RAIL VEHICLE PERFORMANCE MONITOR FOR ACCELERATION
VELOCITY AND DISTANCE (U)
NATIONAL RESEARCH COUNCIL
OF CANADA OTTAWA (ONTARIO) DIV OF M. C M ZWARTS
JUN 83 DME-MI-840 NRC-22447
F/G 14/2
RAIL VEHICLE PERFORMANCE MONITOR FOR ACCELERATION, VELOCITY AND DISTANCE

by

C. M. G. Zwarts

Division of Mechanical Engineering

OTTAWA
JUNE 1983

MECHANICAL ENGINEERING REPORT
MI-840
NRC NO. 22447
RAIL VEHICLE PERFORMANCE MONITOR FOR ACCELERATION, VELOCITY AND DISTANCE

CONTRÔLEUR D'ACCÉLÉRATION, DE VITESSE ET DE DISTANCE DES VÉHICULES SUR RAILS

by/par

C.M.G. Zwarts

Accession For

NTIS GRA&I
DTIC TAB

Unannounced
Justification

By
Distribution/

Availability Codes

Avnil and/or
Dist
Special

A-1

C.A.M. Smith Head/Chef
Railway Laboratory/Laboratoire ferroviaire

E.H. Dudgeon
Director/Directeur
SUMMARY

A measuring technique for velocity, acceleration and distance of rail vehicles is discussed. Velocity and distance are derived directly from an optical digitally encoded sensor. A low noise differentiation method allows very small accelerations of rail vehicles to be obtained from the velocity signal. Description and design details are given of the major circuit functions.

RÉSUMÉ

Une technique de mesure de la vitesse, de l'accélération et de la distance des véhicules sur rails est présentée. La vitesse et la distance sont calculées directement par un capteur optique codé numériquement. Une méthode de différentiation à faible bruit permet de déceler les très petites accélérations à partir du signal de vitesse. Sont fournis une description des principaux circuits et des détails sur leur conception.
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY</td>
<td>(iii)</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.0 SPECIAL CIRCUIT FUNCTIONS</td>
<td>1</td>
</tr>
<tr>
<td>2.1 Phase-Locked Loop Frequency Multiplier</td>
<td>2</td>
</tr>
<tr>
<td>2.2 Frequency Doubler</td>
<td>2</td>
</tr>
<tr>
<td>2.3 Multiplexed Sample-Hold Circuit</td>
<td>2</td>
</tr>
<tr>
<td>2.4 Averaging Sample-Hold Circuit</td>
<td>3</td>
</tr>
<tr>
<td>2.5 Non-Recursive Low-Pass Filter</td>
<td>4</td>
</tr>
<tr>
<td>3.0 VELOCITY</td>
<td>4</td>
</tr>
<tr>
<td>4.0 ACCELERATION</td>
<td>6</td>
</tr>
<tr>
<td>5.0 DISTANCE</td>
<td>7</td>
</tr>
<tr>
<td>6.0 CALIBRATION</td>
<td>8</td>
</tr>
<tr>
<td>7.0 ACCURACY OF MEASUREMENTS</td>
<td>8</td>
</tr>
<tr>
<td>8.0 CONCLUSIONS</td>
<td>8</td>
</tr>
</tbody>
</table>

### ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>11</td>
<td>17</td>
</tr>
</tbody>
</table>

(iv)
<table>
<thead>
<tr>
<th>Figure</th>
<th>Illustration</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Non-Recursive Low-Pass Filter</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>Non-Recursive Low-Pass Filter — Response to Step Input</td>
<td>18</td>
</tr>
<tr>
<td>14</td>
<td>Non-Recursive Low-Pass Filter — Response to Pulse Input</td>
<td>18</td>
</tr>
<tr>
<td>15</td>
<td>Block Diagram of Velocity Circuit</td>
<td>19</td>
</tr>
<tr>
<td>16</td>
<td>Input Signal of Averaging Sample-Hold Circuit</td>
<td>19</td>
</tr>
<tr>
<td>17</td>
<td>Signal Waveforms of Velocity Circuit</td>
<td>20</td>
</tr>
<tr>
<td>18</td>
<td>Differentiation Techniques</td>
<td>21</td>
</tr>
<tr>
<td>19</td>
<td>Block Diagram of Acceleration Circuit</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>Average Difference Quotient Circuit</td>
<td>22</td>
</tr>
<tr>
<td>21</td>
<td>Block Diagram of Distance Circuit</td>
<td>22</td>
</tr>
</tbody>
</table>
RAIL VEHICLE PERFORMANCE MONITOR FOR ACCELERATION, VELOCITY AND DISTANCE

1.0 INTRODUCTION

Whereas the instrumentation discussed in this report has been designed for a specific series of tests to measure brake performance on railway vehicles, it could readily have a more general application such as aiding the locomotive engineer to obtain a more efficient train operation.

