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THESIS

PROBABILISTIC FATIGUE LIFE PREDICTIONS
OF STRUCTURAL COMPONENTS IN
HIGH-CYCLE FATIGUE REGIMES

by

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June 1983

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Probabilistic Fatigue Life Predictions
of Structural Components in
High-Cycle Fatigue Regimes

by

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Submitted in partial fulfillment of the
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ABSTRACT

A principal mode of failure of structural components in mechanical systems is fatigue. One method of predicting the probability of fatigue failure of a structural component is to determine the probability that the calculated cumulative fatigue damage index is greater than the critical damage index at failure. The cumulative fatigue damage index is represented as a random variable, and the critical damage index is represented by the statistical variance of existing experimental data. A FORTRAN computer code using this failure criteria is presented, which calculates the probability of failure for a structural component in the high-cycle fatigue regime under a random stress response environment, using both the Weibull and log-normal statistical distribution models. The Weibull model has been found to be the more conservative model in the low probability of failure region, which is consistent with failure predictions between the two models using the classical failure criteria of cyclic life.

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I. INTRODUCTION

Fatigue is a many faceted phenomenon, influenced by such diverse factors as component surface condition, mean and maximum stress, prestrain, temperature, loading rates, stress concentrations, corrosion, axial orientation of loading, loading sequence and random stresses or strains incurred [Ref. 1] through [Ref. 5]. All of these factors, as well as the material properties of the component undergoing the fatigue process, have an enormous statistical variation in their determined magnitudes. This variation in parametric values has led to the development of reliability analysis techniques for predicting the useful life of a component.

Early prediction models were developed from extensive case history data files. From this information, mean stress and cycles to failure were readily available, so the most prevalent analysis models are based on cyclic stress history techniques. To apply these models to a new design, extensive testing must be accomplished to obtain sufficient data to get reasonable resolution of predicted results. The testing problem becomes even more severe for components operating in the "infinite life" region of stress-life (S-N) curves.

Many structural components (i.e., heat exchangers, offshore structures, etc.) are subject to random loadings, such as flow excitation, with generally small vibration amplitudes

and stress/strain cyclic lives in excess of 10^{10} cycles. For this situation, the fatigue problem is one of high-cycle random fatigue [Ref. 6]. Due to a lack of experimental data and case histories for structures in this regime, fatigue failure prediction models based on the cumulative damage index of the Palmgren-Miner Linear Damage Law are being proposed.

The purpose of this research was to develop a FORTRAN computer code based on the cumulative damage index that could be used to predict the probability of failure for a structural component in the high-cycle fatigue regime under a random stress response environment. The computer code developed calculates the probability of failure for such a component, utilizing both a Weibull and a log-normal statistical distribution procedure. The computer code was tested by using existing experimental data, with the predicted results of the two distributions compared to each other. Additionally, the trends of the predicted values are compared with published descriptions of the behavior of these two distribution models as used in cyclic life analysis.

The predicted probabilities of failure for the two models were compared for both single component structures, and simple, multiple component structures. In the multiple member case, the weakest link analogy was used as the structure failure criteria. In this analogy, the structure is assumed to fail when the weakest component of the structure fails.

This document contains the analytic development of equations used in the computer code, as well as extensive graphical presentation of calculated results. The computer code and a users guide are presented as appendicies. Recommendations for future work, such as incorporation of multiaxial fatigue analysis and effects of maintenance procedures are also given.

II. BACKGROUND

A. FATIGUE

Fatigue has been described as a process of progressive localized permanent structural changes in a material due to strains at some point in that material, which may result in cracks or fracture after a sufficient number of cyclic fluctuations [Ref. 7]. Fatigue can be classified into two categories; low-cycle fatigue and high-cycle fatigue. In the low-cycle fatigue regime, plastic strain predominates and ductility controls performance. In the high-cycle fatigue regime, elastic strain dominates and strength control performance. In the high-cycle fatigue regime, most of the fatigue life is spent in the crack initiation phase, with a relatively short portion of life spent in the crack propagation phase.

In that the analytic technique developed here contains material fatigue strength as one of its control variables, and utilizes stress vice strain analysis, the model is only applicable to high-cycle fatigue life predictions. No applicability to low-cycle fatigue is intended or implied, thus the following discussion is limited to high-cycle fatigue. Many factors affect fatigue life in the high-cycle fatigue regime. Two of these factors are solely stress-dependent functions, and deserve some amplification. Additionally, a brief discussion of S-N curves is presented.

1. Loading Sequence

The loading sequence effect has received extensive investigation. One simple, but descriptive presentation of the effect was tendered by Dowling [Ref. 8]. Figure 2.1 is a representation of the potential orders of stress loading sequence.

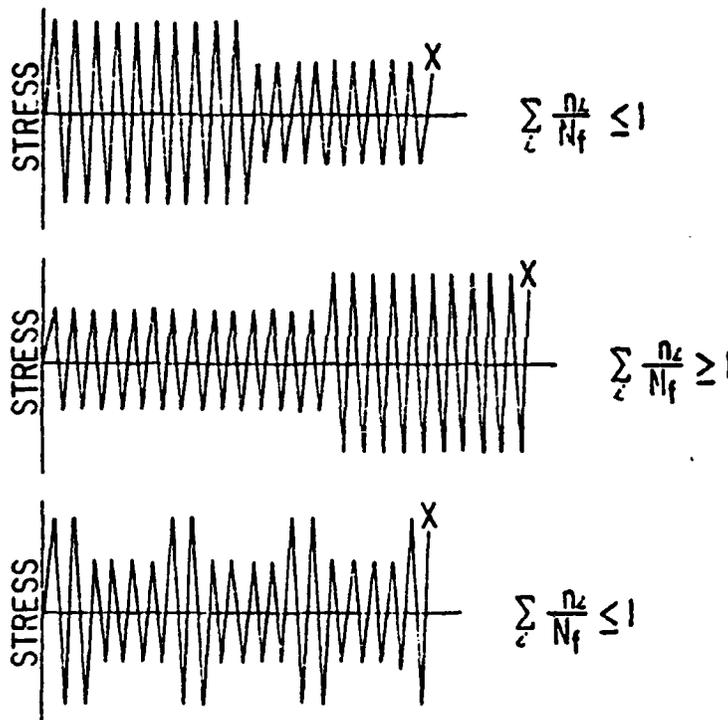


Figure 2.1 SEQUENCE EFFECTS ON CRACK INITIATION (from Ref. 8)

For the high-low stress loading sequence (Fig. 2.1(a)), a crack may initiate in the high stress range, with resultant failure in the low stress range after fewer operating cycles

than would normally be expected. For the low-high loading sequence (Fig. 2.1(b)), application of a large number of cycles at the low stress level does not significantly affect the number of cycles required for failure at the high loading level. For multiple changes in loading sequence (Fig. 2.1(c)), a crack initiated at a high level can propagate in the low level. In all three cases, cracks are assumed to initiate in the high stress loading level, but to propagate in either stress loading level. These cracks are only cracks due to loading sequence, and do not include cracks that would have developed regardless of loading level (i.e., constant amplitude stress loaded structures can develop cracks). Thus, loading sequence has a significant effect on fatigue life.

2. Mean Stress

Realistic fatigue life prediction in cases where the mean stresses are large, relative to the fluctuating stresses, must account for the affect of the mean stresses. Due to the relatively low fluctuating stress amplitudes in the high-cycle fatigue regime, mean stress effects are very important. If the strain levels are high enough to cause repeated plastic straining, mean stresses are rapidly relaxed and mean stress effects can be neglected [Ref. 6]. However, high-cycle fatigue is dominated by elastic strain, not plastic strain, so mean stress effects must be accounted for.

Various investigators have proposed models which estimate the effect of mean stress, σ_0 on fatigue life. Most of these models are based on the following relationship:

$$\Delta \sigma(N) = a(\sigma_0) \Delta \sigma_{eq}(N) \quad (\text{eqn } 2.1)$$

where $\Delta \sigma(N)$ = stress range including the effect of mean stress as a function of cycles, N

$\Delta \sigma_{eq}(N)$ = equivalent stress range that would cause failure in the absence of a mean stress expressed as a function of cycles, N

$a(\sigma_0)$ = mean stress correction coefficient

The four most frequently used models for $a(\sigma_0)$ are graphically depicted in Figure 2.2. Examination of these curves demonstrate a significant difference in predicted safe values for the design stress range. Comparison of the models with experimental data for ductile steels indicates the Goodman and Soderberg curves are too conservative, while the Gerber parabola seems to be the best representation. The Goodman model does correspond rather well with brittle steel data, and as fracture strength approaches ultimate strength, the SAE model approaches the Goodman models predicted value [Ref. 6].

