Experimental Designs for Quantal Response Models

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Experimental design, sensitivity experiment, binary response model, multiple response model, stochastic approximation, efficiency gain, multiple comparison.

Three problems have been studied:

1. A new sequential design procedure has been developed for estimating the percentiles of a quantal response curve which describes the probability of response as a function of stimulus level. It is asymptotically fully efficient and distribution-free. For small samples it outperforms the best Robbins-Monro procedure in an extensive empirical study. The percentages
of saving range from 25% to 60%.

2. It is shown that, by classifying the outcome of a sensitivity experiment into more than two categories, the parameters in a parametric response curve can be estimated more precisely. The theoretical result is obtained via the Missing Information Principle. The empirical study points out where substantial gains can be achieved.

3. Several procedures for the multiple comparison of all linear contrasts in unbalanced situations are compared both empirically and theoretically. Recommendations for the choice of procedures are given.
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1. **Statement of Problems**
   a. Efficient sequential design of sensitivity experiments involving binary responses.
   b. Information gain (in terms of saving the number of runs) from classifying a response into three (or more) ordered categories instead of the ordinary binary responses.
   c. Multiple comparison in unbalanced one-way analysis of variance.

2. **Summary of results**
   a. C.F.J. Wu, the Principal Investigator, has developed a new sequential design procedure for estimating the percentiles of quantal response curve in a sensitivity experiment. Its updating rule is based on an efficient summary of all information available via a parametric model. It turns out to be related to a stochastic approximation scheme and is asymptotically distribution-free. A "logistic-MLE" version of Wu's procedure substantially outperforms in a simulation study all other competing procedures, including the optimal Robbins-Monro procedure. It results in saving between 25% to 60% runs. The results are reported in T.R. No. 42 MSRI, Berkeley. The paper was submitted for publication.
   b. In many sensitivity experiments conducted in the Army laboratories, the outcomes are classified into two categories (penetrate, not penetrate; animal alive or dead; etc.). Very often it is possible to have more refined classification of the outcomes (e.g., penetrate, partially penetrate, not penetrate.) This will require more work on the part of the experimenters. The question is whether it is worthwhile? It is so if it results in more precise estimation of the parameters of interest, which in turn implies that fewer runs
are needed for a given precision. The P.I. and his Research Assistant S.K. Tse have completed theoretical and empirical studies of this issue. They show that for a large class of parametric models and most parameters of interest, the tertiary-response model provides more precise estimation than the binary-response model. The gain ranges from minor to substantial, depending on the experimental design scheme and the values of the unknown parameters. The results will soon appear in a MRC technical report.

4. Mr. S.K. Tse, who was on the R.A. support of DAAG 29-82-K0154, will complete his Ph.D. degree requirements by the summer of 1984.