A^t} \right|^2$$

so that the calibration constants and a variation of the mathematics of section 3 would allow $\Gamma_A$ to be found, in principle. In practice, measurement of an active device would generally require (automatic) tuning to present a (non-linear) DUT with a matched source.

4.5 We speculated in the introduction that modulation with homodyne detection could, in principle, be used with the multistate reflectometer. An arrangement allowing this might take the form of Figure 3.
This provides for a constant level carrier about 30dB above the signal. If the isolation of the modulated signal from the mixer is sufficient and if the source amplitude is maintained sufficiently constant, then this arrangement appears to be described by $V_k = |(d_r + e_k)/(c_k + 1)|$, so that substitution of $V_k = 0_k$ implies that the theory of section 3 is applicable. It would provide increased dynamic range but requires more equipment to do so.

5 ACKNOWLEDGEMENT

The author acknowledges helpful discussions with T E Hodgetts, including one on the possibilities that might arise from the use of a multiplicity of reflectometer detectors, since that topic led to this work (for it would allow a multiport reflectometer providing, say, 360 power ratios to be simulated).

6 CONCLUSION

We have illustrated the relation between the theory of reflectometers that rely on switching to provide phase and amplitude information from two detectors and those that avoid switching by using more than two detectors. This relation leads to a simple multistate reflectometer which appears to offer adequate precision for laboratory measurement of VRC and comparison of RF power meters over at least a waveguide bandwidth at reasonable cost, using thermistors for detection.

7 REFERENCES


In this report, established multiport reflectometer theory is applied to show that a simple switched reflectometer employing two RF detectors can be calibrated in terms of known standards of voltage reflection coefficient. It considers some design aspects of this multistate reflectometer and distinguishes between its possible application and those of the dual six-port network analyser.