Only one sensor is used to measure displacement, velocity and acceleration. It consists of an optical transducer mounted on the end of a car axle, producing 240 pulses per revolution of the wheel. The three parameters therefore are directly related to the diameter of the wheel.

With the sensor producing a pulse train having a frequency proportional to speed, the basic signal processing required to obtain those three parameters is respectively:

- for distance: "integration"
  Each time interval between two pulses represents a certain distance interval; hence, accumulating (=integration) pulses gives the distance travelled.

- for velocity: "frequency to voltage conversion"
  The rate of pulses per second and therefore the frequency are directly proportional to speed. The signal processing required is a linear conversion from frequency to voltage.

- for acceleration: "differentiation"
  The frequency being proportional to speed, some form of differentiation is required to derive acceleration.

Distance and velocity can be obtained using well known circuit techniques. The measurement of extremely low acceleration levels, however, is difficult since the acceleration is obtained by differentiation of the velocity output signal. This tends to sharply increase the inherent noise and any ripple that may be left in the velocity output signal.

The design philosophy for an accurate acceleration measurement is thus necessarily:

1) The signal processing to obtain the velocity should be specifically designed to obtain minimum ripple in the velocity output signal.

2) The differentiation process should be optimized for minimum noise gain beyond the signal frequency spectrum.

This report discusses the special electronic instrumentation developed to satisfy these design criteria. The method described to obtain low noise differentiation, however, is not restricted to this particular application but is of general interest wherever low noise differentiation is required.

2.0 SPECIAL CIRCUIT FUNCTIONS

Several essential circuit functions are discussed somewhat more in detail to allow the remainder of the report to be more readily understood.
2.1 Phase-Locked Loop Frequency Multiplier

The velocity is obtained from a digital signal (pulse train from optical sensor); hence, a certain quantization is implied, which directly affects the ripple in the velocity output signal.

A substantial reduction of the quantization noise can be obtained using a phase-locked loop as a frequency multiplier; the quantization noise being inversely proportional to frequency.

The block diagram is shown in Figure 1. The phase-locked loop should automatically lock in for any value of the input frequency. This requires a digital phase/frequency discriminator in the loop. Its transfer function, for phase and frequency, is shown in Figure 2.

The variations in speed, and hence in input frequency, of railway vehicles are small. The closed loop bandwidth is designed to be small enough to obtain some filtering of possible input frequency jitter but wide enough to allow “quasi” instantaneous response to the slow input frequency variations.

2.2 Frequency Doubler

A further up-conversion of the frequency is obtained by multiplying the output frequency of the frequency multiplies by a factor of two, using both the positive and the negative transitions to produce unipolar pulses. The frequency doubler circuit is shown in Figure 3.

2.3 Multiplexed Sample-Hold Circuit

The sample-hold circuit is a key element in the velocity and acceleration circuits. Moreover, it is used in the transversal low-pass filter.

The desirable features are accurate acquisition and low droop during the hold time.

Conventional sample-hold circuits sample the input signal during a small aperture time interval. The signal, however, is changing during this interval and hence, for accurate acquisition, one has to keep the aperture to a minimum, see Figure 4.

The sample-hold circuit developed, uses a rather different concept. The basic circuit configuration, shown in Figure 5, consists of a bridge arrangement of four MOSFET transistors and two capacitors. The bridge output is unloaded by a very high impedance MOSFET-input unity gain operational amplifier.

The switches $S_1$, $S_3$ and $S_2$, $S_4$ are operated by complementary clock signals, hence whilst one capacitor is in the hold-mode, the other capacitor is simply following the input signal and vice versa. Hold is initiated by a “breaking” contact, hence assuring very accurate acquisition.

There is a unity-gain positive feedback loop from the output to the substrates of the MOSFETs. This loop keeps the voltage drop over the inherent substrate-drain junctions virtually zero, eliminating the major leakage path of the MOSFET transistors. The circuit configuration has an exceptionally low droop, typically 3μV/sec for slowly varying input signals.

