An ANALYTICAL TECHNIQUE
for
SHIP-FENDER INTERACTION

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The design and selection of appropriate fender systems must consider the dynamic interaction between the ship and port or pier structure. Energy absorption characteristics of marine fender systems vary as a function of fender geometry material, load time history, load spatial distribution, local hull stiffness and frequency of loading. The dynamic interaction of marine fender systems with ship's hull is studied in time domain. The frequency dependence of the hydrodynamic coefficients is considered in the form of a simplified convolution.
1. INTRODUCTION

In order to achieve more appropriately designed marine fender systems, an understanding of the vessel-fender dynamic interaction is essential. The dynamic analysis can describe more accurately the fender's energy absorption characteristics and operational performance requirements.

There are several approaches which consider the total energy of a berthing ship to be absorbed by the fender/pier system in addition to the magnitude of the fender reaction force generated. A simple and commonly used approach is the energy method; e.g., Lee [10], Brolsma et al. [1]. Accurate predictions using this method involve knowledge of hydrodynamic coefficients and system stiffnesses for the total system. Empirical values are commonly used in this method. System damping has been ignored in this approach. Alternate statistical approaches consider information obtained from model measurements to determine the design energy value of the berthing impact and fender energy absorption; e.g., Svendsen [13]. With the design risk selected, the design value of the energy to be absorbed by the fender system can be determined. The disadvantage of this method is in obtaining the necessary statistical information for problem solution. Kim [7] proposes an approximate and simpler method. The idea is based on the use of time average of the kinetic energy of the berthing ship during the time interval of fender compression. This method is applicable only to the ship in calm water and berthing broadside.

The time domain solutions of forces and motions have been developed by van Oortmerssen [15] and Fontijn [3]. Both of these methods use Impulse Response Function techniques. It is easy to adopt different external forces and some other factors in the time domain method. The procedure presented herein is based upon a simplified convolution integral for added mass and damping calculations, which eliminates the disadvantage of the expensive calculation in the above time domain solutions. Although the mathematical problem is formulated herein in sway, yaw, surge and roll motions, only the lateral motion results are presented here. The hydrodynamic coefficients, as functions of frequency, can be determined theoretically using two-dimensional strip theory or three-dimensional source distribution method. In order to verify the current technique with Fontijn's results, the same hydrodynamic coefficients are used herein.
2. DYNAMIC RESPONSE OF THE SHIP AND THE BERTHING STRUCTURE

During the berthing, the ship will undergo dynamic motion which has six degrees of freedom; namely, swaying, yawing, rolling, surge, heave and pitch. The heave and pitch motions are of little consequence in energy dissipation and may be neglected. In the following analysis, it is assured that there is no sliding contact along the fender's surface. Also, the coupling effects between each mode have been neglected.

Consider the dynamic equilibrium of the center of gravity of the ship as shown in Figure 1, the equations of motion are:

\[
(m_k + a_k) \ddot{x}_k + b_k \dot{x}_k = f_k \quad k = 1, 2, 4, 6
\]

where \(x_1, x_2, x_4\) and \(x_6\) are the surge motion (\(x_1\) positive forward), sway motion (\(y_1\) positive to port), roll motion (\(\phi\)), and yaw motion (\(\theta\)), respectively. The \(m_k, a_k\) and \(b_k\) are inertia mass, hydrodynamic mass and hydrodynamic damping in the corresponding directions. \(f_k\) represents the external fender reaction force on the ship in the \(k\) direction.

For the berthing structure with the effective mass \(M_s\), the dynamic equations are:

\[
M_s \ddot{x}_s + u_x \dot{x}_s + k_x x_s = -f_1
\]

\[
M_s \ddot{y}_s + u_y \dot{y}_s + k_y y_s = -f_2
\]

where \(K_{x,y}\) and \(u_{x,y}\) are the structure stiffness and damping, respectively. The displacement of the point of contact can be calculated after the dynamic equations (1) to (3) are solved.
If the elastic deformation of the ship hull is under consideration, let $K_h$, $S_h$ and $\mu_h$ define the stiffness, deflection and the damping coefficients of the hull at the point of contact respectively. The equations of motion of the ship hull are:

$$M_h S_{hx} + \nu_h S_{hx} + K_h S_{hx} = f_1$$

$$M_h S_{hy} + \nu_h S_{hy} + K_h S_{hy} = f_2$$

In the case of a very rigid berth, the deflection of berthing structure can be neglected due to the high stiffness of the structure. The effects of the ship's local stiffness can be investigated with this method.

The solution of dynamic equations is carried out by the numerical integration method with appropriate choice of problem parameters.

