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Analytical Prediction of Turbulent Heat Transfer Parameters:

The Third Annual Report

Adrian Bejan
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<td>Abstract (Continue on reverse side if necessary and identify by block number):</td>
<td>The objective of this research is to construct a purely theoretical foundation for the phenomenon of turbulent heat transfer. In the present report it is shown that the buckling theory of inviscid streams and the classical hydrodynamic stability theory are in agreement with respect to the time scale criterion that accounts for transition to turbulence. Two experimental studies that confirm this correspondence are described.</td>
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ANALYTICAL PREDICTION OF TURBULENT HEAT TRANSFER PARAMETERS:
THE THIRD ANNUAL REPORT
December 1983

Adrian Bejan
Associate Professor
Department of Mechanical Engineering
University of Colorado
Boulder, Colorado 80309

Prepared for
M. K. Ellingsworth
Program Monitor
The Office of Naval Research
Arlington, Virginia 22217

Background

This third annual report reviews the research produced by our group during the academic year 1982-1983, in the pursuit of a purely theoretical basis for turbulent flow phenomena and for performing engineering calculations of turbulent transport parameters. The objective of our work during the third year is better understood if one takes a brief look at our objectives and accomplishments during the first two years.

Our group's interest in the fundamentals of turbulent flow was sparked by the idea that any large-Reynolds-number stream (i.e., any stream that is relatively inviscid) possesses a longitudinal length scale ($\lambda_B$) which is proportional to the stream's transversal length scale ($D$). This longitudinal length scale is the widely observed meander wavelength of turbulent streams (jets, wakes, shear layers, plumes). The suggestion that "$\lambda_B/D = \text{constant}$" is a property of all inviscid streams was published in 1981 as the end-result of the buckling theory of inviscid flow columns [1,2]. Whether or not the scaling law recommended by the buckling theory is correct remained to be established on the basis of old and new experiments. One issue we recognized from the start is that any theory that predicts a previously unknown property of turbulent flow deserves to be treated with serious attention, simply because turbulence science as we have it is dominated by empiricism. For this reason, any legitimate advance on the theoretical front is potentially capable of reducing significantly man's reliance on empiricism in dealing with engineering calculations of turbulent flow.

It is with this philosophical outlook that we devoted a good part of the last three years to the task of verifying the validity of the $\lambda_B - D$ scaling law of inviscid flow. During the first year we focused on a series of laboratory experiments designed to visualize the meandering or the buckling of high-Reynolds-number flows and to measure the $\lambda_B/D$ constant. Another, much more rewarding phase
of our experimental effort was to sift through the fluid mechanics literature
and to re-examine classical experimental results in light of our suspicion that
beneath all of them resides the $\lambda_B^D$ property. We documented our experimental
findings individually in the peer-refereed literature [3-5] and in review form
in chapter 4 of my first book [6]. All the experiments examined by us – new and
old – validate the buckling theory prediction that a longitudinal length scale
exists, and that this length scale is proportional to the transversal length
scale of the stream under consideration. It is worth pointing out that since its
publication in 1981 the buckling theory has triggered at least one other experi-
mental study [7], whose conclusions relative to the validity of the $\lambda_B^D$
scaling law is in perfect agreement with ours.

During the second year of this research program we turned our attention to
analytical work that invokes the $\lambda_B^D$ property in order to predict some of
the more frequently used features of turbulent flow. These analytical developments
ranged from predicting the constant-angle growth (i.e., the triangular or conical
shape) of all turbulent mixing regions, to calculating the viscous sublayer
thickness in turbulent boundary layer flow. Samples of this analytical work are
presented in the peer-refereed literature [6,8,9] and throughout the "turbulent
flows" part of my course in convection heat transfer [10]. In all cases, the
$\lambda_B^D$ property is used to derive analytically classical facts known empirically:
this new property is used to partially replace empiricism with theory in our
own comprehension of turbulence.

During the third year of sponsored research, 1982-1983, we could have
continued with more buckling flow experiments and with more analyses of
turbulent flow, and our success and productivity would have been assured. We
chose not to do this (two years of intensive work of this kind were enough to
satisfy our curiosity), instead, we devoted the third year to investigating the
possible relationship between buckling theory and hydrodynamic stability theory. We were able to show that the hydrodynamic stability theory of inviscid flow and the buckling theory of inviscid flow are in fact in agreement with regard to the existence of the $\lambda_B D$ scaling law: as shown in the next section, the agreement between the two theories is easy to establish once "one knows what to look for" in the volume of information generated by hydrodynamic stability analyses, (i.e., once one knows from buckling theory that a certain proportionality of scales might have been overlooked).

**Hydrodynamic stability theory and buckling theory vis-à-vis transition**

A review of analytical results of linear stability analyses of inviscid flows (Table 1) shows that any inviscid stream of thickness $D$ is unstable to disturbances whose longitudinal wavelength exceeds a certain multiple of $D$. For example, a two-dimensional inviscid jet of triangular profile is unstable to wavelengths in excess of 1.714 $D$. Beginning with Rayleigh's paper [11], much has been made in the stability literature of the maximum exhibited by the growth rate of the disturbance. More interesting, however, is the "coincidence" that the neutral wavelength 1.714 $D$ is only 5 percent smaller than the buckling wavelength scale of a two-dimensional stream ($\frac{\pi}{\sqrt{3}}D = 1.81 D$; see Refs. [1,6]). This coincidence seems to be insensitive to the actual shape of the $U(y)$ profile chosen for analysis. For example, in a stack of $D$-thick counterflow jets of sinusoidal profile ($u = U_0 \sin \frac{\pi y}{D}$) the neutral wavelength is 2 $D$, which is only 10 percent greater than the buckling length scale ($\frac{\pi}{\sqrt{3}}$) $D$. The same scaling between flow thickness and neutral wavelength is revealed by the stability analysis of other finite-thickness flows (Table 1).