The clock signal, applied to the gates, causes some charge transfer, via the inherent junction capacitances of the MOSFETs, to the “hold” capacitor. These junction capacitances are not necessarily equal and hence the clock signal, via the small capacitive imbalance of the bridge, “leaks” through to the output as a small amplitude square wave signal in phase with the clock signal. It can be virtually eliminated, however, with a small valued potentiometer in series with the input. The operation of the sample-hold circuit is illustrated in Figure 6 for an arbitrary input signal.
2.4 Averaging Sample-Hold Circuit

The acceleration is derived by obtaining an "average-difference-quotient" of the velocity signal. This implies that the velocity signal is averaged over successive time intervals and that the difference is taken between successive average values. With the magnitude of the accelerations being very small, this should be done very accurately.

The circuit developed to perform these operations (averaging and holding) with high precision is shown in Figure 7. It consists of the multiplexed sample-hold circuit put inside the feedback loop of a precision integrator. Figure 8 shows the same circuit where the multiplexed sample-hold circuit is reduced to its simplest expression to more readily show the operation of the circuit. Notice that there is no direct path between the output of the integrator and the buffer output amplifier of the sample-hold circuit. The output signal is therefore constant during each averaging interval.

The relationship between input and output voltage can be derived as:

\[ E_o \left\{ (n+1)r, nr \right\} = E_o \left\{ nr, (n-1)r \right\} - \frac{1}{RfCf} \int_{(n-1)r}^{nr} E_o \left\{ nr, (n-1)r \right\} dt \]

\[ - \frac{1}{RfCf} \int_{(n-1)r}^{nr} E_i(t) dt \]  

where \( r \) is the averaging time interval and \( E_o \left\{ (n+1)r, nr \right\} \) the output voltage, constant over the time interval \( r = (n+1)r - nr \).

If one selects \( r = RfCf \) then:

\[ \frac{1}{RfCf} \int_{(n-1)r}^{nr} E_o \left\{ nr, (n-1)r \right\} dt = E_o \left\{ nr, (n-1)r \right\} \]

Substituting Equation (2) into Equation (1) gives:

\[ E_o \left\{ (n+1)r, nr \right\} = - \frac{1}{RfCf} \int_{(n-1)r}^{nr} E_i(t) dt \]

From Equation (3) it follows that the output signal is independent of previous output signals.

The operation of the averaging sample-hold circuit can therefore be summarized as follows:

During each time interval \( r \), three actions take place simultaneously:

1) The input signal is integrated and hence averaging takes place.
2) The output signal puts back into the capacitor of the integrator the exact opposite amount of charge, accumulated during the previous interval \( T \), hence automatic resetting occurs.

3) The value of the output signal is sampled at the end of each averaging interval \( T \) and held constant for an interval \( T \), resulting in the hold operation of the circuit.

The operation of the circuit is illustrated in Figures 9 and 10 for a step-input and arbitrary input signal respectively.

2.5 Non-Recursive Low-Pass Acceleration Filter

The small magnitude, low frequency acceleration signal, during coasting or braking, has superimposed on it, much larger amplitude short duration acceleration signals. These latter are caused by train actions such as a “run-in” of the train. Filtering should therefore be given careful attention.

With the acceleration being a sequence of sampled data points, averaged over one second, it was natural to use an analog adaption of a digital filtering technique — “non-recursive low-pass filtering with equal weighting coefficients”, which gives a “true” running average, computed on the number of data points used by the filter.

This filtering technique has several advantages:

1) phase delay is linear with frequency,
2) response time is fast,
3) easy, direct interpretation of the effect of filtering on the input signal.

The filter, shown in Figure 12, consists of an analog delay line, made up of six sample-hold circuits in series. Each sample-hold circuit is producing a one second unit-delay (Fig. 11). The input to the filter is a sampled acceleration signal, averaged over one second. Four outputs are given, selected by a rotary switch, having respectively a 1, 2, 4 and 6 seconds running average time constant.

The operation of the filter is illustrated for a step input in Figure 13 and for a pulse input in Figure 14.

