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STATISTICAL MODELS  
FOR  
ESTIMATING OVERHEAD COSTS

by

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Statistical Models for Estimating Overhead Costs

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## EXECUTIVE SUMMARY

This paper is part of an overall cost management and control project directed towards the major contractors of the Naval Air Systems Command (NAVAIR). The purpose of this report is to analyze overhead costs at two contractors in an attempt to determine, if possible, major causal factors which may affect overhead in these two firms. This report represents only part of the description and analysis of overhead costs at the two selected contractors.

The development and discussion of descriptive models for the two contractors is contained in a Naval Postgraduate School (NPS) Master's thesis which was completed in June 1983 (Stevens (1983)). In addition to developing descriptive models of the two contractors and presenting the data which are used in this report, Stevens applied the PIECOST model to the two contractors and compared those results with the results contained herein. His comparisons indicate that the data categorization and modeling procedures presented in this paper offer a better means for analyzing overhead costs than does PIECOST.

Other work which also forms part of this overall effort is an NPS Master's thesis dealing with the evaluation of compensation levels in the aerospace industry which also was completed in June 1983 (Becker (1983)). Becker did not use any of the proprietary data on which this report is based but, instead, used publicly available data to compare wage

levels among industries, concentrating on the aerospace industry in particular. Additionally, he provided a construct which may be used to pursue further investigation of wage levels in the aerospace industry.

The statistical models for analyzing overhead costs which are presented in this paper have yielded, in general, excellent structural results. Additionally, predictive analyses are undertaken of the best structural models. These predictive analyses show that reasonable predictions are possible for one contractor and that excellent predictions are available for the other. These results indicate that this entire procedure may yield fruitful results when applied to other contractors.

A comparison of the structural results for the two contractors shows that they have statistically indistinguishable variable total overhead costs when using direct personnel as the explanatory variable. This indicates that, despite the differences in overall structure of the two firms, there is a great similarity in the outcomes of the personnel assignment and costing processes.

It is seen that computer-related costs are not explainable using any of the variables available from this sample. Contrary to general perceptions, it is seen that these costs did not account for an increasing proportion of overhead costs over the period of this sample.

The results indicate that overhead, at least for this sample, tends to follow variations in output levels. This suggests that a production-function analysis of these and similar firms may offer an alternative to the more traditional approaches for modeling the cost attributes of these firms. The production-function approach offers the capability to directly estimate and predict all of the interesting cost attributes of the firm. More research is necessary in this area to evaluate the advantages and disadvantages of the alternative approaches for estimating and predicting costs of these and similar firms.

## INTRODUCTION

Current methods for estimating overhead costs generally rely upon the use of estimated overhead rates which are then applied to estimated labor hours or costs in each of several functional categories, such as engineering or manufacturing. Total overhead is then obtained by summing across all the functions. This approach is not entirely satisfactory since changes in operating rates cause changes in overhead rates which are reflected only after a significant lag. For firms in which output fluctuates significantly, this approach can result in poor estimates of overhead costs with corresponding difficulties for product pricing. In instances where the Federal government is the sole purchaser of the product, actual production costs (both direct and indirect) are important inputs into the price and quantity negotiation process. With aerospace contractor overhead comprising 30 to 50 percent of total costs, it is imperative that overhead costs be estimated with greater accuracy. It also may be important that the estimation procedure, in addition to having excellent explanatory and predictive capabilities, be relatively simple and (statistically) parsimonious. This may be necessary in order that the prediction process be routinized for use by persons with diverse backgrounds, e.g., statistical cost estimators and accounting clerks.

An alternative approach to estimating overhead costs is to estimate overhead costs directly and, hence, forego di-

rect reliance upon overhead rates. Two examples of this are provided by Martinson (1969) and Gross and Dienemann (1978). Martinson reclassified overhead costs from the usual functional categories into an input-oriented categorization and then regressed these new categories of overhead costs on various operating variables. Current conventional wisdom holds that the Martinson approach has been unsuccessful in almost all of its subsequent trials.

Gross and Dienemann estimate various categories of overhead costs using direct labor and material costs on a pooled time-series, cross-section sample of aerospace firms. The categories which they used were similar to those used by Martinson. Unfortunately, there is a major technical difficulty with the methodology of Gross and Dienemann. Almost all of their regression models use lagged values of the dependent variable as one of the explanatory variables, yet they report only the Durbin-Watson statistic as the measure of the degree of autocorrelation present in their models. It is well-known (see Judge, et. al. (1980) or Maddala (1977), for example) that the use of lagged values of the dependent variable as an explanatory variable results in an upward bias of the Durbin-Watson statistic (that is, the statistic does not find autocorrelation when it is actually present). Since it is also well-known that the presence of positive autocorrelation in a regression model biases downward the standard errors and biases upward the R-squared

statistic, most of the results of Gross and Dienemann have unknown reliability.

The procedures described below attempt to estimate overhead costs and various input categories of overhead costs from two aerospace contractors as functions of operating variables. This approach is similar to that of Martinson except that the input-oriented categories for overhead costs are different and the set of available operating variables is different. The focus here is on determining the effectiveness of a procedure which can be routinized and, hence, utilized by persons with a relatively low degree of statistical sophistication. Consequently, the number of explanatory variables is purposefully kept to a minimal level.

#### DATA SOURCES AND CHARACTERISTICS

Data were obtained from two major defense aircraft manufacturers with their full cooperation. The data, however, are proprietary and are not releaseable. To preserve the unidentifiability of the data and results, any specific reference to the two manufacturers will be in the form of contractor A and contractor B.