3. HYDRODYNAMIC MASS AND DAMPING

The hydrodynamic mass and damping are important parameters to be considered in the determination of the berthing forces. The hydrodynamic mass is governed by the following factors:

- Ship characteristics (beam, draft, size)
- Under-keel clearance
- Water depth
- Type of berthing structures (open, semi-open, solid)
- Fender characteristics
- Berthing modes
- Berthing velocity.
Most of the methods to determine the hydrodynamic characteristics do not take into account all of the above-mentioned factors. A recent review by Kray [9] has presented a good summary of the state-of-the-art on hydrodynamic mass determination. The most common practice is to have a constant added mass as presented by Vasco Costa [16], Komatsu and Salman [8]. Hayashi and Shirai [4] presented a theoretical formulation of ship added mass in shallow water as a function of the ratio of draft to water depth, the Froude number of the ship and the coefficient of head loss of the counter flow under the hull. Oda [12] conducted experiments to verify their theory. In order to have better agreement in very shallow water berthing, Oda modified the theory by neglecting the shear stress acting on the hull and also used a friction parameter as a function of sway force coefficient, contraction coefficient and water depth to draft ratio. The theory modified by Oda will be used to compare with the existing experiments and present method.

The constant added mass coefficient is not appropriate for berthing of ships in shallow water with different berthing speeds. Oortmerssen [13] and Endo [2] calculated the hydrodynamic characteristics of ship as function of frequencies in the shallow water based on three-dimensional approach. Comparisons have been made with the results for cases available in the literature. Generally speaking, the agreement is good. In the study by Fontijn [3], the two-dimensional hydrodynamic characteristics at low frequencies were modified to fit the experimental data. Time histories of fender force in Fontijn's paper compared well with the present method.

4. EFFECTS OF LOCAL HULL STIFFNESS

In the design and selection of the most appropriate fender system, consideration should not only take into account the energy absorption capacity of the fender systems but also the strength requirement of ship side structure due to berthing impact. The bending rigidity of hull structure in the horizontal direction is extremely high, so that the deformation from the whole ship, assumed as a beam, is not critical. Two kinds of deformation are considered by Kawakami et.al. [5,6]. There are the deformations of hull plate between frames and the deformations of hull plate between frames and transverse bulkheads. The analytical treatment of the hull deformations and stresses due to berthing impact is also presented in the report edited by Matsunami et.al. [11]. In their analysis, the deflections and stresses of ship hull are considered as a static reaction.
When the contacting region between ship hull and fender is small, the load distribution within the contacting area can be generally considered to be uniform. In the present dynamic analysis, the constant contact area on the ship hull is either on the center of the plate or on the beam, but can be simulated anywhere once described.

5. MATHEMATICAL METHOD

As mentioned earlier, a basic difficulty arises in the time domain solution dealing with the frequency-dependent hydrodynamic coefficients. The force derived from drag and added inertia is a complex function of ship movement at each given frequency. The time history of a force denoted by \( f(t) \) can be described by means of the convolution integral as:

\[
f(t) = \int_{-\infty}^{t} K(t-\tau) y(\tau) \, d\tau
\]

(6)

Where \( y(\tau) \) is the time history of the ship motion, e.g., sway. The kernel function \( K(t) \) is the Fourier transformation of the complex transfer function of \( F(\omega) \) with respect to the ship motion, which is given by:

\[
K(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} \, d\omega
\]

(7)

This procedure has been used in a number of investigations to obtain the histories of forces, e.g., Fontijn [3] and van Oortmerssen [15].

In the present application, the convolution operation is implemented by digital computation in a different manner. Tou [14] has shown that, for a single input sinusoidal frequency of amplitude \( a \), the instantaneous value of the output time sequence, \( f(n\Delta t) \), can be predicted by a weighted sequence of \( n \) previous time history steps as follows:
\[ f(n\Delta t) = \sum_{m=1}^{n} w_m \sin \left[ \omega(n-m)\Delta t \right] \]  

(8)

Where:
- \( w_m \) = weighting coefficients
- \( \Delta t \) = time interval

However, the instantaneous values of this function can also be given in terms of an input sinusoid as:

\[ f(n\Delta t) = |F(\omega)| \sin (\omega n\Delta t + \alpha) \]  

(9)