3.0 VELOCITY

The output signal of the optical sensor is a frequency proportional to the velocity of the vehicle. A linear frequency-to-voltage conversion is thus required to obtain a voltage proportional to the velocity. There are several ways to do this. In this particular case, very small values of acceleration are to be obtained from the velocity signal by differentiation. The main concern therefore is to develop a method of converting frequency to voltage so that the “ripp’ e” in the output voltage is kept to a minimum; otherwise, their amplification, resulting from the differentiation, will drown the small acceleration signals.

The block diagram of the circuit is shown in Figure 15. The frequency \( f_1 \), from the optical sensor, is multiplied by 32, using a phase-locked loop frequency multiplier and a frequency doubler. This results in a considerable increase in resolution and hence a substantial decrease in the quantization. The output signal \( E_3 \) of the frequency doubler circuit is the input for a precision monostable. The output signal \( E_4 \) of the monostable is subsequently averaged by the “averaging sample-hold” circuit, which output voltage \( E_5 \) is proportional to the velocity.
The output signal $E_4$ of the monostable consists of a pulse train, amplitude $E$ and width $\tau^*$, having a frequency $f_4$ proportional to the velocity, see Figure 16. Substituting these parameters into Equation (3) derived earlier for the “averaging sample-and-hold” circuit gives:

$$E_5 \{ (n + 1)\tau, n\tau \} = \frac{-1}{R_iC_i} \int_{(n-1)\tau}^{n\tau} E_4(t)dt \quad (4)$$

$$E_5 \{ (n + 1)\tau, n\tau \} = \frac{A E \tau^* f_4}{R_iC_i} \quad (5)$$

where $A$ is the gain of the output buffer amplifier.

The optical sensor is related both to time (frequency $= f_1$) and to speed via the diameter $D$ of the wheel (1 revolution $= n$ pulses).

Hence:

$$1 \text{ revolution/sec} = n \text{ pulses/sec} = 0.1785 \cdot D \text{ miles/hour} \quad (6)$$

Furthermore,

$$f_1 = 5.602 \cdot \frac{n v}{D} \text{ pulses/sec} \quad (7)$$

where $[D]$ = inch and $[v]$ = miles/hour.

$f_4$ is equal to $m f_1$, the relationship between $f_4$ and velocity $v$ is therefore:

$$f_4 = 5.602 \frac{n m v}{D} \text{ pulses/sec} \quad (8)$$

Substituting in Equation (5):

$$E_5 \{ (n + 1)\tau, n\tau \} = \left[ \frac{5.602 \cdot A E \tau^* \cdot n m}{R_iC_i D} \right] \cdot v \text{ volts} \quad (9)$$

Substituting the actual values of the parameters into Equation (9) results in:

$$E_5 \{ (n + 1)\tau, n\tau \} = \left[ 0.1 \right] \cdot v \text{ volts} \quad (10)$$

where $E_5$ is the velocity output signal, updated every $\tau$ seconds, and $v$ the velocity in miles/hour.
4.0 ACCELERATION

The acceleration is obtained indirectly through differentiation of the velocity output signal. Differentiation, in general, is noisy since the transfer gain is increasing linearly with frequency.

Figure 18 illustrates the specific approach taken to obtain low noise differentiation.

The "differential-quotient" of a function $f(t)$ is:

$$\frac{df}{dt} = \lim_{dt \to 0} \frac{f \left( t + \frac{1}{2} dt \right) - f \left( t - \frac{1}{2} dt \right)}{dt}$$

(11)

This operation can be implemented very accurately with an operational amplifier circuit, where $C$ is the input capacitor and $R$ the feedback resistor. However, the noise generated is substantial and additional filtering is required.

In sampled data systems, analog and digital, the "difference-quotient" is implemented based on subsequent data points.

$$\Delta f = \frac{f \left( t + \frac{1}{2} \Delta t \right) - f \left( t - \frac{1}{2} \Delta t \right)}{\Delta t}$$

(12)

The values $f \left( t \pm \frac{1}{2} \Delta t \right)$ represent "instantaneous" sampled values of $f(t)$. Therefore, any noise or signal spikes, occurring at these sampling times can substantially alter the value of the "difference-quotient", hence the signal processing is basically noisy.

The technique developed to obtain low noise differentiation is illustrated in Figure 18 and is referred to as the "average-difference-quotient".

To obtain the "average-difference-quotient"

$$\overline{\frac{\Delta f}{\Delta t}} = \frac{1}{\Delta t} \int_{t-\Delta t}^{t} f(t)dt - \frac{1}{\Delta t} \int_{t}^{t+\Delta t} f(t)dt$$

(13)

the following operations are required:

1) obtaining the average values of $f(t)$ over intervals $\Delta t$,

2) holding each average value for one interval $\Delta t$,

3) taking the difference of the average values of subsequent intervals.