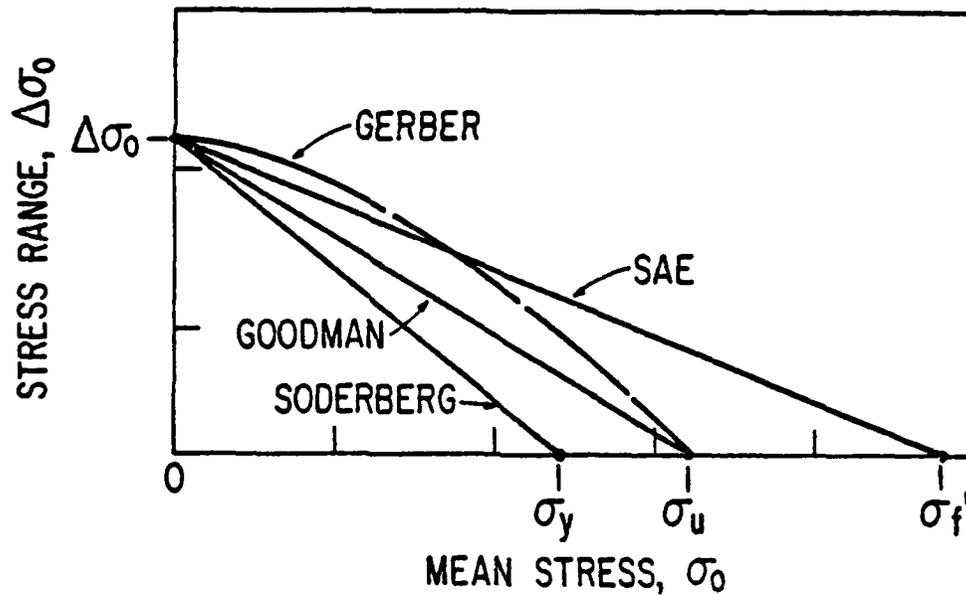


Figure 2.2 PROPOSED MODELS FOR THE EFFECT OF MEAN STRESS
(from Ref. 9)

Goodman model:	$1 - (\sigma_o / \sigma_u)$
Soderberg model:	$1 - (\sigma_o / \sigma_y)$
SAE model:	$1 - (\sigma_o / \sigma_f^1)$
Gerber model:	$1 - (\sigma_o / \sigma_u)^2$

where σ_o = mean stress

σ_u = ultimate tensile strength

σ_y = yield strength

σ_f^1 = fatigue strength coefficient (or true fracture strength)

3. Stress-Life Curves

Stress-life (S-N) curves have been used for deterministic fatigue analysis for decades. Typically, S represents the applied stress, and N represents number of cycles, or life to failure. These plots usually are presented on semilog or log-log coordinates. Most S-N curves have a continuously sloping trace, but some curves (primarily low strength steels), have a discontinuity, with an essentially horizontal segment from 10^6 to 10^7 cycles.

This marked cessation of non-zero slope led to the first definition of endurance (fatigue) limit. The fatigue limit is the limiting value of the median fatigue strength as N becomes very large. The median value is used due to the scatter in test values. Commonly, test data at several stress levels is obtained, and then S-N curves are drawn through the median points, and thus represent fifty percent expected failures [Ref. 10]. This statistical scatter, and subsequent median value representation is an added reason for using probabilistic fatigue analysis, vice deterministic fatigue analysis.

Fatigue is usually classified as either low cycle or high cycle fatigue, with the transition occurring around 10^5 cycles. Most high cycle fatigue test data terminates around 10^7 cycles, and the preponderance of S-N curves in the literature range from 10^0 to 10^7 cycles. This study is concerned with fatigue lives well above the published curves, i.e.,

fatigue lives greater than 10^{10} cycles, so an extrapolation method of available data and curves was needed.

Recent literature classifies fatigue lives in the range of interest as being in the "ultra-high cycle fatigue" region, rather than the usual phraseology such as fatigue life in the "infinite life" region of the S-N curve, because of the low, or poorly defined endurance limit of most materials. Manson [Ref. 11] presents an excellent discussion on all three ranges of fatigue life, and of importance to this study, presents a summary on ultra-high cycle fatigue extrapolation. This summary is based on the work of G. R. Halford, of the ASME Nuclear Pressure Vessel Piping Code Activity. In that reference material is available in which techniques for, and justification for, extrapolating known S-N curves into the region of interest in this study, the assumption of the availability of the appropriate S-N curves for fatigue life predictions is valid.

B. PALMGREN-MINER HYPOTHESIS

In 1945 Miner developed a cumulative linear fatigue damage model, based on criterion suggested by Palmgren in 1924 [Ref. 10]. Within the context of this hypothesis, fatigue failure occurs when the cumulative fatigue damage index reaches one(1). Because of its simplicity, the model has been widely used in fatigue analysis. However, it does not include consideration of the statistical variability existing in fatigue behavior.

The critical value (i.e., the actual value of the cumulative damage index at failure, not the assumed value of 1.0 at failure) is not always close to one (1), but has a range of values from 0.18 to 23, with only a small portion close to one [Ref. 12].

To account for the statistical variability of the critical damage index (denoted as Δ), two probability distribution models have been used. These are the Weibull and the log-normal distributions. Wirsching [Refs. 12, 13] suggests using the log-normal distribution for Δ with a mean of 1.00 and a coefficient of variation of 0.65, based on the collection of experimental data of several investigators. Buch [Ref. 14] also examined numerous sets of existing experimental data, investigating the variation of the critical damage index Δ , for different loading spectra and structural components.

Selecting thirty-seven sets of data from Buch's paper, order statistics [Refs. 15,16] were used to reduce the data and plot it on probability paper. A best fit analysis was conducted, and the Weibull distribution with a mean of 0.9 and a coefficient of variation of 0.67 was found to be the best representation of the data, as shown in Figure 2.3. The Weibull distribution shape parameters b and c were determined to be 0.9791 and 1.3289 respectively.

From the mean and coefficient of variation of Δ , the standard deviation of Δ was determined to be 0.4781. Accepted probabilistic theory indicates 99.7% of all possible values of

any distribution will be contained within the range of the mean plus and minus three standard deviations. Thus the statistical range of the Δ 's from Buch's data is 0.00 to 2.33, excluding the tail regions, while the actual range is 0.01 to 3.40. This is a significant deviation from the value 1.0 of the Palmgren-Miner hypothesis, and demonstrates the need for the development of a probabilistic fatigue life prediction model.

C. PROBABILITY MODEL OF Δ

The thirty-seven sets of Δ and the resultant shape parameters, mean, standard deviation and coefficient of variation were used to test the developed Weibull distribution model. The same mean and coefficient of variation were used in the log-normal distribution so that a legitimate comparison could be made, with the understanding that the data fit the Weibull distribution better than it did the log-normal distribution, so some discrepancy was expected. The mean and coefficient of variation would be the same, regardless of which distribution actually applied, but tail region scatter affects may be different for each distribution.

1. Weibull Distribution

The probability density function $f_x(x)$ is defined as:

$$f_x(x) = \frac{c}{b} \left[\frac{x}{b} \right]^{c-1} \exp \left[- \left(\frac{x}{b} \right)^c \right] \quad (\text{eqn 2.2})$$

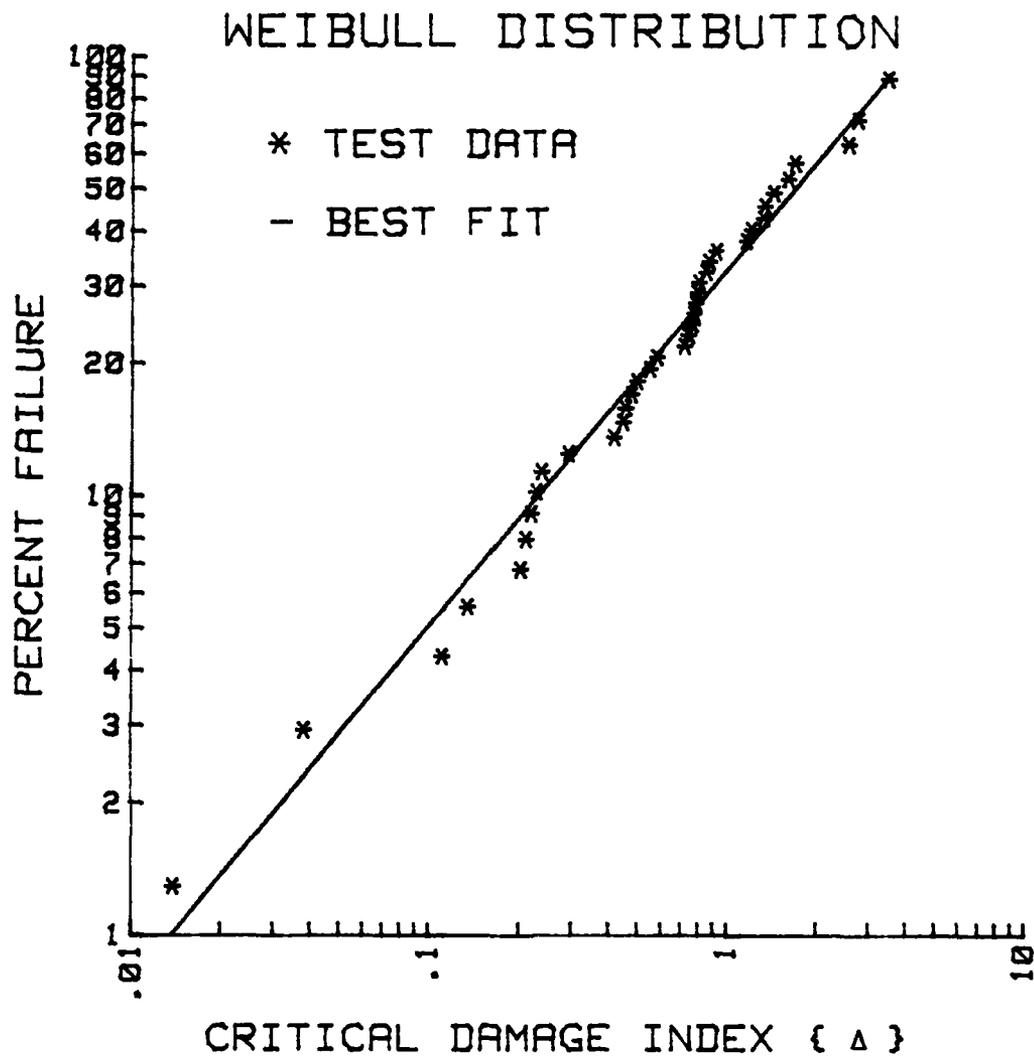


Figure 2.3 WEIBULL DISTRIBUTION OF THE CRITICAL FATIGUE DAMAGE INDEX (Δ) AT FAILURE

The cumulative distribution function $F_x(x)$ is defined as:

$$F_x(x) = 1 - \exp \left[- \left(\frac{x}{b} \right)^c \right] \quad (\text{eqn 2.3})$$

where $x \geq 0$, $b > 0$, and $c > 0$.