Prior to the obtaining of any data, a particular format for collection of overhead cost data was determined in order to assure uniformity of data categories across the different firms. The overhead cost data from the major manufacturing division of each of the two contractors were collected on a

quarterly basis for the years 1978 through 1982 within this defined format. Other additional data pertaining to production and operating characteristics of the divisions of the two manufacturers were also obtained.

The format for overhead costs has five major categories with several subcategories within each category. The first category, labor-related costs, has the subcategories of indirect salaries, fringe benefits and other compensation, other personnel costs, and all other labor-related costs. The next major category, facilities costs, has the subcategories of depreciation, repair and maintenance, leased equipment, utilities, and other facilities-related costs. The operations category has only one subcategory which contains telephone, telegraph, postage, fuels, and outside services. The mixed category, which contains costs having elements of both labor and facilities, has the subcategories of cafeteria, scrap sales, process tests, and independent research and development plus bid and proposal costs. The last major category consists of those costs which are external to the division but internal to the company. The two subcategories are computer services and other external costs. This latter subcategory is comprised of the net allocations both to and from other divisions and the corporate headquarters. Table 1 shows a detailed enumeration of this categorization.

TABLE 1

Cost Categories

- A. Labor-related
  - 1. Indirect Salaries
    - a. Cross-overs
    - b. Significant labor in repair and maintenance
  - 2. Fringe Benefits and Other Compensation  
(includes holiday, sick, and vacation leaves; severance pay; FICA and insurance contributions; savings plan; stock awards; etc.)
  - 3. Other Personnel Costs  
(includes tuition and training costs; suggestion awards; travel and relocation costs; etc.)
  - 4. All Other Labor-related Costs  
(includes temporary personnel; outside hires; etc.)
- B. Facilities
  - 1. Depreciation
  - 2. Repair and Maintenance
    - a. Plant rearrangement
    - b. Repair materials
  - 3. Leased Equipment
  - 4. Utilities  
(includes heating, lighting, etc.)
  - 5. Other Facilities-related Costs  
(includes taxes, insurance, etc.)
- C. Operations  
(includes telephone, telegraph, operating supplies, expendable equipment, postage, fuels, consulting services, protection services, etc.)
- D. Mixed
  - 1. Cafeteria
  - 2. Scrap Sales
  - 3. Process Tests
  - 4. Independent Research and Development and Bid and Proposal Costs (IR&D/B&P)
- E. External to Division - Internal to Corporation
  - 1. Computer Services
  - 2. Other Allocations

The categorization presented in Table 1 is similar to that utilized in the PIECCST model of Martinson (1969). An

attempt to replicate the PIECOST model on this data set indicated that both the data categorization and the modeling procedures generated results inferior to those presented here (Stevens (1983)).

Table 2 shows the indices utilized to convert the various categories of cost data from current to constant fourth quarter 1982 dollars. All indices came from Bureau of Labor Statistics and Bureau of Economic Analysis publications, with the explicit indices enumerated in the table. It is recognized that these indices, along with almost all others, are imperfect, but they were selected in an attempt to provide the best measures of inflation from among all readily-available indices relevant to these particular categories.

The operating data consist of such elements as direct labor hours, direct personnel, indirect personnel, direct material cost, direct labor cost, sales, and square footage of the plant. The direct and indirect personnel are equivalent headcounts, i.e. actual headcounts adjusted for amounts of overtime actually worked. The direct labor cost, direct material cost, and sales figures were converted to constant 1982 dollars by, respectively, the BLS SIC 3721 index, the Producer Price Index (PPI) for materials and components, and the Department of Labor index for transportation equipment published by the Bureau of Economic Analysis. Both sales and square footage eventually caused statistical modeling problems since sales were available only on an annual basis

TABLE 2

## Indices Used to Convert Current to Constant Dollars

Category	Index
A. Labor-related	BLS SIC 3721
B. Facilities	
1. Depreciation	GNPD Structures
2. Repair and Maint.	GNPD Services
3. Leased Equipment	GNPD Durable Equipment
4. Utilities	PCED Electricity and Gas
5. Other	PCED Services
C. Operations	GNPD Services
D. Mixed	GNPD Services
E. External	GNPD Services

BLS SIC 3721 is the Bureau of Labor Statistics price index of wages and fringe benefits for Standard Industrial Classification (SIC) code 3721, which is the aircraft industry.

GNPD is the Gross National Product Deflator for the indicated category and is published by the Bureau of Economic Analysis.

PCED is the Personal Consumption Expenditures Deflator for the indicated category and is also published by the Bureau of Economic Analysis.

from both contractors, while square footage was available on a quarterly basis for only four years from one contractor. Annual figures were used for each quarter in all cases where quarterly data were not available.

Tables 3 and 4 show the detailed format for the data collection process as well as the percentage breakdown of constant dollar costs for each of the five major categories by quarter for each of the contractors. In summarizing the percentage breakdown given in Table 3, it may be seen for contractor A that labor-related costs accounted for between 59 and 68 percent of all overhead costs over the twenty

TABLE 3

## Percentages of Total Overhead Costs by Category

## Contractor A

Year and Quarter	Category				
	Lab	Fac	Ops	Mix	Ext
781	.67	.09	.08	.05	.11
782	.67	.08	.09	.05	.11
783	.68	.09	.08	.05	.11
784	.65	.09	.08	.06	.12
791	.68	.09	.08	.06	.10
792	.66	.09	.08	.05	.12
793	.66	.09	.08	.06	.11
794	.65	.11	.09	.04	.11
801	.67	.10	.08	.06	.10
802	.65	.09	.10	.05	.11
803	.64	.11	.08	.05	.11
804	.65	.10	.10	.05	.11
811	.64	.10	.10	.06	.10
812	.62	.10	.10	.06	.12
813	.61	.10	.08	.06	.15
814	.65	.12	.10	.04	.08
821	.63	.10	.09	.07	.11
822	.64	.11	.09	.05	.12
823	.59	.10	.08	.12	.11
824	.67	.12	.09	.02	.10

