MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A
COORDINATED SCIENCE LABORATORY
DECISION AND CONTROL LABORATORY

STRATEGIES FOR COMPLEX SYSTEMS

University of Illinois at Urbana-Champaign

Approved for public release; distribution unlimited.
The research program was focused on investigating new methods of analysis, synthesis, and optimization of control systems, particularly those which contain disturbance inputs, uncertain parameters, and other modeling uncertainties. The general objective was to develop new methods to improve the performance of control systems by counteracting the effects of these modeling uncertainties and disturbance inputs. The new methods can be classified into several general categories: multivariable feedback design in the frequency domain based on...
comparison sensitivity matrix and robustness concepts, adaptive observers and adaptive control, multiple time-scale and singular perturbations, chained aggregation methods, and incentive controllers for hierarchical systems.
FINAL SCIENTIFIC REPORT

CONTROL STRATEGIES FOR COMPLEX SYSTEMS
FOR USE IN AEROSPACE AVIONICS

by

J. B. Cruz, Jr., P. V. Kokotovic, and W. R. Perkins

Research Supported by

Air Force Office of Scientific Research (AFSC)
United States Air Force
under Grant AFSR-78-3633
Covering the Period 1 July 1978 to 30 June 1983

Submitted to:

Directorate of Mathematical and Information Sciences
Air Force Office of Scientific Research (AFSC)
Bolling Air Force Base, D. C. 20332

Decision and Control Laboratory
Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DTIC
August 1983
This technical report has been reviewed and is
approved for public release (ARDC 77-3633-A).
Distribution is unlimited.
MATTHEW J. KEPPEL
Chief, Technical Information Division
I. Faculty Employed on Grant During Part or All of Five-Year Period

Jose B. Cruz, Jr., Professor, Co-Principal Investigator
William R. Perkins, Professor, Co-Principal Investigator
Petar V. Kokotovic, Professor, Co-Principal Investigator
Tamer Basar, Associate Professor
Juergen Ackermann, Visiting Professor

II. Summary of Research Accomplishments

The research program was focused on investigating new methods of analysis, synthesis, and optimization of control systems, particularly those which contain disturbance inputs, uncertain parameters, and other modeling uncertainties. The general objective was to develop new methods to improve the performance of control systems by counteracting the effects of these modeling uncertainties and disturbance inputs. The new methods can be classified into several general categories: multivariable feedback design in the frequency domain based on the comparison sensitivity matrix and robustness concepts, adaptive observers and adaptive control, multiple time-scale and singular perturbations, chained aggregation methods, and incentive controllers for hierarchical systems.

The results obtained during the five-year period are fully documented in about 80 journal articles and conference papers presented at various international congresses and national meetings, and in several technical reports of the Laboratory. We briefly sketch below the major accomplishments.
A. Sensitivity and Stability of Multivariable Feedback Systems with Large Plant Perturbations

AFOSR has sponsored our work on sensitivity of multivariable systems for a number of years. One of the key concepts we developed which has had significant impact in the development of design concepts for feedback systems is comparison sensitivity. Although it was originally proposed for plant perturbations which might be large, the concept was principally used for small plant perturbations. We had explicit results for nonlinear perturbations which are not necessarily small. In recent years several investigators have focused attention on the stability of multivariable feedback systems when the plant is subject to large perturbations. The principal tool is singular value analysis. We have shown that our earlier work on sensitivity could be expressed in terms of singular values in a manner that displays remarkable similarity in the nature of requirements for stability robustness. We have clarified the relationships between the two sets of sufficient conditions. In particular we have shown that the conditions for maintaining sensitivity, i.e., sensitivity robustness, include the conditions for maintaining stability, i.e., stability robustness. Thus, a system design which is robust in sensitivity with respect to large plant perturbations is necessarily robust in stability with respect to the same large plant perturbations. Thus the requirement for sensitivity margin exceeds that for stability margin. See [28,38,39].

Singular value analysis provides tight bounds when the disturbances or system perturbations are assumed to be completely unknown, except for bounds on their norms. When the dominant plant perturbations and disturbances are structured, the singular value analysis may yield conservative results.
To extend the usefulness of singular value analysis, we have developed a complementary tool, the singular value sensitivity function. By parameterizing the uncertainty, as suggested by the structure of the perturbation, it is possible to gain further insight in the robustness of a feedback system. We have derived formulas for the Frechet derivatives of singular values and we have examined design examples, where knowledge of the singular value sensitivities enabled us to conclude poor robustness along certain directions of parameter perturbations. This information is not deducible from singular value analysis alone. Singular value sensitivities provide information which complements the information obtained from singular values with very little added computation. See [40,56] for details.