The relationship between the shape parameters b and c , and the mean (\bar{x}) and the standard deviation (σ_x) of the random variable x are:

$$\bar{x} = b \Gamma \left[1 + \frac{1}{c} \right] \quad (\text{eqn 2.4})$$

and

$$\sigma_x = b \sqrt{\Gamma \left[1 + \frac{2}{c} \right] - \Gamma^2 \left[1 + \frac{1}{c} \right]} \quad (\text{eqn 2.5})$$

where $\Gamma(a) = \text{Gamma function} = \int_0^{\infty} e^{-t} t^{a-1} dt$.

From equations 2.4 and 2.5, the coefficient of variation, CV is:

$$CV = \frac{\sigma_x}{\bar{x}} = \sqrt{\frac{\Gamma \left[1 + \frac{2}{c} \right]}{\Gamma^2 \left[1 + \frac{1}{c} \right]} - 1} \quad (\text{eqn 2.6})$$

2. Log-normal Distribution

The other, more frequently used, distribution function in fatigue analysis is the log-normal distribution function. It is more prevalently used due to the availability of a closed form mathematical formulation. The probability density function $f_x(x)$ is defined as:

$$f_x(x) = \frac{1}{\sqrt{2\pi} \sigma_x^{\wedge}} \frac{1}{x} \exp\left[-\frac{1}{2} \left(\frac{\ln(x) - \hat{x}}{\sigma_x^{\wedge}}\right)^2\right]; \quad x > 0 \quad (\text{eqn 2.7})$$

where \hat{x} is the mean of $\ln(x)$ and σ_x^{\wedge} is the standard deviation of $\ln(x)$. The cumulative distribution function $F_x(x)$ is defined as:

$$F_x(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln(x) - \hat{x}}{\sigma_x^{\wedge}}} \exp\left[-\frac{u^2}{2}\right] du; \quad x > 0 \quad (\text{eqn 2.8})$$

or

$$F_x(x) = \Phi \left[\frac{\ln(x) - \hat{x}}{\sigma_x^{\wedge}} \right] \quad (\text{eqn 2.9})$$

where $\Phi [\]$ is the standard form of the normal distribution functions. For a random variable x , if x follows a log-normal distribution, then $\ln(x)$ follows a normal distribution [Ref. 17], thus the closed form normal distribution functions can be used. This alleviates the necessity of using numerical integration routines to solve the distribution functions.

The mean, \bar{x} , and the standard deviation, σ_x^{\wedge} , of random variable x are:

$$\bar{x} = \exp \left[\hat{x} + \frac{\sigma_x^{\wedge 2}}{2} \right] \quad (\text{eqn 2.10})$$

and

$$\sigma_x = \exp [2\hat{x} + \sigma_x^2] \{ \exp [\sigma_x^2] - 1 \} \quad (\text{eqn 2.11})$$

Solving for \hat{x} and σ_x in terms of \bar{x} and σ_x :

$$\hat{x} = \ln(\bar{x}^2) - \frac{1}{2} \ln(\sigma_x^2 + \bar{x}^2) \quad (\text{eqn 2.12})$$

and

$$\sigma_x = \sqrt{-\ln(\bar{x}^2) + \ln(\sigma_x^2 + \bar{x}^2)} \quad (\text{eqn 2.13})$$

III. FORMULATION OF ANALYSIS

A. INTRODUCTION

In formulating a probability based analysis of fatigue life for structural components, the measure of fatigue failure can be expressed as the probability of failure, P_f . The failure criterion to be used can be defined such that the design cumulative fatigue damage index (denoted as D), is greater than the critical damage index Δ , at failure. Both damage indexes are subject to statistical scatter, thus the probability of failure is a two variable function. The probability of failure can be defined as:

$$P_f = P_r[D > \Delta] = \int_{-\infty}^{\infty} F_{\Delta}(x) f_D(x) dx \quad (\text{eqn 3.1})$$

or

$$P_f = 1 - P_r[D < \Delta] = 1 - \int_{-\infty}^{\infty} F_D(x) f_{\Delta}(x) dx \quad (\text{eqn 3.2})$$

where F_D , F_{Δ} = the cumulative distribution functions of D and Δ respectively.

f_D , f_{Δ} = the probability density functions of D and Δ respectively.

Due to the paucity of experimental data in the cyclic stress range of interest, a time history simulation model must be used to generate a representative data base for input to the probabilistic analysis model. The simulation model used in conjunction with this study was developed by Y. S. Shin [Ref. 6]. In that a substantial portion of the developed source code consists of this model, pertinent aspects of the model are presented in conjunction with the development of the probabilistic analysis model.

B. CUMULATIVE FATIGUE DAMAGE

As previously discussed, the most widely used criterion for determining fatigue failure for variable stress loads is the Palmgren-Miner hypothesis. The cumulative fatigue damage index, D , is the sum of the damage incurred at each stress level, and can be expressed as:

$$D = \sum D_i = \sum \frac{n_i}{N_i} \quad (\text{eqn 3.3})$$

where n_i = elapsed number of cycles at stress level s_i

N_i = total number of cycles at stress level s_i
 inducing failure in the component

D_i = damage incurred at stress level s_i

For continuous systems, D can be expressed by:

$$D = \int_0^{\infty} \frac{N_o f_s(S)}{N(S)} ds \quad (\text{eqn 3.4})$$

or

$$D = N_0 E \left[\frac{1}{N(S)} \right] \quad (\text{eqn 3.5})$$

where N_0 = the total number of cycles in the design life

$f_s(S)ds$ = fraction of stress peaks between stress
level s and level $s+ds$

$N(S)$ = number of cycles at stress level s

$E[Y]$ = expected value (probabilistic mean) of Y

At failure, $D \geq \Delta$, and from equations 3.4 or 3.5, the total number of cycles to failure (N_T) can be calculated as:

$$N_T = \frac{\Delta}{\int_0^{\infty} \frac{f_s(S)}{N(S)} ds} \quad (\text{eqn 3.6})$$

or

$$N_T = \frac{\Delta}{E\left[\frac{1}{N(S)}\right]} \quad (\text{eqn 3.7})$$

C. RANDOM VIBRATION ANALYSIS

"A stochastic process is employed to analyze the vibration responses statistically, and to determine the type of pressure (e.g., narrow/wide band process) and the peak envelope distribution with statistical parameters (e.g., mean, standard deviation). High-cycle fatigue test data for materials under random vibration are scarce and the commonly available sinusoidal fatigue S-N curves are resorted to as a basis to evaluate the random fatigue life." [Ref. 6]

For a given fatigue life curve, equation 3.6 can be evaluated if $f_s(S)$, the probability density function (PDF) of a

peak stress envelope is known, or can be approximated. In most cases, the PDF cannot be easily derived. However, if the stress history follows a Gaussian process with zero mean stress, closed form solutions for $f_s(S)$ are available.

1. Probability Density Function for Peak Stress Envelopes

The irregularity factor, α , can be defined such that:

$$\alpha = \frac{n_0}{m_0} \quad (\text{eqn 3.8})$$

where n_0 = expected rate of zero (mean stress) crossings

m_0 = expected rate of peaks

If α approaches zero, a very peaky process is indicated, and a Gaussian distribution for $f_s(S)$ can be assumed. Therefore, $f_s(S)$ can be expressed as:

$$f_s(S) = \frac{1}{\sqrt{2\pi} \sigma_s} \exp\left[-\frac{S^2}{2\sigma_s^2}\right] ; \quad -\infty < S < \infty \quad (\text{eqn 3.9})$$

where σ_s is the standard deviation of the stress amplitude.

If α approaches one, a narrow band process is implied, and a Rayleigh distribution for $f_s(S)$ can be assumed. Thus, $f_x(S)$ can be expressed as:

$$f_s(S) = \frac{S}{\sigma_s^2} \exp\left[-\frac{S^2}{2\sigma_s^2}\right] ; \quad 0 \leq S < \infty \quad (\text{eqn 3.10})$$

When the cyclic stress history follows the stationary Gaussian process with zero mean stress [Ref. 18], the PDF for the peak stress envelope can be expressed by:

$$f_s(S) = \frac{\sqrt{1-\alpha^2}}{\sqrt{2\pi} \sigma_s} \exp \left[-\frac{S^2}{2\sigma_s^2(1-\alpha^2)} \right] + \frac{\alpha S}{2\sigma_s^2} \left\{ 1 + \operatorname{erf} \left[\frac{S}{\sigma_s \sqrt{\frac{2}{\alpha}} - 2} \right] \right\} \exp \left[-\frac{S^2}{2\sigma_s^2} \right] \quad (\text{eqn 3.11})$$

where $\operatorname{erf}(x)$ is the Gaussian error function, and can be expressed as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (\text{eqn 3.12})$$

2. Number of Stress Cycles at Stress Level S_i to Cause Failure: N_i

The source data for this analysis can be obtained from existing sinusoidal S-N fatigue curves, constant amplitude testing, or stress limits specified in codes, such as the ASME Pressure Vessel and Piping Codes. For example, if S-N data is available such that

$$S_i = \sigma_f^1 N_i^b \quad (\text{eqn 3.13})$$

then

$$N_i = \left[\frac{S_i}{\sigma_f^1} \right]^{-\frac{1}{b}} \quad (\text{eqn 3.14})$$

where

$$\beta = -\frac{1}{b} ; \beta > 0 \quad (\text{eqn 3.15})$$

In which b is the fatigue strength exponent of the particular material being analyzed. These values have been experimentally determined and tabulated for selected engineering alloys [Ref. 19].