The proportionality of length scales identified in Table 1 tells us that during transition a stream can fluctuate relative to its ambient with a period
Table 1. Minimum wavelength for instability in inviscid flow

(a) free jet \hspace{2cm} \lambda_{\text{min}} = 1.714 D

(b) shear layer \hspace{2cm} \lambda_{\text{min}} = 4.914 D

(c) velocity discontinuity \hspace{2cm} \lambda_{\text{min}} = 0 \ (D = 0)
of order \( \lambda/(U_0/2) \), where \( \lambda \) is the disturbance wavelength and \( U_0 \) the scale of the relative velocity between stream and ambient. And since for instability \( \lambda \) must be greater than a length nearly identical to the buckling wavelength \( \lambda_B \) [1,6], the stream fluctuation time scale will be equal to or greater than the buckling time

\[
\text{t}_{\text{fluctuation}} \geq \text{t}_B = \frac{\lambda_B}{U_0/2} \tag{1}
\]

Since \( \lambda_B \sim D \), the fluctuation period exceeds a minimum value that is proportional to \( D \). The same conclusion is shown graphically in Fig. 1. The domain of possible inviscid instability is situated to the right of the \( t - D \) line represented by eq. (1).

Since "inviscidity" is a flow property, not a fluid property\(^*\), the domain of possible inviscid instability must also be situated to the left of the \( t - D^2 \) parabola on Fig. 1. The \( t - D^2 \) curve has its base in the argument that any stream \((U_0,D)\) started impulsively relative to a stationary ambient becomes viscid during a time given by the scale of transversal viscous communication over a distance \( D/2 \) [1,6],

\[
\text{t}_v = \frac{D^2}{16v} \tag{2}
\]

Thus, the disturbed stream fluctuates as an inviscid stream if

\[
\text{t}_{\text{fluctuation}} \leq \text{t}_v \tag{3}
\]

\(^*\) all fluids have a measurable viscosity, \( \mu \).
Combining eqs. (1) - (13), we learn that the transition is possible as long as

\[ t_B \leq t_{\text{fluctuation}} \leq t_v. \]  

Figure 1 suggests that in any stream-like flow the leading section of the flow is laminar, and that the transition is possible for the first time when the buckling number reaches \( \text{0(1)} \),

\[ N_B = \frac{t_v}{t_B} - 1 \]  

In terms of a local Reynolds number based on local transversal length scale, \( U_0D/\nu \), the \( N_B - 1 \) criterion is written as

\[ \frac{U_0D}{\nu} \geq 10^2. \]

The transition criterion (5,6), derived here based on the scaling trend discovered in some of the results of inviscid stability analyses (Table 1), is identical to the criterion suggested originally by the buckling theory of inviscid streams. Most recently, we tested this criterion against experiments on transition in round laminar plumes [12] and in natural convection boundary layers (wall jets) near vertical walls heated at uniform temperature or uniform heat flux [13]. These experiments are described next only in "abstract" form, as they have both been published in the peer-refereed literature.*

*reprints can be obtained by writing to Adrian Bejan, University of Colorado, Campus Box 427, Mechanical Engineering Department, Boulder, Colorado 80309
References


7. A. Pollard, Private Communication, Department of Mechanical Engineering, Queen's University, Kingston, Ontario, Canada.


Abstract

This paper reports a theoretical and experimental study of the fundamental mechanism responsible for transition in natural convection plume flow. Theoretically, it is argued that the transition occurs when the time of viscous penetration normal to the plume becomes comparable with the minimum time period with which the plume can fluctuate as an unstable inviscid stream. It is also argued that at transition the plume wavelength must always scale with the local plume diameter. The experimental part of the study focused on transition in the axisymmetric air plume above a point heat source. Smoke visualized of the plume shape at transition led to extensive observations that support strongly the transition mechanism proposed theoretically. The transitional plume is seen to meander in plane (two-dimensionally) and with a wavelength which scales with the plume diameter. If excited externally by many such wavelengths, the plume has the property to select the natural wavelength proposed theoretically. The equivalence between the present transition mechanism and the transition predicted by the buckling theory is discussed.

**Abstract**

Hydrodynamic stability analysis of an inviscid wall jet shows that instability is possible above a characteristic disturbance wavelength which is proportional to the jet thickness. This scaling is the basis for an argument that transition occurs when the fluctuating time period of the unstable (inviscid) wall jet is of the same order as the viscous diffusion time normal to the jet. The transition must occur when the jet Reynolds number is of the order of $10^2$. Published observations of transition along a heated vertical wall are reviewed in order to test the validity of the proposed scaling argument. Specifically, numerous observations on buoyant jets near isothermal walls, near constant-heat-flux walls, and in enclosures with vertical isothermal walls are shown to support the validity of the transition mechanism proposed.
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