B. Sensitivity Reducing Compensators Using Observers

We developed the concept of comparison sensitivity for multivariable systems several years ago as a tool for assessing the benefits of feedback. Linear optimal state regulators were found by Kreindler to automatically satisfy our sensitivity criterion. Naeije and Bosgra extended Kreindler's result to output feedback controls using dynamic compensators. Implementing a full state feedback law using an observer to estimate the unmeasured states will not satisfy the output sensitivity reduction criterion in general. We developed an extension of Naeije and Bosgra to the particular case of output feedback system using state observers [9,22]. An interactive software has been developed and applied to a simple aircraft control example [A2]. The design procedure using observers is an improvement over the design with arbitrary compensator dynamics for the following reasons. First, the design
of the observer is well-known and by placing the poles of the observer the designer is selecting poles of the overall feedback system. Second, the dynamic order of the reduced order observer is less than the maximum bound on the dynamic order of the compensator designed by the methods of Naeije and Bosgra. Finally, the use of observers leads to a useful interpretation of the sensitivity weighting matrix [9,22].

C. Sensitivity Adaptive Feedback with Estimation Redistribution

We developed an approach to the synthesis of dynamic controllers for systems containing unknown random parameters. This approach, called SAFER control [10,11,12,23,30,31] allocates individual parameter estimation costs for a given total parameter estimation cost, so as to minimize the primary control cost function. This is achieved by appropriate choice of controller gains from the dynamic controller and choice of weighting coefficients for the sensitivity functions which are related to the parameter estimation accuracy. Some simplification can be effected when output sensitivities rather than state sensitivities are used. The general output SAFER problem is still quite complicated and the algorithm is still numerically complicated.

We refined and simplified our approach to the synthesis of dynamic controllers. We obtained significant algorithm and computational simplifications for the SAFER control of single-input single-output discrete-time systems. Using a one-step ahead minimum mean-square error criterion, as in the self-timing regulator case, and a sensitivity adaptation over a single stage in the future, a simple SAFER algorithm has been obtained. A
significant contributing factor to the simplification is the effective use of only two sensitivity functions to capture the effects of all parameters in the system. The SAFER concept was applied to a model for a magnetic suspension system to obtain some numerical and simulation experience with the method.

D. Parameter Space Design of Robust Control Systems

A method has been developed for the design of robust control systems. Robustness is with respect to large plant parameter variations, sensor failures, and quantization effects in the controller. Single-input linear time-invariant plants, represented in discrete-time form, are considered, and it is assumed that the plant matrices depend on a physical parameter vector. The controller structure is assumed to be in state feedback form. The feedback gains in the state feedback are the free design parameters. It is assumed that desirable features of the dynamic behavior of the control system can be specified by a region in the eigenvalue plane. The acceptable region in the eigenvalue plane can be mapped into a region of coefficients of acceptable characteristic polynomials. By pole assignment, the space of characteristic polynomial coefficients can be mapped to acceptable regions in the control parameter space. These sets of mappings are the basis for both graphic and algebraic computer-aided design methods. Constraints such as limits on actuator gains, robustness with respect to large plant parameter variations, and robustness with respect to sensor failures are readily reflected into the control parameter space. The method has been applied to the design of a crane control system and a robust stabilization for the
short period longitudinal model of an F4-E aircraft with canards. See [16], [25], and [50] for details.

E. Output Feedback Compensator Design

Based on a modified output regulator problem, a design oriented methodology has been developed for the synthesis of output feedback compensators retaining $\xi(1 \leq \xi \leq n)$ optimal eigenvectors from an nth order reference state feedback regulator. Viewing $\xi$ as a design parameter, Medanic, et al., [41] have shown that the case $\xi > r$ leads to a dynamic compensator of dimension $(\xi - r)$ whose parameters can be determined by solution of an associated output feedback pole-placement problem. Using an iterative dyadic pole-placement procedure, an algorithm has been devised recently which determines the solution of this pole-placement problem without a priori assumptions on the compensator dimension. The methodology also can be extended to the class of stabilizable systems and the required compensator shown to possess a separation property. Details of the procedure may be found in [41].

Already the approach has attracted the attention of practitioners, with control engineers at General Electric finding the approach more effective than any other existing approaches in solving a difficult design of a controller for a NASA power-generating windmill and a coal gasification plant.

F. Variable Structure Model Following Control Systems

A new design concept for adaptive model-following control systems capable of shaping the error transient responses was developed using the theory of variable structure systems and sliding mode. It is shown that
the resulting model-following control system exhibits adaptive properties inherent in adaptive model-following systems designed by existing methods. An aircraft control problem which has been approached using various model-following techniques is considered and a performance comparison with the present design is made.