Substituting equation 3.14 into 3.7,

$$\Delta = N_T E\left[\frac{S^\beta}{(\sigma_f')^\beta}\right] = \frac{E[S^\beta]}{(\sigma_f')^\beta} N_T \quad (\text{eqn 3.16})$$

For a given stress history the expected value S , $E[S^\beta]$ can be estimated from:

$$E[S^\beta] = \frac{1}{m} \sum_{J=1}^m S_J^\beta \quad (\text{eqn 3.17})$$

B. CYCLIC STRESS/STRAIN ANALYSIS

When stress histories follow other than a stationary Gaussian process, or have a non-zero mean stress value, closed form solutions are not available for the PDF. The use of the random vibration approach to determine $E[S^\beta]$ is extremely difficult. To alleviate this problem, cycle counting algorithms are used. These algorithms are very useful if cyclic stress/strain histories for the component are known. The major disadvantage of using cycle counting methods is

that a tremendously large amount of data is required to obtain reliable fatigue life predictions. To compensate for the lack of data, a typical stress/strain time history data block is estimated [Ref. 20]. The generated data block is assumed to repeat itself ad-indefinitum, so that fatigue life prediction calculations for high-cycle fatigue can be accomplished.

1. Cycle Counting Algorithms

The object of cycle counting methods is to compare actual, irregular load histories with S-N curves developed from uniformly repeated simple load cycles, which are readily available.

"All good counting methods must count a cycle with the range from the highest peak to the lowest valley and seek to count other cycles in a manner that maximizes the ranges that are counted. This rule can be justified either by assuming that damage is a function of the magnitude of the hysteresis loop, or by considering that in fatigue (as in many other fields) intermediate fluctuations are less important than the overall differences between high points and low points."

"All good counting methods count every part of every overall range once and only once. They also count smaller ranges down to some predetermined threshold once and only once. Three counting methods that achieve this objective are well documented in the literature: range-pair, rainflow, and racetrack." [Ref. 10]

All three methods can be used with, or without mean stress considerations. The racetrack method is considered the most suitable for condensing actual load histories. The rainflow method is the most popular method, and can be used with load histories, or strain histories [Ref. 10].

2. Rainflow Cycle Counting Method

The rainflow cycle counting method is used in this study, because it can identify the stress range associated with low frequency components, and the linear damage law summation procedure allows addition of damage index values from large and small stress ranges [Ref. 6].

The "rainflow counting" method is so named because it reminded its developers, M. Matsuishi and T. Endo [Ref. 21], of rain flowing down a series of pagoda roofs. Rules are imposed on this downward flow so that cycles and half cycles can be differentiated and counted.

In application, a sample stress time history, $X(t)$, is converted to a point process of peaks and troughs as shown in Figure 3.1, with the peaks assigned even numbers, and the troughs assigned odd numbers. The time axis is oriented vertically, with the downward direction being the positive direction. A "rainflow" is started at each peak and each trough. When a rainflow started at a trough comes to a peak, the flow is terminated if the magnitude of the opposite trough is less than the originating trough (e.g., Fig. 3.1(b); paths 1-4, 5-6, 7-8 and 9-10). For a path originating at a peak, flow is terminated by a peak of greater magnitude than the originating peak (e.g., paths 2-3, 4-9). If the rain flowing down a path intercepts flow from a previously counted path, counting of the current path is terminated so that each path is

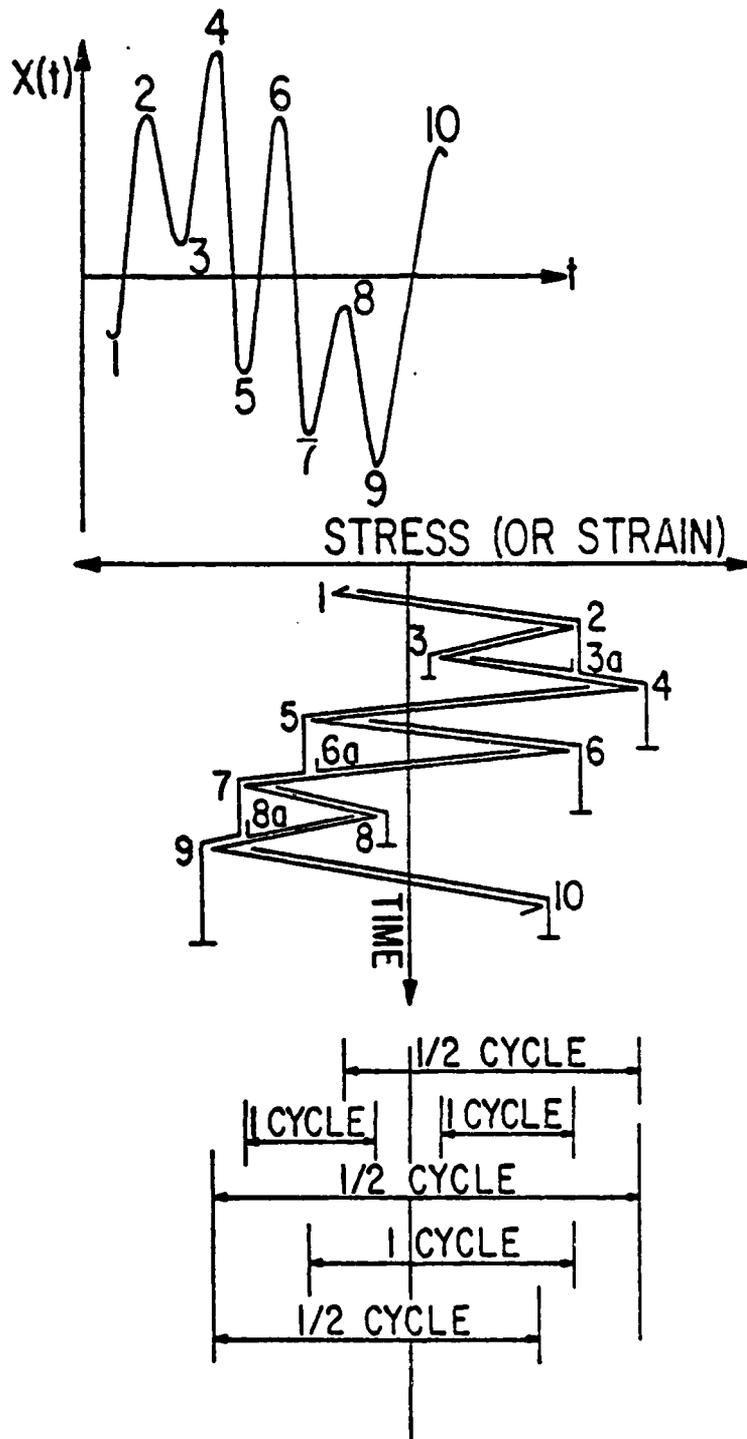


Figure 3.1 EXAMPLE OF RAINFLOW CYCLE COUNTING METHOD
(from Ref. 22)

only counted once (e.g., paths 3-3a, 6-6a, 8-8a). A new path (i.e., cycle count) is not started until the path under consideration is terminated by one of the above rules.

E. WEIBULL DISTRIBUTION MODEL

The probability density function $f_x(x)$ and the cumulative distribution function $F_x(x)$ were defined as equations 2.2 and 2.3. The probability of failure, P_f , was defined as equations 3.1 or 3.2. For this study, equation 3.1 was chosen. Substituting Δ and D into $F_{x=\Delta}(x)$ and $f_{x=D}(x)$ respectively, the new cumulative distribution function can be expressed as:

$$F_{\Delta}(x) = 1 - \exp \left[- \left(\frac{x}{b_{\Delta}} \right)^{c_{\Delta}} \right] \quad (\text{eqn 3.18})$$

and the new probability density function can be expressed as:

$$f_D(x) = \frac{c_D}{b_D} \left(\frac{x}{b_D} \right)^{c_D-1} \exp \left[- \left(\frac{x}{b_D} \right)^{c_D} \right] \quad (\text{eqn 3.19})$$

where b_{Δ} , c_{Δ} = Weibull distribution shape parameters for the critical damage index

b_D , c_D = Weibull distribution shape parameters for the design damage index

From these equations, the probability of failure can now be expressed as:

$$P_f = \int_0^{\infty} F_{\Delta}(x) f_D(x) dx \quad (\text{eqn 3.20})$$

Solution of this integral requires the use of numerical integration techniques. The program developed uses the NONIMSL subroutine DQSF, which is a combination of Simpson's rule and Newton's three-eighths rule. This routine is a fourth order precision algorithm with fifth order truncation error, and was deemed to be accurate enough for the purposes of this study.

