FOREIGN TECHNOLOGY DIVISION

TRANSIENT PROCESSES IN AN INDUCTOR SYSTEM CONSISTING OF A FLAT COIL AND A MULTILAYER CONDUCTING MEDIUM

by

V.N. Bondaletov, V.P. Gel'yetov, Ye.N. Chernov

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TRANSIENT PROCESSES IN AN INDUCTOR SYSTEM CONSISTING OF A FLAT COIL
AND A MULTILAYER CONDUCTING MEDIUM

V. N. Bondaletov, V. P. Gel'yetov, Ye. N. Chernov

Introduction

Flat inductors placed over a conducting medium are used in a whole series of electrical engineering devices (during electroinductive flaw detection, induction heating, magnetic pulse treatment of metals, etc.).

Steady electromagnetic processes in these fixed inductor systems are usually calculated in the approximation of a plane electromagnetic wave. The stationary electromagnetic field of an annular coil with a sinusoidal current placed over a multilayer medium was also found by solving the heterogeneous Helmholtz equation for the vector potential of the magnetic field.

The transient electromagnetic processes which take place during the discharge of a capacitive store to an inductor, mainly characteristic of magnetic pulse deformation of metals and dynamic induction acceleration of conductors, are calculated in the approximation of a plane electromagnetic wave. However, the assumption of the presence of a single field strength component does not always turn out to be true in practice. Therefore, in this work we calculate the transient processes for the more general case - a flat disk coil with a current
placed over a multilayer conducting medium. We will use a coil with a surface current uniformly distributed over the dimensions and placed horizontally over an unbounded medium (Fig. 1) as the calculation model.

The field of a real inductor can then be found by adding the fields of the individual disks.

1. Calculation Method

The initial equations for solving any electrodynamic problems are the Maxwell equations, which, disregarding the bias currents, we will write as follows for isotropic media:

\[
\begin{align*}
\text{rot} \mathbf{H} &= \gamma \mathbf{E} + \mathbf{j}_{\text{om}}, \\
\text{rot} \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}.
\end{align*}
\]

where \( \gamma \) is the specific conductivity of the medium; \( \mathbf{j}_{\text{om}} \) is the density of the extraneous currents, which are controlled by an external source.

We will make the calculation by the successive application of the integral equations.

We will use the Laplace transformation to eliminate differentiation through time \( t \).

Then

\[
\begin{align*}
\text{rot} \mathbf{A}(p) &= \gamma \mathbf{E}(p) + \mathbf{j}_{\text{om}}(p), \\
\text{rot} \mathbf{E}(p) &= -\mu_0 \mu_r \mathbf{H}(p), \\
\mathbf{H}(p) &= \frac{1}{\mu_0} \text{rot} \mathbf{A}(p).
\end{align*}
\]
where $\tilde{A}(p)$ is the expression for the vector potential (according to Laplace); $\mu$ is the relative magnetic permeability of the medium.

Solving system of equations (2) analogously to [3], we arrive at a heterogeneous Helmholtz equation for expressing the vector potential:

$$V^2\tilde{A}(p) + k^2\tilde{A}(p) = -\mu \mu_0 \tilde{J}_{\text{m}}(p),$$  \hspace{1cm} (3)

where $k^2 = -\mu \mu_0$.

In the future we will bear in mind that all of the transformations are made with transforms of the functions, without stipulating this specially.

We will use a cylindrical coordinate system with the z-axis directed normal to the surfaces of the layers and coinciding with the axis of the disk for the calculation; then (3) changes into a second-order equation in partial derivatives relative to the only (because of the axial symmetry) component $A_y$:

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial A_y}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 A_y}{\partial z^2} + (k^2 - \frac{1}{r})A_y = -\mu \mu_0 \tilde{J}_{\text{m}}.$$  \hspace{1cm} (4)

We will eliminate the differential operations on the coordinate $r$ according to [3] using the Hankel integral transformation with a kernel in the form of the first-order Bessel function $J_1(\alpha r)$:

$$\tilde{A} = \int r J_1(\alpha r) A(r, p, z) dr.$$  \hspace{1cm} (5)

where $\tilde{A}$ is the transformed vector potential; $r$ is the weighting function; $J_1(\alpha r)$ is the kernel; $\alpha$ is the transformation variable.