In general there are two classes of design methods of adaptive model-following control (AMFC) systems. Landau based his method on hyperstability concept proposed by Popov. Other designs utilize Lyapunov methods. While the primary concern of these design methods is to guarantee that the error between the states of the model and the controlled plant goes to zero, the transient behavior of this error is not prescribed. Only some qualitative discussions are provided on the relationship between the adaptation gains and the speed of the norm of this error.

The adaptive control laws derived using the Lyapunov method for single-input-single-output model reference adaptive systems are discontinuous control laws. These control laws belong to a particular class of discontinuous feedback laws called variable structure control. Feedback systems with variable structure control laws are called variable structure systems (VSS). The salient feature of VSS is that the so-called sliding mode exists on a switching surface. While in sliding mode, the feedback system becomes less sensitive to system parameter variations and disturbance inputs. The connection of VSS and adaptive model reference system is through sliding mode. The advantage of designing AMFC systems by the theory of VSS is that the transient response of the model plant error can be prescribed by the design. We have developed a design procedure for multiinput model-following systems which retains the error transient shaping capability as in the single-input
design by utilizing design methods for VSS. We have applied this method to an aircraft control problem.

The plant of this problem represents the three degrees-of-freedom linearization longitudinal state equations of a conventional subsonic aircraft, a Convair C-131B. The model in this case is chosen to be the estimated dynamics of a large supersonic aircraft. This problem has been considered in various model-following papers and it was used in comparing the performance of VSMFC systems to AMFC systems and LMFC systems.

Simulations indicate that a variable structure model-following control law significantly improves the error transient behavior in comparison to that for an adaptive model following control or a linear model-following control. Details are given in [15]. See [13] and [14] also.

G. A Newton-Laypunov Design for a Class of Nonlinear Regulator Problems

In contrast to the well-developed theory of the linear regulator problem, there are relatively few results on the nonlinear regulator problem. The main difficulty lies in solving the Hamilton-Jacobi equation arising in such problems for the optimal feedback control. In previous work, based on the assumptions that the nonlinearities are weak and the linearized problem is controllable (stabilizable) and observable (detectable), feedback controls are obtained using matched asymptotic expansions. Numerical computation of the series expansions involves tensor equations, and the domain of stability depends on the truncation of the series expansion of the control.

We have extended some results from linear regulators to a class of nonlinear regulators using an iterative scheme. In particular, we obtain
analogs of the stabilizing solution to the Riccati equation and the Newton-Lyapunov method for computing the Riccati solution in nonlinear regulators. The iterative scheme differs from earlier ones in that it successively generates improving controls while maintaining a fixed domain of stability. Exponential stability which is crucial in previous work is not essential here.

We consider a class of nonlinear regulators where the system is linear in the control and the cost function to be minimized is a quadratic form of the control. Due to the structure of the problem the Hamilton-Jacobi (H-J) equation yields a feedback control law with a simple structure. We have shown that the stabilizing solution of the H-J equation is the unique optimal solution. At each stage of the iteration, we improve the feedback control which possesses a domain of stability not smaller than that of the initiating control. The controls are successively solved for from a system of linear partial differential equations, which is an analog of the matrix Lyapunov equation appearing in the iterative solution of the Riccati equation for linear regulators. Furthermore, the improvement in the cost function is quadratic. The uniqueness result guarantees that if convergence occurs, the design method yields the optimal solution.

The numerical solution to the partial differential equations is computed using the method of characteristics which deals with an equivalent system of ordinary differential equations. The result is a feedback control map. In practice, to reduce the amount of data storage and computation, suboptimal schemes such as polynomial approximations, can be used. For further details see [1],[36].
H. Error Analysis of Identifiers and Adaptive Observers

Applicability of model reference adaptive systems are severely hampered by the unrealistic assumption of the exact match of the plant and the reference model order. We are in the process of developing a singular perturbation approach to remove this obstacle. The basic idea of our approach is as follows: The difference in the model-plant order causes the equations for the adaptation error and the adjustment law to be singularly perturbed. The reduced order design corresponds to the situation when the orders match. The presently existing Lyapunov or hyperstability theories provide the stability and convergence criteria for this ideal case. The model-plant mismatch violates these criteria, but our perturbation method can be employed to find bounds on the mismatch within which stability will be preserved.