Figures may not sum due to rounding.

quarters. Facilities costs ranged between 8 and 12 percent, operations costs varied from 8 to 10 percent, mixed costs covered 4 to 12 percent and external costs ranged from 8 to 15 percent. It should be noted that external costs remained between 10 and 12 percent of total overhead costs except for the third and fourth quarters of 1981 when they were 15 and 8 percent, respectively. Similarly, mixed costs were always 7 percent or below, except for the last three quarters of

1982 when they rose from 5 percent to 12 percent and then declined to 2 percent. There were no other apparent trends in the data of contractor A. One anomaly was the rather large shift in labor-related expenses in the final three quarters of 1982. The percentages of total overhead costs changed from 64 to 59 to 67 over these quarters.

TABLE 4

Percentages of Total Overhead Costs by Category

Contractor B

Year and Quarter	Category				
	Lab	Fac	Ops	Mix	Ext
781	.63	.12	.09	.08	.08
782	.61	.13	.09	.10	.08
783	.63	.12	.09	.08	.08
784	.66	.14	.09	.04	.07
791	.63	.11	.10	.08	.08
792	.63	.12	.09	.08	.08
793	.62	.13	.10	.07	.08
794	.62	.13	.11	.04	.09
801	.60	.11	.11	.09	.08
802	.60	.11	.12	.08	.09
803	.60	.13	.12	.07	.09
804	.60	.16	.13	.03	.09
811	.57	.13	.12	.08	.09
812	.58	.12	.12	.08	.09
813	.57	.14	.12	.07	.10
814	.57	.15	.12	.04	.11
821	.59	.12	.12	.07	.10
822	.57	.12	.12	.08	.11
823	.57	.13	.12	.08	.10
824	.55	.15	.13	.05	.11

Figures may not sum due to rounding.

For contractor B, Table 4 shows that labor-related costs ranged between 55 and 66 percent of total overhead costs over the entire sample. Facilities costs ranged between 11 and 16 percent, operations costs varied from 9 to 13 percent, mixed costs covered 4 to 10 percent, and external costs ranged from 7 to 11 percent of total overhead costs. The general trend for contractor B over the sample period has been to reduce the proportion going to labor-related costs and to increase the proportion going to external, primarily computer, costs. Although this trend is apparent, it was not found to be statistically significant. Using a test based on the number of runs of signs of first differences (Gibbons (1976)), the significance probability for the one-tailed test against the presence of a trend was 0.39.

#### MODELING QUARTERLY OVERHEAD COSTS

Sequential cost and operating data, as with most other time series data resulting from firm operations, can be expected to exhibit some level of autocorrelation. This is because firm expenditures from period to period are not totally random but tend to change relatively smoothly. Consequently, the error process of a time series-based statistical model of the costs of a firm does not exhibit the desired (normal) random structure but, instead, exhibits a structure in which errors in one period tend to be related to errors in other periods. Although the presence of some

form of autocorrelation in the residuals of a regression model does not create any problems in obtaining unbiased estimates of the regression coefficients themselves, it does result in biased estimates of the standard errors of the regression coefficients. Hence, any hypothesis tests which rely upon either the standard errors or functions of the standard errors may result in erroneous conclusions. This includes the standard t tests for the statistical significance of the difference of the regression coefficient value from zero. Consequently, it is desirable to obtain not only unbiased estimates of the regression coefficients but also unbiased estimates of their standard errors.

First order autocorrelation occurs when the errors of the model are related to the errors in the adjacent, prior periods. The errors are said to follow a first order autoregressive, or AR(1), process. Yearly cost and operating data tend to have errors which follow an AR(1) process. The use of quarterly data, however, may cause the autocorrelation to take on a special form. Instead of standard, first order autocorrelation, one would expect to encounter a special form of fourth order autocorrelation (Wallis 1972). Plots of the raw data confirmed that this form of autocorrelation is potentially present since, within each year, there was a clearly discernible tailing off of expenditures toward the final quarters. This pattern is a typical one for organizations which operate in an environment of known, binding

budgets with all funds available at the beginning of the budget period.

The general model utilized in this analysis is of the form

$$y_t = X_t \beta + \varepsilon_t, \quad (1)$$

$$\varepsilon_t = \rho \varepsilon_{t-4} + \eta_t, \quad t=1, \dots, T \quad (2)$$

where  $X_t$  is, in general, a  $T \times k$  matrix and  $\beta$  is a  $k \times 1$  vector. The  $y_t$  are overhead costs, either total or some category, and the  $X_t$  are operating variables, such as direct personnel. The error component of the model,  $\varepsilon_t$ , has the specific structure indicated by equation (2), where  $\eta_t$  has the zero-mean and constant-variance properties usually assumed for the error component of a regression model. Note that this model assumes a special form of the general fourth-order autoregressive (AR(4)) process. The general AR(4) process can be written as

$$\varepsilon_t = \rho_1 \varepsilon_{t-1} + \rho_2 \varepsilon_{t-2} + \rho_3 \varepsilon_{t-3} + \rho_4 \varepsilon_{t-4} + \eta_t. \quad (3)$$

The form of the AR(4) process used here assumes that the effects of the prior three quarters are negligible compared to the effect of the corresponding year-earlier quarter.