Then the problem in question is reduced to an ordinary second-order differential equation relative to the unknown twice-transformed function $\tilde{A}(\alpha)$:

$$\frac{d^2 \tilde{A}}{d\alpha^2} - q^2 \tilde{A} = -\mu \mu_0 \tilde{J}_{\text{m}}.$$  \hspace{1cm} (6)
where \( \eta = \sqrt{\lambda - \mu_1} \) is the root with the positive real part.

The general solution for nonhomogeneous equation (6) is known [4]:

\[
\mathbf{A} = \frac{\mu_1}{2\eta^2} \left[ \exp(qz)(\mathbf{B} - \int_0^z \mathbf{A}(\xi) d\xi) + \exp(-qz)(\mathbf{C} + \int_0^z \mathbf{A}(\xi) d\xi) \right],
\]

where \( \xi \) is the integration variable along direction \( z \); \( \mathbf{B} \) and \( \mathbf{C} \) are arbitrary constants determined from the boundary conditions.

As we know, the conditions of continuity of the vector potential and the tangential component of the magnetic field strength must be satisfied on the interface of the media:

\[
\mathbf{A}_s(\mathbf{r}, z) = \mathbf{A}_{s1}(\mathbf{r}, z),
\]

\[
\mu_s \frac{\partial \mathbf{A}_s}{\partial z} = \mu_{s1} \frac{\partial \mathbf{A}_{s1}}{\partial z}
\]

with \( Z = Z_s \).

After finding the transformed function \( \hat{\mathbf{A}}(\mathbf{r}, z) \), the unknown vector potential can be found by using the successively inverse transformations of Hankel:

\[
\mathbf{A}(\mathbf{r}, z) = \int \hat{\mathbf{A}}(\mathbf{r}, z) Z(\alpha) d\alpha
\]

and Laplace:

\[
\mathbf{A}(\mathbf{r}, z) = \frac{1}{2\pi i} \int e^{i\beta z} A(\beta) d\beta.
\]

2. Field of Disk Coil Located Over Two-layer Medium, Plate, Half-Space

We will find the expression for the vector potential in the sufficiently general case when a disk coil is located over a layer of thickness \( d \) with conductivity \( \sigma \) lying on conducting half-space \( \mu_s \).

We will set \( \mu_1 = \mu_2 = \mu_3 = 1 \), which is valid for nonferromagnetic
For the upper half-space \((z > 0)\), considering that \(\kappa = 0\) and \(g = \lambda\), from (7) we obtain

\[
\tilde{A}_z = \frac{\mu_0}{2\pi x} \left[ \exp(\lambda z) \left( \beta - \int_{\text{tm}} \exp(-\lambda \xi) d\xi \right) + \exp(-\lambda z) \left( \beta + \int_{\text{tm}} \exp(\lambda \xi) d\xi \right) \right].
\]  

(11)

In view of the absence of extraneous currents, we will have the following for the conducting layer \((z < 0)\):

\[
\tilde{A}_z = \frac{\mu_0}{2\pi x} \left[ \beta_2 \exp(g_z z) + \beta_3 \exp(-g_z z) \right].
\]  

(12)

For the lower half-space

\[
\tilde{A}_z = \frac{\mu_0}{2\pi x} \beta_3 \exp(g_z z),
\]  

(13)

since \(C_3 = 0\), for when \(z = 0\), the field should be bounded.

The field should also be bounded as \(z \to -\infty\); therefore, from (11) we can determine:

\[
\beta = \int_{\text{tm}} \exp(-\lambda \xi) d\xi.
\]  

(14)

We will consider the extraneous current density to be independent of \(r\) and equal to

\[
j_{\text{tm}}(r) = \left( \frac{j_0(r')}{r} \right) \delta(r) \text{ when } z = h; r \gg 0
\]

at

\[
j_{\text{tm}}(r) = 0 \text{ when } z + h; r \gg r_0.
\]  

(15)

Then, with consideration of (16), we obtain the following for transformed value \(J_{\text{tm}}(r)\):

\[
J_{\text{tm}} = 2 \delta(z - h) \int r^2 \varphi'(r) dr = 2 \delta(z - h) F(r).
\]  

(16)
where \( \delta(z-h) \) is the Dirac delta function
\[
F(x) = \int_{-\infty}^{\infty} \delta(x) \, dx.
\]

Substituting (16) in (14), we will find the constant \( B_1 \):
\[
B_1 = \int_0^T F(x) \exp(-\lambda \xi) \delta(z-h) \, d\xi = \int_0^T F(x) \exp(-\lambda h).
\]  