In [42], [57], and [74] we have established stability bounds for several identifiers and adaptive observers. Furthermore, we have obtained error estimates which indicate a crucial phenomenon: in the presence of model-plant mismatch the choice of the input signal is much more critical than in the ideal matchable case. Rich persistently exciting signals, thus far believed to be the best, can have disastrous effects on performance. Using our error estimates we need only an order of magnitude knowledge of the dynamic range of the phenomena which are neglected (and thus cause the model-plant mismatch), in order to indicate how to select input signals to minimize the adaptation error. Instead of earlier richness conditions, we have introduced the notion of dominant richness, which quantifies the spectral content of the inputs needed for robust identifiers and adaptive observers.
I. Robust Redesign of Adaptive Feedback Control

Our current research in robustness of adaptive schemes is focused on the model-plant mismatch in adaptive feedback control. This is a more difficult problem than the identification problem because we no longer possess the freedom to choose the input signal. However, our approach can be extended to this case allowing an implicit form of the dominant richness condition. The main risk of adaptive freedom with model-plant mismatch is the interaction of adaptively induced nonlinear high gain with neglected high frequency parasitics. To reduce this risk some form of filtering or some forgetting factors are needed. When such factors are introduced, the adaptation error is not zero even in the ideal case when the model matches the plant. Hence the need for a quantitative trade-off between the ideal performance and robustness in nonideal situations.

Our singular perturbation approach has led to a redesign procedure in which this trade-off can be made. Since the global stability is no longer possible, the goal is to extend the region of boundedness $R$. Every solution originating in $R$ converges to a residual ball $B$, which should be as small as possible. The useful operational range is $R$ minus $B$. Our approach is to express the estimates of $R$ and $B$ in terms of adaptation parameters and the model uncertainty parameter $\varepsilon$. In this way a scheme can be designed to enlarge the operational range. This is the topic of our current research, first results of which are presented in [46, 58].

J. A Control Problem with Structural Choices

In the usual optimal control problem it is assumed that the structure of the plant is fixed and that the control variable is the only means
by which the evolution of the state or output can be influenced. There are several cases in practice where the structure of the plant may be amenable to changes which are at the decision maker's disposal. We considered the problem whereby, during the operation of the system, the decision maker, in addition to applying his usual control, may switch from one structure to another at instants of time that he chooses. Potential applications in addition to flight control are organizational control resource allocation, and hierarchical control.

We derived necessary conditions for optimality where the plant can be switched to one of two possible configurations described by linear state equations and a quadratic performance index with weights which are keyed to the switched mode of the plant. Also, we obtained sufficient conditions for which a switched linear state feedback control is optimal.

We extended the problem formulation to the stochastic case with disturbances in the plant and noisy measurement, and we showed that the separation principle does not hold. Furthermore, we showed that the stochastic control problem can be solved in two steps, the first step involving the solution of a classical linear quadratic Gaussian problem and the second step involving the solution of a deterministic singular control problem [32].

K. Time-Optimal Control of a Class of Singly Perturbed Nonlinear Systems

Theoretical studies of nonlinear singularly perturbed optimal controls have been devoted to unconstrained problems. A few results dealing with control constraints are restricted to linear time-invariant systems. In contrast, the most interesting applications have been to nonlinear
systems with constrained states and controls. In such complex problems the advantages or order reduction and separation of time scales achieved by singular perturbation methods are manifold, leading to conceptual, computational and control implementation simplifications.

Our work has been directed toward the development of an analytical and computational methodology to deal with nonlinear constrained problems. The time-optimal problem is a typical representative of trajectory optimization problems. It is well-known that many other cost functionals can be transformed into this format.

The time-optimal control of a class of nonlinear singularly perturbed systems possesses the two time-scale property that the optimal control is made of a control in a slow time-scale followed by a control in a fast time-scale. Based on this property a near time-optimal is defined. Two examples illustrating the computation of the near-optimal control and a simple iterative technique have been developed [7,21].

L. Singular Perturbations and Time Scales

Singular perturbation methods are among powerful analytical tools for control system design. A limiting factor, however, is the need to have the system in explicit singular perturbation form with a small parameter \( \epsilon \) multiplying some of the derivatives. A much wider class of dynamic systems possesses similar two-time-scale properties, but does not appear in the explicit singular perturbation form. We have made substantial progress in developing a modeling methodology for identifying the time scales from weakly coupled subsystems. Basic ideas of this methodology are presented in [45] and have been extended to nonlinear networks and flexible structures.
A practically important class of feedback control systems appearing as nonstandard singular perturbation forms are high-gain systems with actuator and sensor parasitics. We have developed a perturbation method to analyze interaction of high gain modes with parasitic modes and to guarantee a stable design. Details are found in [68].

Our work on singular perturbations has also resulted in a survey [67] of more than 320 references published since our 1976 survey.