After selection of the independent variable(s) for a particular model, the general procedure was to first perform an

OLS regression on the untransformed data and test for the presence of the above form of the AR(4) process in the residuals. Following Wallis (1972), the test statistic can be written as

$$d_4 = \frac{\sum_{t=5}^T (e_t - e_{t-4})^2}{\sum_{t=1}^T e_t^2} \quad (4)$$

where

$$e_t = y_t - \hat{y}_t,$$

$$\hat{y}_t = X_t \hat{\beta}, \text{ and}$$

$\hat{\beta}$  is the estimator of  $\beta$  obtained from the OLS regression indicated by equation (1). This test and test statistic is an exact analog to the Durbin-Watson test and test statistic which are used for an AR(1) process. Tables of the upper and lower critical points of the distribution of  $d_4$  are given by Wallis for the above type of model. The critical points for a ninety-five percent confidence level with twenty observations and a single explanatory variable (plus a constant) are .924 (lower) and 1.102 (upper). Values of  $d_4$  which are larger than the upper critical point indicate the absence of this AR(4) process in the residuals. Values of  $d_4$  which are smaller than the lower critical point indicate

the presence of this AR(4) process in the residuals. Values of  $d_4$  which fall between the upper and lower critical points indicate inconclusive results.

If the test reveals the presence of this AR(4) process in the residuals, then the model must be reestimated using a transformed version of the original data. The data are transformed as

$$* \quad y_t = y_t (1 - \rho_4^2) \quad , \quad t=1,2,3,4 \quad , \quad \text{and} \quad (5)$$

$$* \quad y_t = y_t - \rho_4 y_{t-4} \quad , \quad t=5, \dots, T. \quad (6)$$

Each of these transformations requires an estimate of  $\rho_4$ . Although there are a number of ways to estimate this parameter, only the three most straightforward techniques were selected here because of the potential requirement that this entire procedure be replicable by persons with relatively low levels of statistical sophistication. Judge, et. al. (1980) derives these three estimators for the case of an AR(1) process.

The first estimate of  $\rho_4$  is

$$* \quad \rho_4 = \frac{\sum_{t=5}^T e_t e_{t-4}}{\sum_{t=1}^T e_t^2} \quad , \quad (7)$$

where  $e_t$  is the residual from the OLS regression in equation (1). This estimator is the sample correlation coefficient when the population autocorrelation process is given by equation (2). The second estimate of  $\rho_4$  is

$$\tilde{\rho}_4 = \frac{\sum_{t=5}^T e_t e_{t-4}}{\sum_{t=5}^T e_t^2} \quad (8)$$

This estimator is identical to that obtained as the estimator of  $\rho_4$  in the regression indicated by equation (2) and is bounded from below by  $\rho_4^*$ . The third estimate of  $\rho_4$  is

$$\hat{\rho}_4 = 1 - .5d_4 \quad (9)$$

This estimator is derived from equation (4) via equation (2) and asymptotic arguments. Note that the value of this estimator is easily obtainable from the value of the test statistic calculated from equation (4).

Each of these estimators was calculated for regressions involving the total and major categories of overhead costs. The calculation of  $\tilde{\rho}_4$  from a regression yielded an estimated standard error for this estimator. In all cases, the three estimators were well within two standard errors of each other using the estimated standard error of  $\tilde{\rho}_4$ . Because of its ease of calculation,  $\hat{\rho}_4$  is the recommended estimator and only its values are reported.

After each model was reestimated using the transformed data of equations (5) and (6) and the estimator of  $\hat{\rho}_4$  given by equation (9), the model was checked for the presence of first order autocorrelation using the Durbin-Watson statistic. In all cases of interest and in almost all other cases, this check indicated that there was no first order autocorrelation still present in the regression residuals after the removal of the special form of fourth order autocorrelation presented above.

### STRUCTURAL ANALYSIS

The procedures outlined in the previous section will be illustrated in detail using total overhead costs from each of the two contractors. All statistical results will be reported to three significant digits. Following these illustrations, the final results for the major categories for each contractor will be presented and discussed.

All of the results reported here utilize total direct personnel as the explanatory variable. Other explanatory variables such as direct labor hours and direct labor cost also produced reasonable results. In a few cases, those results were marginally superior to the results reported here, but direct personnel outperformed the others over the entire range of cost categories.

There exists a perception that, both in general and in the two cases considered here, computer costs are a growing

proportion of total overhead costs. This perception exists despite the statistical result reported above which showed that there was no trend over this particular five year period for these two firms. In an effort to verify this perception, several attempts were made to model computer costs. Computer costs were treated as functions of, alternatively, total personnel, direct labor hours, direct labor costs, and several other operating variables. All such attempts yielded very poor statistical results. Additionally, rates of change of computer costs were treated as functions of several operating variables as well as rates of change of those variables. Again, all such attempts yielded very poor statistical results.

Table 5 presents the results of these procedures applied to the regression of total overhead costs for contractor A (TOTOHA) upon total direct personnel for contractor A (DIRPERA). The results of the regression on the original, untransformed data indicate very poor results. The adjusted R-squared is near zero and the F-statistic is far below the five percent critical value of 4.41, which indicates that the regression equation is explaining very little beyond the mean of the dependent variable. Additionally, the standard errors of the two coefficients are relatively large in comparison to the coefficient estimates. The value of the Durbin-Watson statistic indicates that no clear conclusion may be drawn concerning the presence of first-order autocorrela-

TABLE 5

Model: TOTOHA = a + b DIRPERA

## Untransformed Data

Standard Error of the Regression:	15000.
Adjusted R-Squared:	.0122
F-Statistic:	.770
Durbin-Watson Statistic:	1.26
Estimate of a:	156000.
Standard Error:	75200.
Estimate of b:	5.30
Standard Error:	6.04

Estimate of $d_4$ :	.431
Estimate of $\rho_4^*$ :	.538
Estimate of $\tilde{\rho}_4$ :	.777
Standard Error:	.196
Estimate of $\hat{\rho}_4$ :	.784

## Transformed Data

Standard Error of the Regression:	8030.
Adjusted R-Squared:	.941
F-Statistic:	307.
Durbin-Watson Statistic:	1.92
Estimate of a:	18000.
Standard Error:	3420.
Estimate of b:	13.8
Standard Error:	.787

tion since it falls between the upper and lower five percent critical points of 1.41 and 1.20.