Each sampled data point is the average of $f(t)$ over a time interval $\Delta t$, resulting in a considerable noise rejection allowing hence the average slope to be measured very accurately.
It can be seen that operations 1 and 2 are implemented by the velocity circuit, which was specifically designed with this particular technique of low noise differentiation in mind.

To obtain the acceleration from the velocity signal it is only necessary to implement operation 3. This requires a "multiplexed sample-hold" circuit and a differential input amplifier. The circuit schematic is shown in Figure 20.

The acceleration signal is filtered by the "non-recursive low-pass filter", with selectable integration time constants, to add flexibility and to further improve on the attenuation of short duration acceleration signal variations.

The output signal of the velocity circuit is:

$$E_5 \left\{ (n + 1)\tau, n\tau \right\} = 0.1 \cdot \bar{v} = 0.1 \cdot \frac{1}{\tau} \int_{(n-1)\tau}^{n\tau} v(t)dt$$

(14)

The acceleration derived by the technique described above is:

$$E_6 \left\{ (n + 1)\tau, n\tau \right\} = A_2 A_3 \left[ E_5 \left\{ (n + 1)\tau, n\tau \right\} - E_5 \left\{ n\tau, (n - 1)\tau \right\} \right]$$

(15)

where \(A_2\) and \(A_3\) are the gain factors of respectively the difference amplifier and the output buffer amplifier.

The output signal of the "non-recursive low-pass filter" is:

$$E_7 \left\{ n\tau, (n - 1)\tau \right\} = \frac{1}{p} \sum_{q = 1}^{p} \left[ E_6 \left\{ (n + 1 - q)\tau, (n - q)\tau \right\} \right]$$

(16)

where \(p\) is the number of stages of the delay line.

The gain factors \(A_2\) and \(A_3\) were selected to obtain for the static transfer gain for the acceleration output signal: \(1\) volt/ft/sec\(^2\).

5.0 DISTANCE

The optical sensor gives \(n\) pulses per revolution. For a given wheel diameter \(D\), each input pulse represents a small distance interval. Accumulation of the pulses gives therefore the distance \(S\) travelled.

A digital scaling can be used to obtain suitable engineering units. For the particular experiments, for which this instrument was designed, an output pulse is produced for every 10 revolutions of the wheel. (The output pulse is used to activate the event marker on a chart recorder.)

With \(n = 240\) pulses/revolution and having one output pulse for every 10 revolutions it follows that a digital scaling factor of 2400 is required. The diameter \(D\) of the wheel is \(D = 33.26\) inches, hence each output pulse corresponds to a distance interval \(\Delta S\) of 87.07 ft.
6.0 CALIBRATION

The calibration of the distance and velocity outputs is relatively simple. For the distance calibration it is required to measure the diameter of the wheel. The velocity output can be readily calibrated by substituting a variable frequency generator for the optical sensor.

The acceleration channel is calibrated by applying a very low frequency triangular signal from a function generator to the input of the “averaging sample-and-hold circuit”. The velocity and acceleration output signals are displayed simultaneously on a 10 inch wide strip chart recorder. From the slope of the triangular output signal one can calculate the amplitude of the acceleration signal. The gain in the acceleration channel is adjusted subsequently to give an output amplitude equal to the calculated value for the amplitude. (The acceleration signal is a square wave due to the action of differentiation.)

7.0 ACCURACY OF MEASUREMENTS

The three parameters measured are derived from an optical sensor, mounted on the axle of a wheel. Lateral movements of the vehicle and variations in track width cause a small variation in the rolling diameter due to its conicity. A precise figure for the magnitude of this variation is difficult to give. It is estimated to translate into a measuring error of about 0.1% for the three longitudinal parameters derived from the angular input information.

The actual measurement of the wheel diameter is estimated to be accurate to 0.1%.

These errors are independent of the speed. The relative accuracy of the input information, prior to the actual signal processing, is therefore 0.2%.

The signal processing in the distance channel does not have an inherent error, consisting essentially of dividing by $N$. The measuring accuracy for distance is therefore 0.2%.

The inherent error of the velocity circuit is very small, 0.02% of full scale. The relative accuracy of the velocity information is hence within 0.3%.