We will use boundary conditions (8) to find constants \( B_2, B_3, C_1, C_2 \). Substituting the values of the vector potentials from (11-13) and their derivatives, on the interface of the media we will have:
\[
\begin{align*}
\frac{1}{\mu} (B_2 \cdot C_1) &= \frac{1}{\mu} (B_2 \cdot C_2), \\
\frac{1}{\mu} (B_3 \exp(q_2, d) \cdot C_1 \exp(q_2, d) - B_1 \exp(-q_2, d) \cdot C_2 \exp(q_2, d)), \\
\frac{1}{\mu} (B_3 \exp(q_2, d) \cdot C_2 \exp(q_2, d) - B_1 \exp(-q_2, d) \cdot C_2 \exp(q_2, d)).
\end{align*}
\]  

Solving this system, we will obtain:
\[
\begin{align*}
B_2 &= \frac{10 (a \cdot q_2) \exp(q_2, d)}{2} \cdot B_1, \quad B_3 = \frac{6}{\mu} \exp(q_2, d) \cdot B_1, \\
C_1 &= \frac{10 (a \cdot q_2) \exp(-q_2, d)}{2} \cdot B_1, \quad C_2 = \frac{6}{\mu} \exp(-q_2, d) \cdot B_1.
\end{align*}
\]  

Substituting these values, and also (17), in (11-13), we will obtain the expression for the transformed vector potentials:
\[
\begin{align*}
\tilde{A}_1 &= \frac{8}{z_2} \int_0^T F(x) \exp(q_2, d), \\
\tilde{A}_2 &= \frac{8}{z_3} \int_0^T F(x) \exp(-q_2, d), \\
\tilde{A}_3 &= \frac{8}{z_3} \int_0^T F(x) \exp(q_2, d) \exp(q_2, d).
\end{align*}
\]
Expressions (20-22) can be used as the initial expressions, e.g., for calculating the pressure during magnetic pulse deformation of thin-walled articles in a metal die.

If a conducting plate of thickness $d$ is located on an insulating base, then $H = 0; \quad \rho = \lambda$, and the expressions for the transformed vector potentials assume the form:

$$F(x) = \frac{\mu_0 \rho}{4 \pi} \frac{\exp(-q \rho)}{(q^2 + \alpha^2)^{3/2}}$$

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In the limiting case $\lambda \rightarrow \infty$, which corresponds to the system of a disk coil over a half-space, expressions (23-25) are considerably simplified:

$$F(x) = \frac{\mu_0 \rho}{4 \pi} \frac{\exp(-q \rho)}{(q^2 + \alpha^2)^{3/2}}$$

3. Transient Processes in the System "Disk Coil - Conducting Half-Space" When a Sinusoidal Decaying Current is Connected to the Source

The connection of an inductor system to a sinusoidal decaying current source is typical of induction casting of massive conductors, as well as of magnetic pulse treatment of metals. In this case, the current density

$$\mathbf{J}(t) = \frac{\mathbf{E}}{\rho} \exp(-at) \sin(\omega t) - \frac{\mathbf{E}}{\rho} \exp(-bt)$$
where \( a \) is the attenuation decrement and \( w \) is the circular current frequency.

Substituting the value \( J_0(\eta) \) in formulae (26), (27) and using inverse transformations (9), (10), we obtain the following expressions for the original vector potentials:

\[
A_x(r, \alpha, \lambda) = \frac{1}{2} \int \frac{\exp(\eta \lambda)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{J_0(\eta)}{\eta} r \, d\eta.
\]

\[
A_y(r, \alpha, \lambda) = \frac{1}{2} \int \frac{\exp(\eta \lambda)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\beta J_1(\eta)}{\eta} r \, d\eta.
\]

\[
A_z(r, \alpha, \lambda) = \frac{1}{2} \int \frac{\exp(\eta \lambda)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\gamma J_2(\eta)}{\eta} r \, d\eta,
\]

We will introduce the variable \( \nu = \lambda r \); then for \( F(\lambda) \), according to [5, p. 657], we will have:

\[
F(\lambda) = r^\nu \int_{r'} \frac{\exp(\nu \eta)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\beta J_1(\nu \eta)}{\eta} r' \, d\eta = \frac{1}{r} \Phi(\eta).
\]

where

\[
r' = \frac{1}{\nu}; \quad \Phi(\eta) = J_1(\eta) H_0(\eta) - \frac{1}{\nu} \eta J_0(\eta).
\]

\( H \) is the Struve function.