Upon testing this model for the presence of the special form of fourth-order autocorrelation discussed above, the null hypothesis of no fourth-order autocorrelation is clearly rejected since the calculated value of  $d_4$  is below the upper and lower five percent significance points of 1.102

and 0.924 (see Wallis (1972)). The three alternative estimates of  $\rho_4$  are calculated and can be seen to be statistically close. The data were then transformed as described above and the model was reestimated.

The regression results for the transformed data show that there is indeed a great deal of information contained in this model of overhead costs. The R-squared value is approaching unity, and the F-statistic indicates that this model contains significantly more information than the mean of total overhead costs. The standard errors of the estimated coefficients are relatively small in comparison to the coefficients and the Durbin-Watson statistic indicates that no first-order autocorrelation remains in this model. In summary, the regression model using transformed data yields excellent results, but the adjustment for this special form of autocorrelation clearly is necessary in order to obtain these results.

Table 6 presents the results of the procedures described in the previous section when they are applied to the regression of total overhead costs for contractor B (TOTOHB) upon total direct personnel for contractor B (DIRPERB). The results are very similar to those presented in Table 5 for contractor A. Very poor results were obtained using untransformed data, the presence of this special form of fourth-order autocorrelation was indicated clearly by the test, and excellent results were obtained using transformed data.

TABLE 6

Model:  $TOTCHB = a + b \text{ DIRPERB}$ 

	Untransformed Data
Standard Error of the Regression:	9311.
Adjusted R-Squared:	.444
F-Statistic:	16.2
Durbin-Watson Statistic:	2.06
Estimate of a:	-22700.
Standard Error:	55800.
Estimate of b:	15.7
Standard Error:	3.90
Estimate of $d_4$ :	.326
Estimate of $\rho_4$ :	.621
Estimate of $\tilde{\rho}_4$ :	.706
Standard Error:	.138
Estimate of $\hat{\rho}_4$ :	.837
	Transformed Data
Standard Error of the Regression:	4920.
Adjusted R-Squared:	.966
F-Statistic:	546.
Durbin-Watson Statistic:	1.44
Estimate of a:	5300.
Standard Error:	2270.
Estimate of b:	13.0
Standard Error:	.556

Table 7 presents the results of the modeling of all the major categories of overhead costs for contractor A. The first model in Table 7 reproduces the results using transformed data from Table 5. This model may be written as

$$TOTOHA = 18000 + 13.8 \text{ DIRPERA},$$

and, since all costs are measured in thousands of dollars, it may be interpreted as indicating that there is a fixed

component of total overhead costs (when a function of direct personnel) of approximately \$18 million, with each additional direct person costing about \$13,800 in total overhead costs.

The second model in Table 7 gives the results using transformed data of the regression of labor-related overhead costs (LABORA) on direct personnel. This model explains labor-related overhead costs at least as well as the previous model explains total overhead costs. It indicates that there is a fixed component of labor-related overhead costs of approximately \$9.39 million and that each additional direct person costs about \$9,590 in labor-related overhead costs.

The third model of Table 7 gives the results of the regression of facilities-related overhead costs (FACA) on total plant-wide square feet (SQFO TA). The test for fourth-order autocorrelation indicated that it was not present in this model; the reported results are based on untransformed data. This model does not explain the data as well as the first two models. A reasonable explanation may be that square footage was not available by quarters for the first year of the sample (the same value was used for each of the first four quarters), so that the variation in facilities costs was necessarily unexplained in those quarters. The R-squared value states that the model is able to explain only about 80 percent of the total variation of facilities

TABLE 7

## Regressions for Contractor A

Model:  $TOTOHA = a + b \text{ DIRPERA}$

Standard Error of the Regression:	8030.
Adjusted R-Squared:	.941
F-Statistic:	307.
Durbin-Watson Statistic:	1.92
Estimate of a:	18000.
Standard Error:	3420.
Estimate of b:	13.8
Standard Error:	.787

Model:  $LABORA = a + b \text{ DIRPERA}$

Standard Error of the Regression:	5640.
Adjusted R-Squared:	.942
F-Statistic:	311.
Durbin-Watson Statistic:	1.54
Estimate of a:	9390.
Standard Error:	2640.
Estimate of b:	9.59
Standard Error:	.544

Model:  $FACA = a + b \text{ SQFOTA}$

Standard Error of the Regression:	1220.
Adjusted R-Squared:	.803
F-Statistic:	78.3
Durbin-Watson Statistic:	2.24
Estimate of a:	-9260.
Standard Error:	3500.
Estimate of b:	6.84
Standard Error:	.774

Model:  $OPSA = a + b \text{ DIRPERA}$

Standard Error of the Regression:	1530.
Adjusted R-Squared:	.645
F-Statistic:	35.4
Durbin-Watson Statistic:	2.74
Estimate of a:	5110.
Standard Error:	855.
Estimate of b:	.879
Standard Error:	.148

Model:  $MIXEDA = a + b \text{ DIRPERA}$

Standard Error of the Regression:	4080.
Adjusted R-Squared:	.107
F-Statistic:	2.17
Durbin-Watson Statistic:	3.00
Estimate of a:	2730.
Standard Error:	2260.
Estimate of b:	.580
Standard Error:	.394

Table 7 (continued)

Model: $EXTDIVA = a + b DIRPERA$	
Standard Error of the Regression:	3420.
Adjusted R-Squared:	.0666
F-Statistic:	1.28
Durbin-Watson Statistic:	2.16
Estimate of a:	4970.
Standard Error:	17100.
Estimate of b:	1.56
Standard Error:	1.38

costs. The model indicates that there is a negative fixed cost component for facilities costs as a function of square footage. This is an implausible result but will be dealt with more fully below. However, it also indicates that each additional square foot results in about \$6.84 of additional facilities costs.

