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ITEM #19, ABSTRACT, CONTINUED: The major effort in the last few months of the period was on the solution of ordinary differential equations (ODEs) by a Japanese visitor, Professor Shunro Watanabe. Professor Watanabe has begun to develop a subsystem of MACSYMA that solves ODEs by converting them to P-functions, originally studied by Riemann. One example of this approach to solving ODEs is given within. One goal of this work is to solve a large percentage of Kamke's equations using this general approach.
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PALG5 FASL DSK SWATAN being loaded
Loading done
we solve

\[
\begin{bmatrix}
0 & 1 & \text{INF} \\
\text{INF} & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\(1/4\)

\[
[ - - - ]
\]

\(y = (x - 1) \quad P \quad [ 3 \quad 2 \quad 12 ] (x)
\]

PALG4 FASL DSK SWATAN being loaded
Loading done

POHAS2 FASL DSK SWATAN being loaded
Loading done

\[
\begin{align*}
K1 \quad T & \left(2 \sqrt{T + T + 1} + \sqrt{3} (T + 1)\right) \quad (x - 1) \\
& + \quad 3 \quad 1/12 \\
& \quad (T - 1)
\end{align*}
\]

\[
\begin{align*}
K2 \quad T & \left(\sqrt{3} (T + 1) - 2 \sqrt{T + T + 1}\right) \quad (x - 1) \\
& + \quad 3 \quad 1/12 \\
& \quad (T - 1)
\end{align*}
\]

where \(t = ^{x(1/3)}\)

Time = 13449 msec.

\[
\begin{align*}
K1 \quad T & \left(2 \sqrt{T + T + 1} + \sqrt{3} (T + 1)\right) \quad (x - 1) \\
& + \quad 3 \quad 1/12 \\
& \quad (T - 1)
\end{align*}
\]

\[
\begin{align*}
K2 \quad T & \left(\sqrt{3} (T + 1) - 2 \sqrt{T + T + 1}\right) \quad (x - 1) \\
& + \quad 3 \quad 1/12 \\
& \quad (T - 1)
\end{align*}
\]

Time = 15278 msec.

BATCH DONE

(C10) closefile(buffer, save);
(C3) showtime true
Time = 5 msec.

(C4) batch(example12):

(C5) k406:16*(x-3-1)-2*diff(y,x,2)+27*x*y=0;
Time = 41 msec.

\[ \frac{2}{3} \frac{2}{2} \frac{dY}{dX} \quad \frac{16}{2} (x-1) \quad + \quad 27 x y = 0 \]

(C6) k406t:48*(x-1)-2*diff(y,x,2)+32*(x-1)-2*diff(y,x)+32*x*y=0;
Time = 50 msec.

\[ \frac{2}{2} \frac{2}{2} \frac{dY}{dX} \quad \frac{48}{2} (x-1) \quad \frac{x}{dX} \quad + \quad 32 (x-1) \quad + \quad 9 x y = 0 \]

(C7) loadfile(pmain_fasl):

PMAIN FASL DSK SWATAN being loaded
Loading done
Time = 426 msec.

(D7) DONE

(C8) load2(k406t):

\[ \frac{dY}{dX} \quad \frac{2}{2} \quad \frac{2}{2} \quad \frac{3}{3} \quad \frac{y}{dX} \]
we solve \[ \frac{2}{2} \frac{3}{3} \frac{x}{2} \quad \frac{16}{16} \frac{x}{x} \quad \frac{-32}{-32} \frac{x}{x} \quad + \quad 16 x \]

SOLVE FASL DSK MACSYM being loaded
Loading done

PHYPGM FASL DSK SWATAN being loaded
Loading done
the type is hypergeometric
the solution may be written by Riemann's P-functions as follows

\[
\begin{bmatrix}
0 & 1 & \inf \\
1 & 3 & 1 \\
\vdots & \vdots & \vdots \\
\end{bmatrix}
\]

\[ y = P \begin{pmatrix} 3 & 4 & 3 \end{pmatrix}(X) \]

PHYPGM FASL DSK SWATAN being loaded
Loading done
END
FILMED
3-84
DTIC