Integrating (29), (3) on a complex plane, after simple transformations, with consideration of (31), we obtain the following expressions for the vector potentials:

\[
A_x(r, \alpha, \lambda) = \frac{1}{\lambda} \int \frac{\exp(\nu \eta)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\beta J_1(\nu \eta)}{\eta} r \, d\eta.
\]

\[
A_y(r, \alpha, \lambda) = \frac{1}{\lambda} \int \frac{\exp(\nu \eta)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\gamma J_2(\nu \eta)}{\eta} r \, d\eta.
\]

\[
A_z(r, \alpha, \lambda) = \frac{1}{\lambda} \int \frac{\exp(\nu \eta)}{\eta^2} \frac{1}{\sqrt{2 \pi}} \frac{\lambda J_0(\nu \eta)}{\eta} r \, d\eta.
\]
Having the expressions for the vector potentials, we can determine all the values which characterize the electromagnetic field in the system in question.

4. Magnetic Field Pressure on a Massive Conductor

We will calculate the currents in a massive conductor and the magnetic field pressure. This problem arises, for example, when straightening large articles by the magnetic pulse method. The magnetic field pressure is usually determined from the volume density of the pondermotive forces:

\[ \mathbf{\ddot{J}} = [\mathbf{j} \times \mathbf{B}] \]  

(34)

where \( \mathbf{j} \) is the volume density of the currents in the conductor; \( \mathbf{B} \) is the magnetic field induction.

The pressure along the z-axis (Fig. 1) is obtained as

\[ P = \int_{\mathcal{V}} B_z \rho \, dz \]  

(35)

where

\[ \mathcal{V} = \int_{\mathcal{V}} B_s \rho \, dz \; ; \; B_s = -\frac{\partial A_s}{\partial z} \; ; \; \rho = -\frac{\partial A_s}{\partial z} \]  

\( A_s \) is the depth of penetration;

\[ \kappa = \sqrt{\frac{\mu}{\mu_0}} \]  

is the depth of penetration;
Considering that during magnetic pulse deformation of metals, \( \frac{B_0}{h} \approx 1 \) and the current damping is small, after simplification, we can obtain the following from (33):

\[
Z(t_2, t) = \frac{A_0}{\mu_0} \exp(-\alpha t) \exp \left( \frac{\alpha t}{\Delta} \right) \sin \left( \frac{2\pi t}{\Delta} \right),
\]

(36)

\[
B_{0P}(t_2, t) = A_0 \exp(-\alpha t) \exp \left( \frac{\alpha t}{\Delta} \right) \sin \left( \frac{2\pi t}{\Delta} \right).
\]

(37)

\[ A = -A_0 \exp(-2\pi t) \sin \delta. \]

(38)

where \( A_0 = \frac{B_0(r)}{2\mu_0} \).

Thus, the dependence of the pressure on time is the same as in the case of a plane electromagnetic wave. However, the pressure distribution on the surface does not remain constant, since

\[
A_0(r) = \frac{1}{2\mu_0} \int_{r_0}^{r} \exp(-\alpha \Delta) \exp \left( \frac{-\alpha \Delta}{\Delta} \right) \sin \left( \frac{2\pi \Delta}{\Delta} \right) dr.
\]

(39)

The dependences \( B_0(r) \) and \( P_0(r) \), obtained by numerical integration for the case \( h' = 0.1 \), are given in Fig. 2.
Conclusions

The solution of the Maxwell equation for the "flat disk coil - multilayer conducting medium" inductor system with an arbitrary dependence of the exciting current on time, obtained by the successive application of the integral Laplace and Hankel transformations, made it possible to obtain the general expressions for the vector potentials in different media. The expressions for the vector potentials were also obtained with the connection of a "coil - conducting half-space" system to a decaying sinusoidal current source.

The inverse transformations were made by numerical integration.

REFERENCES

2. Соболев В.С., Янковский Л.И. Наводные и экраны дуговых... "Наука", 1967.
4. Гринберг Г.А. Каберные вопросы математической теории электрических и магнитных явлений. Изд-во АН СССР, 1948.
5. Гранетти И.С., Рябов И.М. Сборник интегралов, сумм, рядов и производных. М., "Физматгиз", 1962.
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