The fourth model of Table 7 presents the results of the regression of operations-related overhead costs (OPSA) on direct personnel. The reported results are based upon transformed data since the presence of fourth-order autocorrelation was confirmed. This model is able to explain only about 64 percent of the total variation in operations costs, and the Durbin-Watson statistic falls in the inconclusive region. However, the coefficient estimates are and their standard errors are quite good and indicate that there is a fixed component of approximately \$5.11 million and a variable component of approximately \$0.88 for operations-related overhead costs as a function of direct personnel.

The final two models of mixed (MIXEDA) and external (EXTDIVA) overhead costs in Table 7 show very poor fits to the data. In addition to very low R-squares and very low F-statistics, only very low confidence may be placed in the coefficient estimates. Hence, there is very little information contained in these last two models. It should be noted that the model for mixed overhead costs contained a significant amount of fourth-order autocorrelation while the model for external overhead costs did not. The results reported are those using transformed data for the former model and untransformed data for the latter. These results for external costs are not surprising since these are costs which have been allocated both to and from these units. Therefore, these external costs would be correlated with other variables only through the allocation bases.

Direct personnel was not the only independent variable used in attempting to model the various categories of overhead costs. Both direct labor hours (DLHOURA) and direct labor costs (DLCOSTA) proved to yield similar structural results to those of direct personnel for most of the overhead categories. The following are the equations resulting from using total overhead costs and labor-related overhead costs as dependent variables:

$$\text{TOTOHA} = 22300 + 0.0282 \text{ DLHOURA},$$

$$\text{TOTOHA} = 23400 + 1.92 \text{ DLCOSTA},$$

$$\text{LABORA} = 12400 + 0.0196 \text{ DLHOURA}, \text{ and}$$

$$\text{LABORA} = 12700 + 1.34 \text{ DLCOSTA}.$$

The equation fits were very similar to those in Table 7 for the corresponding models. It should be noted that, as expected, direct labor hours and direct labor costs are highly correlated.

Table 8 presents the results of the modeling of all the major categories of overhead costs for contractor B. These results are similar to those in Table 7. In Table 8, there were three cases in which fourth-order autocorrelation was significantly present and which required transformation of the data and reestimation. These cases were the models for total overhead costs, labor-related overhead costs, and mixed overhead costs. The results reported for the remaining three models are based upon the use of untransformed data. The statistical results for all six of these models are, at the very least, acceptable and, in general, are better than those in Table 7 for contractor A.

There are two potential difficulties in the use of several of these models. First, there are negative intercepts for five of the models. These negative intercepts imply the existence of negative fixed costs for those models. As discussed above, this is implausible but not impossible. It should be noted that the models are being fit to data which are very far from the origin. Therefore, these reported results are fully valid for the relevant range of the data. Given that five of the six models yield this result, this author is inclined to believe that this is the proper interpretation of these results.

TABLE 8

## Regressions for Contractor B

Model:  $TOTCHB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	4920.
Adjusted R-Squared:	.966
F-Statistic:	546.
Durbin-Watson Statistic:	1.44
Estimate of a:	5300.
Standard Error:	2270.
Estimate of b:	13.0
Standard Error:	.556

Model:  $LABORB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	4190.
Adjusted R-Squared:	.956
F-Statistic:	413.
Durbin-Watson Statistic:	1.80
Estimate of a:	-3560.
Standard Error:	2630.
Estimate of b:	8.99
Standard Error:	.442

Model:  $FACB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	1870.
Adjusted R-Squared:	.544
F-Statistic:	23.7
Durbin-Watson Statistic:	1.58
Estimate of a:	-28600.
Standard Error:	11100.
Estimate of b:	3.81
Standard Error:	.782

Model:  $OPSB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	1730.
Adjusted R-Squared:	.801
F-Statistic:	77.5
Durbin-Watson Statistic:	1.49
Estimate of a:	-69100.
Standard Error:	10400.
Estimate of b:	6.39
Standard Error:	.726

Model:  $MIXEDB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	1790.
Adjusted R-Squared:	.467
F-Statistic:	17.6
Durbin-Watson Statistic:	1.80
Estimate of a:	-4.60
Standard Error:	697.
Estimate of b:	1.00
Standard Error:	.240

Table 8 (continued)

Model: $EXTDIVB = a + b DIRPERB$	
Standard Error of the Regression:	1950.
Adjusted R-Squared:	.661
F-Statistic:	38.1
Durbin-Watson Statistic:	1.81
Estimate of a:	-53800.
Standard Error:	11700.
Estimate of b:	5.05
Standard Error:	.819

A second difficulty is that square footage was available for contractor B only on an annual basis. Use of the annual numbers on a quarterly basis produced an equation with an R-squared of .43, with other results similar to the model reported in the table. This indicates that square footage on a quarterly basis may provide a significantly superior model to the one reported.