F. LOG-NORMAL DISTRIBUTION MODEL

If Δ and D follow log-normal distributions, $\ln(\Delta)$ and $\ln(D)$ follow normal distributions. Therefore, define random variable Z as:

$$\ln(Z) = \ln(\Delta) - \ln(D) \quad (\text{eqn 3.21})$$

Assuming that Δ and D are independent random variables, the mean and the variance of $\ln(Z)$ are expressed by:

$$\hat{Z} = \hat{\Delta} - \hat{D} \quad (\text{eqn 3.22})$$

and

$$\sigma_{\hat{Z}}^2 = \sigma_{\hat{\Delta}}^2 + \sigma_{\hat{D}}^2 \quad (\text{eqn 3.23})$$

where \hat{Z} , $\hat{\Delta}$, \hat{D} = mean of $\ln(Z)$, $\ln(\Delta)$, and $\ln(D)$

$\sigma_{\hat{Z}}$, $\sigma_{\hat{\Delta}}$, $\sigma_{\hat{D}}$ = variance of $\ln(Z)$, $\ln(\Delta)$, and $\ln(D)$

Hence,

$$\begin{aligned} P_f &= P_r[\ln(D) > \ln(\Delta)] = P_r[\ln(Z) < 0] \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\frac{\hat{Z}}{\sigma_{\hat{Z}}}} \exp\left[-\frac{t^2}{2}\right] dt = \phi\left[-\frac{\hat{Z}}{\sigma_{\hat{Z}}}\right] \quad (\text{eqn 3.24}) \end{aligned}$$

The log-normal distribution shape parameter is simpler to obtain than the Weibull shape parameters. For the log-normal model, it is simply the ratio of the means of the two random variables. Within this study, the shape parameter will be called the central safety factor. The central safety factor (n) is expressed as:

$$n = \frac{\bar{\Delta}}{\bar{D}} \quad (\text{eqn 3.25})$$

Using equations 2.12 and 2.13, $(\hat{\Delta}, \sigma_{\hat{\Delta}})$ and $(\hat{D}, \sigma_{\hat{D}})$ can be expressed in terms of the central safety factor and the coefficients of variation CV_{Δ} and CV_D as:

$$\hat{\Delta} = \ln(\bar{\Delta}^2) - \frac{1}{2} \ln(\sigma_{\Delta}^2 + \bar{\Delta}^2) = \ln \sqrt{\frac{\bar{\Delta}^2}{1+CV_{\Delta}^2}} \quad (\text{eqn 3.26})$$

$$\sigma_{\hat{\Delta}} = \sqrt{-\ln(\bar{\Delta}^2) + \ln(\sigma_{\Delta}^2 + \bar{\Delta}^2)} = \sqrt{\ln[1+CV_{\Delta}^2]} \quad (\text{eqn 3.27})$$

$$\hat{D} = \ln(\bar{D}^2) - \frac{1}{2} \ln(\sigma_D^2 + \bar{D}^2) = \ln \sqrt{\frac{\bar{D}^2}{1+CV_D^2}} \quad (\text{eqn 3.28})$$

$$\sigma_{\hat{D}} = \sqrt{-\ln(\bar{D}^2) + \ln(\sigma_D^2 + \bar{D}^2)} = \sqrt{\ln[1+CV_D^2]} \quad (\text{eqn 3.29})$$

Substituting equations 3.26 through 3.29 into equations 3.22 and 3.23,

$$\bar{Z} = \ln(n) + \ln \sqrt{\frac{1+CV_D^2}{1+CV_{\Delta}^2}} \quad (\text{eqn 3.30})$$

and

$$\sigma_z^2 = \ln [(1 + CV_D^2)(1 + CV_\Delta^2)] \quad (\text{eqn 3.31})$$

Hence, equation 3.24 can be written as [Ref. 23]:

$$P_f = \Phi \left[- \frac{\ln(n) + \ln \sqrt{\frac{1+CV_D^2}{1+CV_\Delta^2}}}{\sqrt{\ln[(1+CV_D^2)(1+CV_\Delta^2)]}} \right] \quad (\text{eqn 3.32})$$

Where $\Phi []$ is the standard argument for normal distributions, as previously discussed. The program developed uses the IMSL subroutine MDNOR, which evaluates the absolute value of the standard argument and returns the probability of survival. The probability of failure is then defined as one minus the probability of survival.

G. WEAKEST LINK ANALOGY

In the weakest link analogy, a multiple member structure is assumed to fail when its weakest member fails. That is, the remaining members cannot take on, without failing themselves, the load born by the failed member. For a multiple component structure, the probability of failure can be defined as:

$$P_f = P_r \left[\bigcup_{i=1}^N (\Delta_i - D_i) \leq 0 : i = 1, 2, 3, \dots, N \right] \quad (\text{eqn 3.33})$$

or

$$P_f = P_r \left[\bigcup_{i \in I} (\Delta_i - D_i) > 0 : I = 1, 2, 3, \dots, N \right] \quad (\text{eqn 3.34})$$

where Δ_i = the critical damage index of the i-th member

D_i = the design damage index of the i-th member

$\bigcup_{i \in I} [(\Delta_i - D_i) \leq 0]$ is the occurrence of at least one event $(\Delta_i - D_i) \leq 0$ for any "i"

N = the number of members

$\bigcap_{i \in I} [(\Delta_i - D_i) > 0]$ is the occurrence of all events $(\Delta_i - D_i) > 0$ for all "i"

Assuming " Δ_i " and " D_i " are mutually independent, and the design damage index is the same for all members ($D_i = D$), then

$$P_f = 1 - \int_{-\infty}^{\infty} \prod_{i=1}^N [1 - F_{\Delta_i}(x)] f_D(x) dx \quad (\text{eqn 3.35})$$

where $F_{\Delta_i}(x)$ = cumulative distribution function of Δ_i

$f_D(x)$ = probability density function of D

If the cumulative distribution function Δ_i is the same for all members,

$$P_f = 1 - \int_{-\infty}^{\infty} [1 - F_{\Delta}(x)]^N f_D(x) dx \quad (\text{eqn 3.36})$$

where $F_{\Delta}(x)$ is the cumulative distribution function of Δ
 $f_D(x)$ is the probability density function of D
 As an approximation, an upper bound on P_f can be expressed
 as [Ref. 24]:

$$P_f = 1 - \int_{-\infty}^{\infty} \left[1 - \sum_{i=1}^N F_{\Delta_i}(x) + \sum_{i=1}^N \sum_{j=1}^N F_{\Delta_i}(x) F_{\Delta_j}(x) \dots \right] f_D(x) dx$$

(eqn 3.37)

$$\leq \sum_{i=1}^N \int_{-\infty}^{\infty} F_{\Delta_i}(x) f_D(x) dx = \sum_{i=1}^N P_{f_i}$$

then

$$\max_i [P_{f_i}] \leq P_{f_{\text{exact}}} \leq \sum_{i=1}^N P_{f_i} \quad (\text{eqn 3.38})$$

where P_{f_i} = the probability of failure of the i -th member.

If the critical damage index and the design damage index
 are the same for all members, then equations 3.36 and 3.37
 reduce to:

$$P_f \leq N \int_{-\infty}^{\infty} F_{\Delta}(x) f_D(x) dx = N P_{f_i} \quad (\text{eqn 3.39})$$

and

$$P_{f_1} \leq P_{f_{\text{exact}}} \leq N P_{f_1} \quad (\text{eqn 3.40})$$

where P_{f_1} = the probability of failure of any one member.

IV. COMPUTATIONAL PROCEDURES

To evaluate the probability of failure for a given structural component, a constant amplitude sinusoidal S-N curve (equation 3.13), statistical parameters of Δ , and either the spectral density function or a representative time history of stress response must be known. The overall computational procedure is shown in Figure 4.1, depicting the step-by-step procedure to calculate the probability of fatigue failure. In the following analysis, it is assumed that the spectral density response curve is available.

A. DETERMINE CRITICAL DAMAGE INDEX SHAPE PARAMETERS

For any Weibull distribution, the cumulative distribution can be defined as:

$$F(x) = 1 - \exp \left[- \left(\frac{x}{\xi} \right)^\eta \right] \quad (\text{eqn 4.1})$$

where ξ = Weibull scale parameter

η = Weibull shape parameter

No probability plotting paper exists for direct plotting of distribution data which contains a shape parameter. However, for the Weibull distribution, the shape and scale parameters may be transformed to equivalent scale and location parameters, thus Weibull probability plots can be constructed [Ref. 16].

From equation 4.1, transformation yields:

$$\frac{1}{1-F(x)} = \exp \left[\left(\frac{x}{\xi} \right)^\eta \right] \quad (\text{eqn 4.2})$$

and then,

$$\ln \ln \left[\frac{1}{1-F(x)} \right] = \eta \ln(x) - \eta \ln(\xi) \quad (\text{eqn 4.3})$$

Therefore, a plot of any Weibull variate $\ln \ln[1/[1 - F(X)]]$ plots as a straight line against the natural logarithms of the observations.

Rewriting equation 4.3 as:

$$w = a + bz \quad (\text{eqn 4.4})$$

where $W = \ln \ln[1/[1 - F(x)]]$

$$z = \ln(x)$$

$$b = \eta$$

$$a = -\eta \ln(\xi)$$

then,

$$\hat{\eta} = b \quad (\text{eqn 4.5})$$

and

$$\hat{\xi} = \exp \left[- \frac{a}{b} \right] \quad (\text{eqn 4.6})$$

Where the values a and b are obtained as the y intercept and slope of the plotted line. Within this study, b of equation 4.4 is redesignated c with appropriate subscripts (i.e., c_Δ or c_D), and a of equation 4.4 is redesignated b with appropriate subscripts (i.e., b_Δ or b_D).