The present design permits velocities to be measured over a range from 1/4 mile/hour up to 70 miles/hour.

The inherent error in the acceleration signal processing is difficult to measure, being so small and requiring a very linear triangular calibration signal at very low frequencies ($\approx 0.02c/sec$). Using a common laboratory function generator, one can nevertheless readily determine the (quasi) static acceleration gain factor to within 1%.

The “noise” amplitude in the acceleration output signal, prior to filtering, is about 1 mV. With a gain factor of one volt/ft/sec$^2$, this translates into an equivalent magnitude of uncertainty of the acceleration of $10^{-3}$ ft/sec$^2$ or 30 $\mu$g. The subsequent filtering by the “non-recursive low pass filter” reduces this noise level even more.

8.0 CONCLUSIONS

A measuring technique for velocity, acceleration and distance of rail vehicles has been discussed.

The instrument has been used to evaluate braking performance of rail vehicles but could be used in general as a train performance monitor.

The low noise method of differentiation and the circuits to implement it are of particular interest and should find applications in other areas where low noise rate information is required.
FIG. 1: PHASE-LOCKED LOOP FREQUENCY MULTIPLIER

FIG. 2: FREQUENCY/PHASE TRANSFER FUNCTION OF PHASE COMPARATOR
FIG. 3: FREQUENCY DOUBLER
FIG. 4: CONVENTIONAL SAMPLE-HOLD OPERATION

FIG. 5: MULTIPLEXED SAMPLE-HOLD CIRCUIT
FIG. 6: MULTIPLEXED SAMPLE-HOLD WAVEFORMS
FIG. 7: AVERAGING SAMPLE-HOLD CIRCUIT
FIG. 8: SIMPLIFIED AVERAGING SAMPLE-HOLD CIRCUIT
FIG. 9: SIGNAL WAVEFORMS OF AVERAGING SAMPLE-HOLD CIRCUIT FOR STEP INPUT SIGNAL
FIG. 10: RESPONSE OF AVERAGING SAMPLE-HOLD CIRCUIT TO ARBITRARY SIGNAL

\[ E_{\text{OUT}} = \frac{1}{R_i C_f} \int_{t_0}^{t_0 + \tau} E_{\text{IN}}(t) \, dt \]
FIG. 11: UNIT DELAY FUNCTION

FIG. 12: NON-RECURSIVE LOW-PASS FILTER
FIG. 13: NON-RECURSIVE LOW-PASS FILTER - RESPONSE TO STEP INPUT

FIG. 14: NON-RECURSIVE LOW-PASS FILTER - RESPONSE TO PULSE INPUT
FIG. 15: BLOCK DIAGRAM OF VELOCITY CIRCUIT

FIG. 16: INPUT SIGNAL OF AVERAGING SAMPLE-HOLD CIRCUIT
FIG. 17: SIGNAL WAVEFORMS OF VELOCITY CIRCUIT
"DIFFERENTIAL - QUOTIENT"
\[
\frac{df}{dt} \bigg|_p = \lim_{dt \to 0} \frac{f(t + \frac{1}{2} dt) - f(t - \frac{1}{2} dt)}{dt}
\]

"DIFFERENCE - QUOTIENT"
\[
\frac{\Delta f}{\Delta t} \bigg|_p = \frac{f(t + \frac{1}{2} \Delta t) - f(t - \frac{1}{2} dt)}{\Delta t}
\]

"AVERAGE - DIFFERENCE - QUOTIENT"
\[
\frac{\overline{\Delta f}}{\Delta t} \bigg|_p = \frac{\frac{1}{\Delta t} \int_t^{t+\Delta t} f(t) dt - \frac{1}{\Delta t} \int_t^{t-\Delta t} f(t) dt}{\Delta t}
\]

FIG. 18: DIFFERENTIATION TECHNIQUES
**FIG. 19: BLOCK DIAGRAM OF ACCELERATION CIRCUIT**

**FIG. 20: "AVERAGE-DIFFERENCE-QUOTIENT" CIRCUIT**

**FIG. 21: BLOCK DIAGRAM OF DISTANCE CIRCUIT**
A measuring technique for velocity, acceleration and distance of rail vehicles is discussed. Velocity and distance are derived directly from an optical digitally encoded sensor. A low noise differentiation method allows very small accelerations of rail vehicles to be obtained from the velocity signal. Description and design details are given of the major circuit functions.