As in the case of contractor A, both direct labor hours (DLHOURB) and direct labor cost (DLCOSTB) also were used as independent variables, and they yielded similar structural results to those in Table 8. The following equations resulted from use of the two major overhead cost categories:

$$TOTOHB = 20100 + 0.0268 DLHOURB,$$

$$TOTOHB = 16000 + 1.75 DLCOSTB,$$

$$LABORB = -2130 + 0.0196 DLHOURB, \text{ and}$$

$$LABORB = 5350 + 1.22 DLCOSTB.$$

The fits were very similar to the corresponding models in Table 8 .

These structural results may be used to compare overhead costs experienced by the two contractors. This comparison will be made only for total overhead costs and labor-related overhead costs since these represent the four models which yielded conclusively good results. The two models for total overhead costs are

$$\text{TOTOHA} = 18000 + 13.8 \text{ DIRPERA} \text{ and}$$

$$\text{TOTOHB} = 5300 + 13.0 \text{ DIRPERB.}$$

It may be seen that the regression for contractor B lies everywhere below the regression for contractor A; not only does contractor B have a (significantly) lower fixed cost but also it has a (not significantly) lower variable cost.

The two models for labor-related overhead costs are

$$\text{LABORA} = 9390 + 9.59 \text{ DIRPERA} \text{ and}$$

$$\text{LABORB} = -3560 + 8.99 \text{ DIRPERB.}$$

A potential difficulty is that the intercept for contractor B is negative. However, in accordance with the discussion above, this is not a serious problem given the range of applicability of the model. Although this intercept for contractor B is not significantly different from zero, it is significantly lower than the intercept for contractor A. It is true here also that the regression for contractor B lies everywhere below the regression for contractor A.

The reader should be aware that these comparisons imply only that, with the same number of direct personnel, contractor B experiences lower total overhead costs and lower labor-related overhead costs than contractor A. These comparisons do not imply that contractor B has lower overhead costs in the two categories than contractor A, regardless of the circumstances. This observed difference is at least partially due to the different personnel classification systems used by the two contractors.

### PREDICTIVE ANALYSIS

Since the results shown in Tables 7 and 8 using total overhead costs and labor-related overhead costs for both contractors were of such high quality, it was determined that a predictive test of these regressions for each contractor should be undertaken. Recall that labor-related costs account for almost two-thirds of all overhead costs. The general procedure was to fit the regression model to a sample of only the first four years (sixteen observations), predict the last year (four observations), and compare the predicted to the actual values of overhead cost.

The regression model using transformed data was estimated exactly as above except that only the first sixteen observations were used. Based upon these estimated results, the last four observations were predicted via the equation

$$y_t = \hat{\rho}_4 y_{t-4} + (X_t - \hat{\rho}_4 X_{t-4}) \hat{\beta}, \quad t=17, \dots, 20, \quad (10)$$

where  $X$  is defined as in equation (1) and  $\hat{\beta}$  and  $\hat{\rho}_4$  are the values obtained from the estimation based on the first sixteen observations. These predicted values of overhead costs were then compared to the observed values of overhead costs using (1) a Pearson correlation coefficient, (2) the root mean squared forecast error, (3) the mean absolute percentage error, and (4) Theil's decomposition of the forecast error.

Table 9 presents the results of fitting the above models for total overhead costs and labor-related costs of contractor A using the procedure just described. When compared to the results based upon all twenty observations as shown in Table 7, the use of the first sixteen observations results in only a very slight degradation of the models' power to explain the data. There have been changes in the estimates of the coefficients, but these changes have not been significant. (The five percent critical value of the F-statistic with sixteen observations and the given model is 4.60, and the upper and lower five percent critical points for the corresponding Durbin-Watson statistic are 1.37 and 1.10.) Based upon this estimation, the last four values of the dependent variable are then predicted via equation (10). This prediction technique necessarily requires knowledge of the independent variable.

TABLE 9

## Estimation and Prediction for Contractor A

Model: TOTOHA = a + b DIRPERA

Standard Error of the Regression:	8100.
Adjusted R-Squared:	.951
F-Statistic:	294.
Durbin-Watson Statistic:	1.90
Estimate of a:	24400.
Standard Error:	4820.
Estimate of b:	13.7
Standard Error:	.798

## Prediction Results

Correlation coefficient between actual and predicted values	.660
Root mean squared error divided by the mean of the actual values	.0717
Mean absolute percentage error (in percent)	6.12
Theil's decomposition of forecast error	
Fraction due to bias	.759
Fraction due to regression	.0336
Fraction due to residual variance	.207

Model: LABORA = a + b DIRPERA

Standard Error of the Regression:	6080.
Adjusted R-Squared:	.944
F-Statistic:	254.
Durbin-Watson Statistic:	1.70
Estimate of a:	11600.
Standard Error:	3620.
Estimate of b:	9.55
Standard Error:	.599

## Prediction Results

Correlation coefficient between actual and predicted values	.355
Root mean squared error divided by the mean of the actual values	.0506
Mean absolute percentage error (in percent)	4.44
Theil's decomposition of forecast error	
Fraction due to bias	.788
Fraction due to regression	.0129
Fraction due to residual variance	.199

The four predicted values are then compared with the actual values. The Pearson correlation coefficient is a measure of the linear association between the actual and predicted values of overhead costs. The value of .66 indicates that there is a reasonable tendency for the predicted values of total overhead costs to follow closely the actual values. The value of .355 shows that there is much less of a tendency for the predicted values of labor-related overhead costs to follow closely the actual values.

A measure of the size of the forecast errors is given by the ratio of the root mean squared error to the mean of the four actual values to be predicted. In the case of total overhead costs, the root mean squared error is just over 7 percent of this mean and shows that the forecast errors are small relative to the actual values. This measure is even smaller, 5 percent, for labor-related overhead costs. A second measure of the size of the forecast errors is the mean absolute percentage error. This measure for both models indicates that the forecast errors are small relative to the actual, observed values.