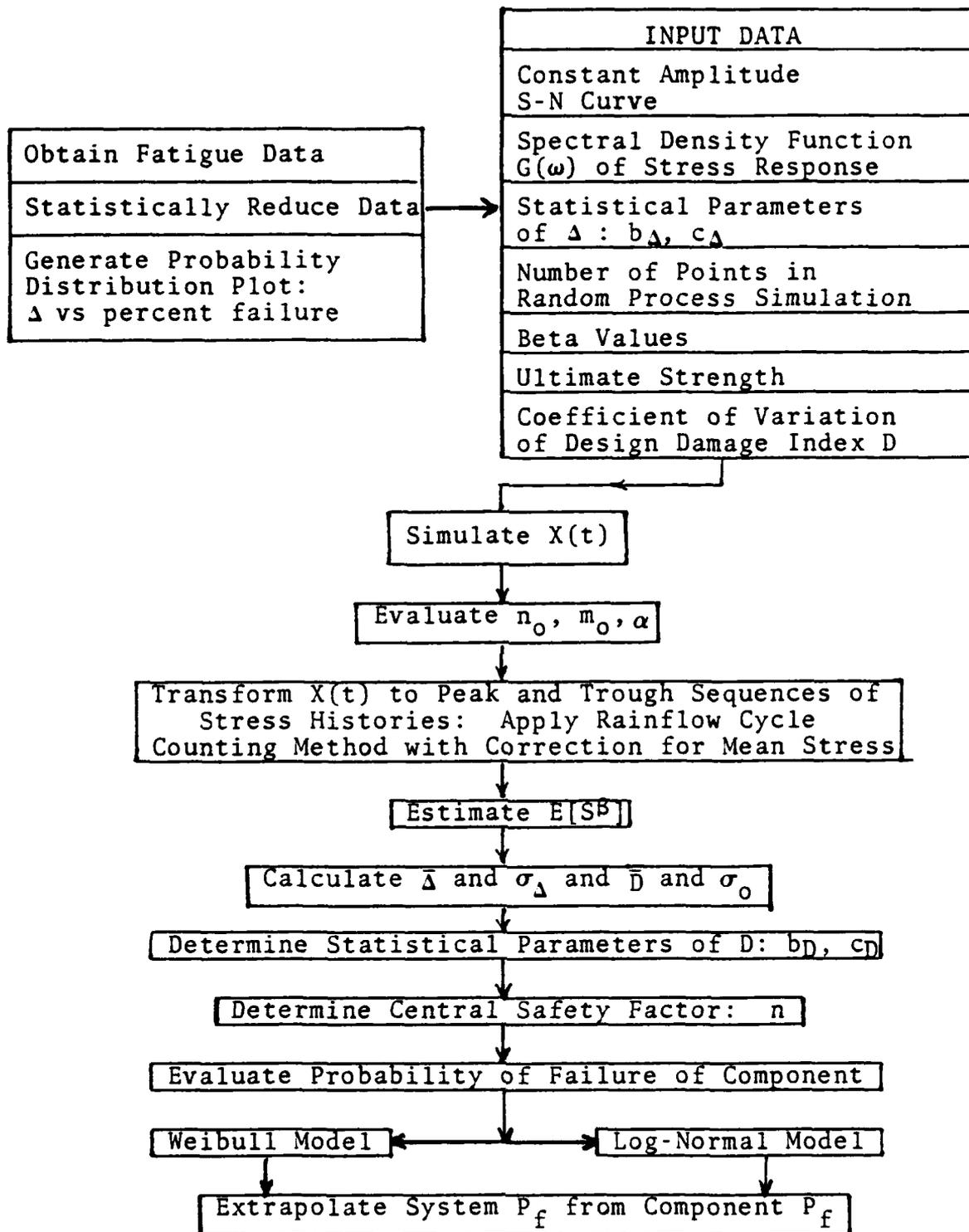


Figure 4.1 COMPUTATIONAL PROCEDURE FLOWCHART

B. SIMULATE STRESS TIME HISTORY X(T)

For a given spectral density function $G(\omega)$, a sample of $X(t)$ is simulated using the following expression [Ref. 25]:

$$X(t) = \sum_{i=1}^j \sqrt{2G(\omega_i) \delta\omega_i} \cos(\omega_i t + \phi_i) \quad (\text{eqn 4.7})$$

and

$$\omega_u = \sum_{i=1}^i \omega_i \quad ; \quad 0 < \omega_i < \omega_u \quad (\text{eqn 4.8})$$

where ϕ_i is a random phase angle, uniformly distributed in the interval $(0, 2\pi)$.

$\delta\omega_i$ = i-th frequency interval

ω_i = frequency at the center of $\delta\omega_i$

ω_u = effective frequency range in $G(\omega)$

This simulation of stress history is for a stress history with zero mean stress. Parametric studies of the effect of mean stress can be accomplished by using the relationship:

$$X(t_i) = X(t_i) + THM \quad (\text{eqn 4.9})$$

where THM is the mean stress amplitude to be studied.

C. CALCULATE n_0 , m_0 AND α

When the spectral density function is known, n_0 , m_0 , and α can be calculated as follows:

$$\eta_0 = \frac{1}{2\pi} \left[\frac{\int_0^\infty \omega^2 G(\omega) d\omega}{\int_0^\infty G(\omega) d\omega} \right] \quad (\text{eqn 4.10})$$

and

$$m_0 = \frac{1}{2\pi} \left[\frac{\int_0^\infty \omega^4 G(\omega) d\omega}{\int_0^\infty \omega^2 G(\omega) d\omega} \right] \quad (\text{eqn 4.11})$$

then α can be calculated using equation 3.8.

D. CALCULATE STRESS RANGES

Transform $X(t)$ to peak and trough time history and apply the rainflow counting method to determine the stress ranges. In the developed source code, subroutine RNFLW counts stress ranges from troughs to peaks, and subroutine RNDRP counts stress ranges peaks to troughs. The mean and standard deviation of each stress range is then calculated by subroutine STAT. These statistical parameters are then averaged to obtain the overall mean and standard deviation of the simulated stress ranges.

E. ESTIMATE $E[S^8]$

The expected value of S^8 with correction for the effects of mean stress can be estimated using one of the mean stress models previously discussed. For this study, the Goodman

model was chosen. The expected value of S^β corrected for mean stress can be expressed as:

$$E [S^\beta] = \frac{1}{K} \sum_{i=1}^K \left[\frac{S_i}{1 - \frac{\sigma_{o_i}}{\sigma_u}} \right]^\beta \quad (\text{eqn 4.12})$$

where K = number of stress cycles counted by the rainflow cycle counting method

σ_{o_i} = mean stress of the i -th cycle

F. CALCULATE $\bar{\Delta}$ AND σ_Δ , AND \bar{D} AND σ_D

From the Weibull parameters b_Δ and c_Δ obtained from the curve-fit procedured previously discussed, the mean, standard deviation, and the coefficient of variation for Δ can be determined via equations 2.4, 2.5, and 2.6. As a result of the rainflow cycle counting of the simulated stress response time history for the given mean and standard deviation of the design fatigue life, the mean and the standard deviation of the design damage index D can be calculated by [Ref. 23]:

$$\bar{D} = \frac{E [S^\beta] \bar{N}_D}{(\sigma_f')^\beta} \quad (\text{eqn 4.13})$$

and

$$\sigma_D = \frac{E [S^\beta]}{(\sigma_f')^\beta} \sigma_{N_D} \quad (\text{eqn 4.14})$$

where \bar{N}_D = mean design number of stress cycles

σ_{N_D} = standard deviation of design number of cycles

G. DETERMINE DESIGN DAMAGE INDEX SHAPE PARAMETERS

The mean and standard deviation of the design damage index previously calculated are fed to the subroutine WEIBR. This subroutine uses an iterative procedure on equation 2.6 with initial guess values of the shape parameters to generate a coefficient of variation. This generated CV is compared to the actual CV determined from $CV = \sigma_D/D$. The initial guess values are then modified and the iteration repeated until the generated CV is within one percent of the actual CV. Once this criterion is met, the calculated shape parameters are returned to the main program.

H. CALCULATE PROBABILITY OF FATIGUE FAILURE

1. Weibull Model

Using equation 3.20, the probability of failure is calculated via numerical integration. In that integration to infinity is not possible, probabilistic theory of distribution curves is utilized to establish the upper limit of integration. From probabilistic theory, it is accepted that 99.7% of the area under a distribution curve is contained within the range of the mean plus and minus three standard deviations of the mean.

The numerical integration routine used, DQSF, uses the mean of either the design damage index, or the critical damage

index as the upper limit of integration for the first iteration. The upper limit of integration is then incremented with one standard deviation of the appropriate damage index and the integral re-evaluated. This iterative procedure is continued until either the predicted probability of failure of the current iteration is increased less than one percent over the previous iteration, or ten standard deviations have been added. This procedure allows maximum penetration into the distributions tail region, and usually accounts for all but $6E-07$ percent of the area under the distribution curve.

2. Log-Normal Model

Using equation 3.32 and the subroutine MDNOR, the probability of survival is calculated. The probability of failure is then calculated as one minus the probability of survival.