Where \( \alpha \) is the phase angle of the output relative to the input. Equations (8) and (9) set equal to each other give a single equation relating the transfer function \( F \) to the weighting coefficients \( w_m \) for a given frequency \( \omega \). When dealing with inputs containing a number of frequencies, this method leads to a set of simultaneous equations for the value of \( w_m \) in terms of the values of the transfer function at each frequency. These equations can be solved by using a least squares approach for a set of weighting coefficients, \( w_m \). When applying this method, the operation is over the past time history of the ship motion. The convolution integral is then replaced by a truncated summation:

\[ f(n\Delta t) = \sum_{m=1}^{n} w_m y \left[ (m-n)\Delta t \right] \]  

(10)

After the evaluation of the added inertia force and drag force, the differential equations are solved by Runge-Kutta step-by-step integration procedure.
6. RESULTS

The analytical technique described herein has been applied to the case in Fontijn's paper [3]. The main dimensions of the ship used in the model are: waterline length = 2.438 m, beam = 0.375 m, draft = 0.15 m, block coefficient = 1.0, ship mass = 137.24 kg, water depth = 0.2 m. The added mass and damping coefficients were determined by Fontijn using 2-D theory with three-dimensional effects on the low frequency range. The same hydrodynamic coefficients are used in order to have a fair comparison. Two types of fenders are considered in the centric berthing: a linear fender of constant stiffness, \( k_1 \), equal to 140 kg/m, and a nonlinear fender force represented by \( k_1 \Delta y_f + k_2 (\Delta y_f - d_f) \), in which \( k_1 = 64 \text{ kg/m} \), \( k_2 = 113 \text{ kg/m} \), \( d_f = 0.00664 \text{ m} \) and \( \Delta y_f \) is the deflection of fender.

Figure 2 shows the comparison between present method with Fontijn's calculated and measured fender forces as a function of time. This analytical method gives good agreement with the experiment. Hydrodynamic coefficients have great influence on the fender response. The use of the existing results is the first step in the development of this method. The nonlinear fender stiffness is calculated for the same ship in Figure 3. Fender force and ship motions for eccentric berthing on linear fenders are given in Figures 4 and 5. The ship characteristics and water depth are same as centric impact. Fender stiffness, \( k_1 \), equals to 65 kg/m. In these figures, the quantity \( m_o \) is defined as \( \rho L E D m_0 (\rho L E D e_0^2 + m_0)^{-1} \). Theoretically, the program has been formulated to accept characterization of any fender system for the calculation of energy absorption. Various generic fender system algorithms have been developed which will be part of further program development.

Hayashi and Shirai's method [4] is considered a simple way to calculate the added mass coefficient. They took into account the shallow water, ship speed and characteristics. That method was modified by Oda [12] to have a better agreement with experiments. Based on the modified theory, fender force on the same model used in this paper is presented in Figure 6. The approaching phase shows better agreement than the detaching phase; Oda shows the similar behavior in his results.

If one takes into consideration the ship hull, then the fender force depends on the hull stiffness and mass. For the model considered above, we assume the local hull is simulated by aluminum plate with size 0.3 m by 0.3 m and 0.006 m thickness; total weight on the plate is 15 kg. The fender reaction is almost never affected by the hull under these circumstances (stiffness is about 100,000 kg/m). If the hull stiffness is reduced to 1000 kg/m (academic purpose), then the maximum fender force changes about 6.5 percent.
7. CONCLUSIONS

The frequency dependence of the hydrodynamic coefficients can be easily represented by a simplified convolution integral. This method has good agreement with existing theory and experiment. The hydrodynamic coefficients significantly affect the vessel-fender interaction. More experimental and theoretical studies are needed in order to include additional environmental effects. Local hull effects do not significantly affect the dynamic response problem. This method provides a means of analyzing the dynamic ship/fender problem in a simplified cost-effective manner.

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9. REFERENCES


DEFINITION SKETCH of PLAN SECTION

FIGURE 1

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Figure 2: Time History of Fender Force: Linear Fender

\( \frac{1}{v/\sqrt{k_f m}} \) vs. \( 1/\sqrt{k_f/m} \)

Theory
Exp
Theory
Theory

\( \nu/\sqrt{\dot{h}} = 0.0251 \)
TIME HISTORY of FENDER FORCE:
NON-LINEAR FENDER

\[ \frac{1}{\sqrt{k_1/m}} \]

\[ \text{Theory} \]

\[ \text{Exp} \}

\[ \text{Fontijn} \]

\[ v/hn = 0.0139 \]

\[ \text{Figure 3} \]

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FIGURE 4
TIME HISTORIES OF ECCENTRIC BERTHING
\( e_0/L = 0.167 \)
FIGURE 5
TIME HISTORIES of ECCENTRIC BERTHING

\(e_0/L = 0.5\)
TIME HISTORY of FENDER FORCE: COMPARISON

**FIGURE 6**

- Present Method
- Hayashi et al