M. Nonlinear Singly Perturbed Control Problems

We have applied singular perturbation techniques to a class of nonlinear, fixed-endpoint control problems to decompose the full order problem into three lower-order problems, namely the reduced order problem, and the left and right boundary layer problems. The boundary layer problems are linear-quadratic, and, contrary to previous singular perturbation works, the reduced problem has a simple formulation. The solutions of these lower order problems are combined to yield an approximate solution to the full nonlinear problem. The full order problem is shown to possess an asymptotic series solution. See [18] for details.

A near optimal feedback stabilization of a class of nonlinear singularly perturbed systems has been developed. Through the construction of a Lyapunov function we have shown that the feedback control can stabilize large initial disturbances of the fast variable. This has the advantage that we can find either a bound for the singular perturbation parameter, given the desired domain of stability, or a domain of stability enclosing large values of the fast variable, for a given value of the singular perturbation parameter. See [19] for details.
N. Generalized Singly Perturbed Systems

Using various geometric and algebraic techniques, a generalized class of singularly perturbed systems has been examined. It has been shown that descriptor variable systems (in the sense of Luenberger) can be viewed as limiting cases of these generalized singular systems. This important connection between these two classes of systems has led to several results, including state feedback, pole placement, and quadratic regulator solutions for descriptor systems.

Finally, a useful new slow-fast subspace (geometric) decomposition for generalized singularly perturbed systems has been found. This decomposition leads directly to pole placement designs using feedback of appropriate slow and/or fast state variables [37,70,73].

O. Chained Aggregation

Chained aggregation was introduced originally as a technique to provide candidate structures for low order models of large systems based on information patterns. Our research has shown that chained aggregation and the resulting Generalized Hessenberg Representation (GHR) provide far greater insight into system structure than was suspected originally. This insight is invaluable in developing closed-loop feedback designs.

The basic geometry of chained aggregation is developed in [47,63] in terms of unobservability subspaces $\mathcal{X}$. The consequences of this subspace structure for control design is explored in detail in [47], where the "three-control-component design" (TCCD) is introduced and developed. The TCCD provides a convenient hierarchical approach to decentralized control synthesis.
At a more fundamental level, the basic system structure as revealed by chained aggregation has led to new system decomposition techniques [61]. It is demonstrated that special information structures lead to particular decompositions in interconnected systems.

Some beginnings have been made in extending these ideas to important classes of nonlinear systems [75]. This is a promising field for further research. An aircraft landing example in [75] nicely illustrates the idea of the approach. A 7th order nonlinear design problem is decomposed into two substantially simpler problems, one linear and one nonlinear.

Finally, connections with our research activity in singular perturbations is made in [62,76]. An interesting new approach to "near unobservability" is introduced here, in terms of canonic angles between subspaces. A relationship between near unobservability and time scales is explored. Thus it is seen once again that chained aggregation and the GHR provide a convenient basis in which a number of important system properties are explicit.

P. Incentive Schemes for Leader-Follower Control Problems

The Stackelberg solution concept first introduced by von Stackelberg for static games (1934,1952), and then extended and applied to dynamic games with static information for the leader (Chen and Cruz 1972, Simaan and Cruz 1973, Cruz 1978) has recently attracted considerable attention in the control literature (Ho 1980, Basar and Selbuz 1979, Papavassilopoulos and Cruz 1979, 1980, Tolwinski 1981, Basar 1982). This recent activity pertains to the case when the leader has access to dynamic information which involves the
follower's past actions. By using indirect methods, the leader's problem can be viewed as one where a strategy is determined which could induce the follower to a certain behavior which is most preferable for the leader.

Recently we have derived conditions for the existence of optimal affine incentive schemes for Stackelberg games with partial information [69]. This result was obtained using a geometric approach. Related issues and results are discussed in [33,34,51,65,66]. A survey of the concepts relevant to coordination and control in systems with multiple controllers appears in [27,53].

Q. Self-Tuning Strategies for Multicontroller Problems

Strategies for multicontroller systems described in the literature pertain to systems whose models are exactly known. Even for stochastic systems, the underlying probability distributions are assumed to be known exactly. We have investigated the difficult problem of determining strategies for multicontroller systems, when some system parameters are known. On-line parameter estimation is combined with adaptive multicontroller strategies. We have focused on one-stage performance criteria which are vastly simpler compared to multistage criteria. However, this is a practically important class of problems even for single controller problems. We have obtained results for self-tuning Nash strategies [55] and also for self-tuning Leader-Follower strategies [79]. Partial results have been obtained for systems with decentralized information [72].
III. Publications


CSL Technical Reports