V. NUMERICAL EXAMPLES

A. INTRODUCTION

In order to test the developed source code, a sensitivity analysis of several variables affecting the predicted probability of failure was conducted. In that this study is a direct expansion of published material on deterministic analysis of fatigue life prediction [Ref. 6], the control variables were chosen to be the same as in that previous study, with the exception of the time history data block duration. The original study centered on fatigue life predictions for light water reactor components subjected to low amplitude, low frequency stress fields induced by flow excitation.

1. Power Spectral Density of Stress Response

To simulate this low frequency, low amplitude stress response, a power spectral density (PSD) response function was assumed. The original study tested both a single peak and a multipeak PSD, and determined that a single peak PSD used to simulate a multipeak PSD at the dominant frequency with equivalent RMS stress yields a conservative fatigue life prediction, thus is safe for use as an approximation.

In that a single peak PSD can be safely used, all test runs were made using the same single peak PSD. The PSD used is depicted in Figure 5.1. The area encompassed within this simulation yields a σ_{RMS} of 2 ksi, which is one of five values

previously tested. This PSD has a calculated irregularity factor α of 0.935, implying that the PSD is a quasi-narrow band distribution, and that the Rayleigh distribution can be used to solve equation 3.4 for the value of the design damage index.

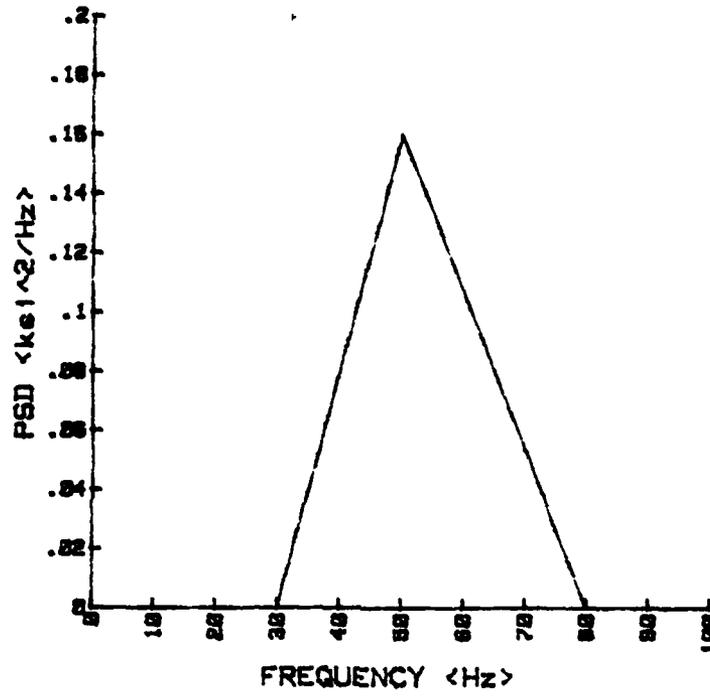


Figure 5.1 RELATIVE POWER SPECTRAL DENSITY RESPONSE

2. Time History Data Block

Having chosen the PSD, equation 4.5 was solved for this PSD, creating the simulated stress time history depicted in Figure 5.2. In the original study, a time history data block of one second was used. For this research, a two second data block was chosen for the second analytic study. For the first

study, the time history data block duration was used as a control variable, with the data block time duration ranging from one to twenty seconds.

The fundamental premise of a time history data block is that it repeats itself ad-indefinitum, and thus, can be used to simulate lives in the infinite life region of the S-N curves. Symmetry of the data block about its time axis midpoint insures repetition of the block. The two second block appears to be more nearly symmetric than data blocks of a shorter time span.

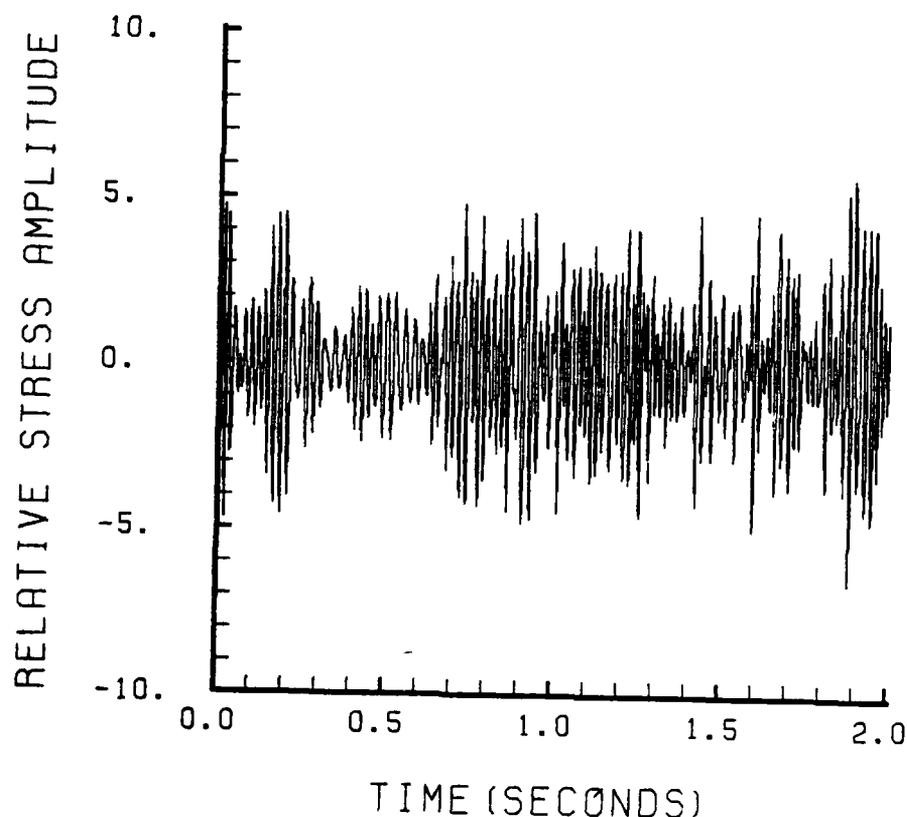


Figure 5.2 SIMULATED RELATIVE TIME HISTORY STRESS RESPONSE

To confirm this observation, data blocks up to twenty seconds in duration were tested. It was found that even time blocks are essentially symmetric about their time axis midpoint, while the odd time blocks are not as close to being symmetric, thus their repetition time is longer than the time interval represented by the data block. None of the twenty examined data blocks exhibited exact symmetry, but all of the even blocks were essentially symmetric, and since the two second block had the shortest repetition cycle, it was chosen as the time history data block duration cycle for use in the second study. Although being further from true symmetry than the even data blocks, the odd time data blocks were retained in the first study because all of the data generated as a result of using those data blocks fell within the acceptance specifications of "Chauvenet's criterion" [Ref. 26; pp. 65-67].

3. Fixed Input Parameters

The remaining input variables that were to be permanently fixed were taken from Shin's paper [Ref. 6], with the exception of the Weibull parameters b_{Δ} and c_{Δ} , which were determined from the data in Buch's paper [Ref. 14], as previously discussed. A summary of the input variables that were fixed throughout all testing is provided as Table I (variable names are fully defined in Appendix A).

TABLE I
FIXED INPUT PARAMETRIC VALUES

<u>VARIABLE</u>	<u>VALUES(S)</u>				
NP	3				
NE	5				
DT	.002				
DF	1.0				
THM	0.5				
SU	60.0				
EXPX(NE)	12.0	12.5	13.0	13.5	14.0
WF(NP)	30.0 50.0 80.0				
SP(NP)	0.00 0.16 0.00				
CL	1.0				
CU	10.0				
BDEL	0.9791				
CDEL	1.3289				

4. Sensitivity Variables

The remaining variables, i.e., the number of points in the random process simulation, the coefficient of variation of the design damage index and the number of stress cycles were the sensitivity variables. Additionally, the beta values were sensitivity variables, but the same five values were used throughout the study, so they are listed as fixed parametric values. In the first analytic study, the coefficient of variation was tested at 0.2, 0.5, and 0.8, and the design cyclic life was fixed at 1.8×10^{10} cycles. This study was to evaluate the sensitivity of each model to the design damage index coefficient of variation.

The second study was of the effect of cyclic life on the models. For coefficients of variation of 0.2 and 0.8,

cyclic life was varied from 10^6 to 10^{13} cycles, while the time history data block was fixed at two seconds (i.e., $DT = 0.002$ seconds, and $IN = 1000$). Plots of probability of failure vs cycles applied were generated for a single component, typically ranging from 10^9 to 10^{12} cycles. Additionally, plots of fifty member structures were generated, with lives ranging from 10^6 to 10^{11} cycles, depending on the coefficient of variation studied [Ref. 24].

B. REDUCTION OF BUCH'S DATA

In order to test the prediction models, critical damage index parameters (i.e., mean, standard deviation, coefficient of variation and Weibull distribution shape factors) were required. Buch [Ref. 14] consolidated test data of several investigators, with the most comprehensive data available being for aluminum alloy 2024-T3 sheet specimens. Thirty-seven sets of these damage index values were reduced and plotted utilizing the methods presented in Chapter IV, section A, and shown in Figure 2.3. The Weibull parameters b and c were then obtained via equations 4.5 and 4.6.

It is recognized that the S-N curves for aluminum alloys do not have a classical "fatigue endurance limit" as do S-N curves for some steels, thus the infinite life region is poorly defined. However, the applied PSD with a σ_{RMS} of 2.0 ksi is below the approximate fatigue limit range for reversed bending of aluminum alloys. The fatigue limit range is listed as 8.0

to 18.0 ksi [Ref. 27; p. 5-9], thus the "infinite life" assumption was deemed valid. Additionally, a review of the simulated relative stress response plot (Figure 5.2) shows that the maximum stress level incurred is about 6.0 ksi, safely below the lower limit of the approximate fatigue limit range.

