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**ABSTRACT**
This report summarizes progress on certain theoretical questions arising in the study of dynamic and quasi-static fracture and contact problems for linearly viscoelastic material. Specifically, results for two classes of problems are described:

A. Determining the angular dependence of the stress field in front of a dynamically propagating Mode III semi-infinite crack in an infinite general...
Determining the stresses, displacements and friction coefficient for the quasi-static problem of a rigid indentor sliding with Coulomb friction over the surface of a power-law inhomogeneous linearly viscoelastic half-plane.
This report is a summary of work completed by the Principal Investigator, Jay R. Walton, under AFOSR M 82-0152 for the year 4-1-82 through 3-31-83. Two separate problem areas were studied. The following sections A and B describe the progress made in each area.

A. The investigation of dynamic viscoelastic fracture was continued. In addition to the problem of constructing closed-form expressions for the dynamic stress intensity factor, attention was also focused upon the question of determining the angular dependence of the stress field in the neighborhood of a dynamically propagating crack. Specifically, the angular dependence was determined for the dynamic, steady state propagation of a semi-infinite Mode III (anti-plane strain) crack through an infinite, isotropic and homogeneous linearly viscoelastic body. This angular dependence was shown to have the form $\sigma_{23}(r,\theta) = f(v,\theta)K/\sqrt{r}$, where $(r,\theta)$ are local polar coordinates at the crack tip, $K$ is the dynamic viscoelastic stress intensity factor (already calculated in earlier work performed under this AFOSR grant) and $f(v,\theta)$ is a universal function of $\theta$ and the crack speed $v$. Indeed, $f(v,\theta)$ is the same function that occurs in the analysis of the corresponding problem for elastic material with a value for the shear modulus equal to the glassy viscoelastic modulus. The study of the angular dependence of the stress field for more complicated crack problems is still in progress.

B. An investigation of contact and fracture problems for inhomogeneous linearly viscoelastic material was initiated. Recently, the Principal Investigator completed a successful analysis of a frictional contact problem for power-law inhomogeneous viscoelastic material. Specifically, a simple closed-form solution was derived for the quasi-static sliding with Coulomb friction of a rigid indenter over a linearly viscoelastic half-plane for which Poisson's ratio is constant but for which the shear modulus is of the form $\mu(t,y) = \mu_0(t/t_c)^{-\lambda}(y/d)^\lambda$ where $0 \leq \alpha < 1$, $0 \leq \lambda \leq 1$ and $\mu_0$, $t_c$ and $d$ are constants. The half-plane is defined by $y \geq 0$. Thus the shear modulus has a power-law dependence on distance from the half-plane surface and models a material that stiffens with

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depth. Such a material inhomogeneity can occur in a variety of settings. The two which generated initial interest in this problem involved attempts to model the sliding of a pipeline over the sea bottom and the frictional interaction of a rigid asperity over a polymeric material or rubbery material in which the heat generated produces a softening of the material near the body's surface. Of principal interest was the determination of the effect of material inhomogeneity (through the parameters $\lambda$ and $d$) on the friction coefficient. It was demonstrated that the friction coefficient has a simple functional dependence on $\lambda$ and $d$ and that its magnitude can vary greatly with $\lambda$ and $d$. 

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