In a plot of the predicted values against the actual values, the spread of values around the line of perfect forecasts (where the predicted values equal the actual values) yields information on the possible inadequacies of the forecasts. Theil's decomposition allows this information to be broken up into three elements and shows the proportions of

the forecast error which are due to (1) bias, (2) regression, and (3) residual variance. The bias proportion indicates the extent to which the average predicted value is different from the average actual value, the regression proportion indicates the extent to which a regression of the actual values on the predicted values follows the line of perfect forecasts, and the residual variance proportion is the remainder of the forecast error. As long as the root mean squared error is low, small proportions due to bias and regression are desirable. A detailed discussion of the Theil decomposition is available in Maddala (1977). The results for both total overhead costs and labor-related overhead costs indicate that most of the error is due to bias. This is not very desirable, but it should be noted that the forecasts have completed only one cycle of the underlying AR(4) process so these decomposition results are not indicative of any long term results.

Table 10 presents the results of fitting the above models for total overhead costs and labor-related costs of contractor B using the estimation and out-of-sample prediction procedure described above. Again, there is only a slight degradation in the models' power to explain the data when using only sixteen observations instead of twenty. No significant changes occurred in the coefficient estimates.

The correlation coefficients between actual and predicted values for both models are exceedingly close to unity. The

TABLE 10

## Estimation and Prediction for Contractor B

Model:  $TOTOHB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	5190.
Adjusted R-Squared:	.967
F-Statistic:	441.
Durbin-Watson Statistic:	1.36
Estimate of a:	4570.
Standard Error:	2610.
Estimate of b:	13.1
Standard Error:	.623

## Prediction Results

Correlation coefficient between actual and predicted values	.934
Root mean squared error divided by the mean of the actual values	.0269
Mean absolute percentage error (in percent)	2.38
Theil's decomposition of forecast error	
Fraction due to bias	.774
Fraction due to regression	.118
Fraction due to residual variance	.107

Model:  $LAEORB = a + b \text{ DIRPERB}$

Standard Error of the Regression:	4570.
Adjusted R-Squared:	.956
F-Statistic:	329.
Durbin-Watson Statistic:	1.71
Estimate of a:	-4130.
Standard Error:	3020.
Estimate of b:	9.05
Standard Error:	.499

## Prediction Results

Correlation coefficient between actual and predicted values	.962
Root mean squared error divided by the mean of the actual values	.0175
Mean absolute percentage error (in percent)	1.35
Theil's decomposition of forecast error	
Fraction due to bias	.324
Fraction due to regression	.214
Fraction due to residual variance	.462

ratios of root mean squared errors to the means of the actual values are very small, as are the mean absolute percentage errors. While the Theil decomposition for total overhead costs is similar to those in Table 9 above, this decomposition for labor-related overhead costs is clearly moving in the desired direction. Hence, the predictive capability for these two categories of overhead costs for contractor B appears to be quite good.

This prediction procedure requires estimates of direct personnel in order to generate the estimates of overhead costs. There are at least two alternative ways of generating these estimates of direct personnel. A first approach is to use the estimates of direct labor hours which are currently used by contractor and government estimators to produce estimates of all direct costs. Using data concerning amount of overtime worked, these estimates of direct labor hours can then be converted into estimates of direct personnel. Alternatively, the above procedure could be derived using direct labor hours, also a good predictor of total and labor-related overhead costs, as the explanatory variable. The estimates of direct labor hours could then be directly input into the prediction process.

A second general approach to estimating direct personnel is to use some other even more readily-available variable to attempt to predict direct personnel. The most logical and most available is units of output. In the case of one of

the contractors, the most straightforward approach of regressing direct personnel on units of output of type 1, units of output of type 2, etc., produced a surprisingly high R-squared statistic of .84. In general, however, some assumptions about the production technology will be necessary in order to utilize this approach. Also, this approach requires a larger sample size than that utilized in the above approach since it estimates a larger number of coefficients. Work is continuing in this area.

#### **SUMMARY**

The statistical models for analyzing overhead costs which have been presented in this paper have yielded, in general, excellent structural results. Additionally, predictive analyses were undertaken of the best structural models. These predictive analyses showed that reasonable predictions are possible for one contractor and that excellent predictions are available for the other. These results indicate that this entire procedure may yield fruitful results when applied to other contractors.

A comparison of the structural results for the two contractors showed that they have statistically indistinguishable variable total overhead costs when using direct personnel as the explanatory variable. This indicates that, despite the differences in overall structure of the two firms, there is a great similarity in the outcomes of the personnel assignment and costing processes.

It was seen that computer-related costs are not explainable using any of the variables available from this sample. Contrary to general perceptions, it was seen that these costs did not account for an increasing proportion of overhead costs over the period of this sample.

It should be noted that labor-related costs for these two firms accounted for the majority of total overhead costs. In such relatively labor-intensive operations, it is natural that personnel-related variables should be a strong determinant of total overhead costs as well as labor-related overhead costs. Therefore, more extensive use of capital, especially automated machinery, than was observed in this sample may result in personnel-related variables being less powerful determinants of total overhead costs than occurred here.

The above results indicate that overhead, at least for this sample, tends to follow variations in output levels. This suggests that a production-function analysis of these and similar firms may offer an alternative to the more traditional approaches for modeling the cost attributes of these firms. The production-function approach offers the capability to directly estimate and predict all of the interesting cost attributes of the firm. More research is necessary in this area to evaluate the advantages and disadvantages of the alternative approaches for estimating and predicting costs of these and similar firms.

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