Within Buch's paper, a limited amount of data on steels was presented. Although the data points were too few in number to obtain a representative data base for probabilistic reduction, the discussed trends were the same as those for the aluminum alloys. Therefore, the specific distribution obtained from the aluminum data is representative of the statistical scatter for most metals, and was deemed acceptable for use as an illustrative example. This observation was repeated in a review of Wirsching's work [Refs. 12,13] and [Refs. 28,29]. The trends of all materials studied, except a smooth specimen composite, were virtually identical.

The mean of the critical damage index for all of the materials was within the range of 0.836 to 1.72, excluding the composites, with coefficients of variation of approximately 0.65. This narrow band led Wirsching to use an assumed mean of 1.00 and CV of 0.65 for all probabilistic studies, regardless of the material involved. In that the precedent has been set for assuming a "typical" statistical distribution is valid for most materials, using fatigue strength exponents associated with steels and a distribution formulated from failure of aluminum alloy samples was accepted as a reasonable combination of

parameters. This combination is possible only because of the wide range of statistical scatter in the available experimental data, and this approximation is presumed to be invalid for data sets having small coefficients of variation.

C. SENSITIVITY TO COEFFICIENTS OF VARIATION

The coefficients of variation of the design damage index studied were 0.2, 0.5 and 0.8. $CV_D = 0.2$ is representative of a distribution with relatively little statistical scatter, while $CV_D = 0.5$ yields a standard deviation of fifty percent of the mean value and represents a fairly large statistical scatter. However, statistical scatter up to seventy percent is common, with scatter up to ninety-eight percent in rare cases, for fatigue studies [Ref. 28]. Thus, $CV_D = 0.2$ is deemed to be conservative, $CV_D = 0.5$ is midrange, and $CV_D = 0.8$ is representative of most "worst case" scatter situations.

The cyclic life values reported ranged between 1.5 and 1.8×10^{10} cycles [Ref. 23], for central safety factors between one and three. Actual values studied ranged from 10^9 to 10^{13} cycles, with central safety factors ranging from near zero to over one thousand. Common engineering safety factors are in the range of one to three, frequently as high as five and occasionally as high as ten, so the range of central safety factors from one to three received the most scrutiny, with some study of the range one to ten. Generated data outside this range was not plotted. The study used a control life of 1.8×10^{10}

cycles, which corresponds to ten years continuous operation of a component subjected to the PSD specified. This value was derived from the deterministic fatigue life prediction determined by the deterministic portion of the source code.

For each coefficient of variation studied, the time history data block was varied from one to twenty seconds by changing the number of points in the random process simulation. In increments of 500 points, the point process simulation was varied from 500 to 10000 points. The time increment was fixed at 0.002 seconds per point. By varying the time data block, the stress response function $X(t)$ was changed. This, in turn, changed the number of simulated cycles counted by the rainflow counting method, thus $E[S^{\beta}]$ and D were varied.

For each value of D , the corresponding central safety factor (equation 3.25) and the predicted probability of failure for each probability distribution model were calculated. Graphical presentations of the probability of failure versus the central safety factor for each distribution, separately and together, were generated.

As can be seen in Figure 5.3, the Weibull model is sensitive to the coefficient to variation up to a central safety factor value of about 2.25. Beyond this value of n , all three CV_D 's studied converge and predict the same probability of failure. Any further change in the time history data block duration has no affect on the separation. Although not plotted,

data runs for central safety factors between ten and twenty were conducted, with no separation in predicted values.

Figure 5.4 presents the results of the log-normal distribution. The log-normal model is sensitive to the coefficient of variation throughout the entire range of central safety factors studied. Below a central safety factor value of about 2, $CV_D = 0.2$ is the most conservative and $CV_D = 0.8$ is the least conservative. At a central safety factor value of about 2, the predicted values cross over, with $CV_D = 0.8$ being the most conservative, and $CV_D = 0.2$ being the least conservative. For all values of n , $CV_D = 0.5$ tracks between the limits established by the other two coefficients of variation.

When the Weibull and log-normal models are compared, each predicts approximately the same probability of failure for central safety factors near one. This compatibility of models was expected, and was one of the acceptance criteria for the source code developed. In the central regions of fatigue data distributions, the log-normal and Weibull distributions are often indistinguishable [Ref. 30]. In this specific case, $n = 1.0$ means $D = \Delta$, and Δ was fixed at 0.9, the mean of the distribution being studied.

Additionally, there is almost always a significant difference in the predicted values within the tail regions of the distribution, with as much as a one order magnitude difference when the actual value is twenty percent or more below the mean value of the index [Ref. 30]. Figures 5.5, 5.6, and 5.7

demonstrate this trend is exhibited by the developed code predictions for each coefficient of variation studied. The predicted probability of failure is nearly equal for the two models for a central safety factor of one, quickly diverge, and then settle out with a near constant separation. Figure 5.8 is included to demonstrate that this separation exists well into the tail region, but decreases as the elapsed life becomes smaller in magnitude.

In that the variable component of the central safety factor is the cumulative design damage index D , and D is a function of elapsed cycles, the higher the value of n , the earlier in fatigue life the model is analyzing. Therefore, the prediction models meet the expected separation of predicted values in the tail region criteria established by the models predecessors.

Based on the coefficient of variation sensitivity analysis, the developed source code does what it was expected to do. That is, it generates essentially equivalent predicted probabilities of failure for the Weibull and log-normal model in the central region of the distribution, and demonstrates the expected divergence of predicted values as the tail regions are entered. In all reference material reviewed, the log-normal model is considerably less conservative than the Weibull model in the tail regions of any distribution, whether the original data fit the Weibull or log-normal distribution better.

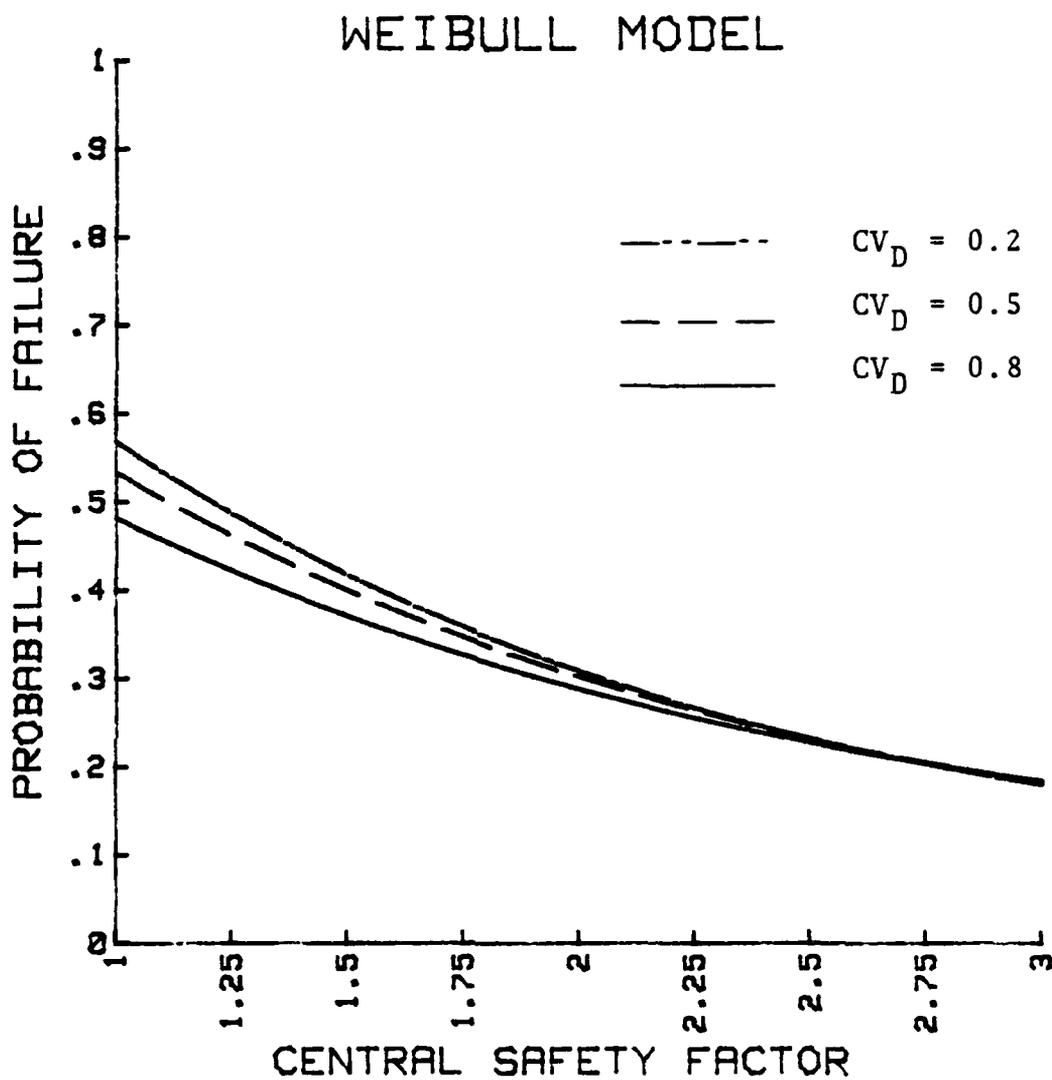


Figure 5.3 WEIBULL MODEL COEFFICIENT OF VARIATION SENSITIVITY

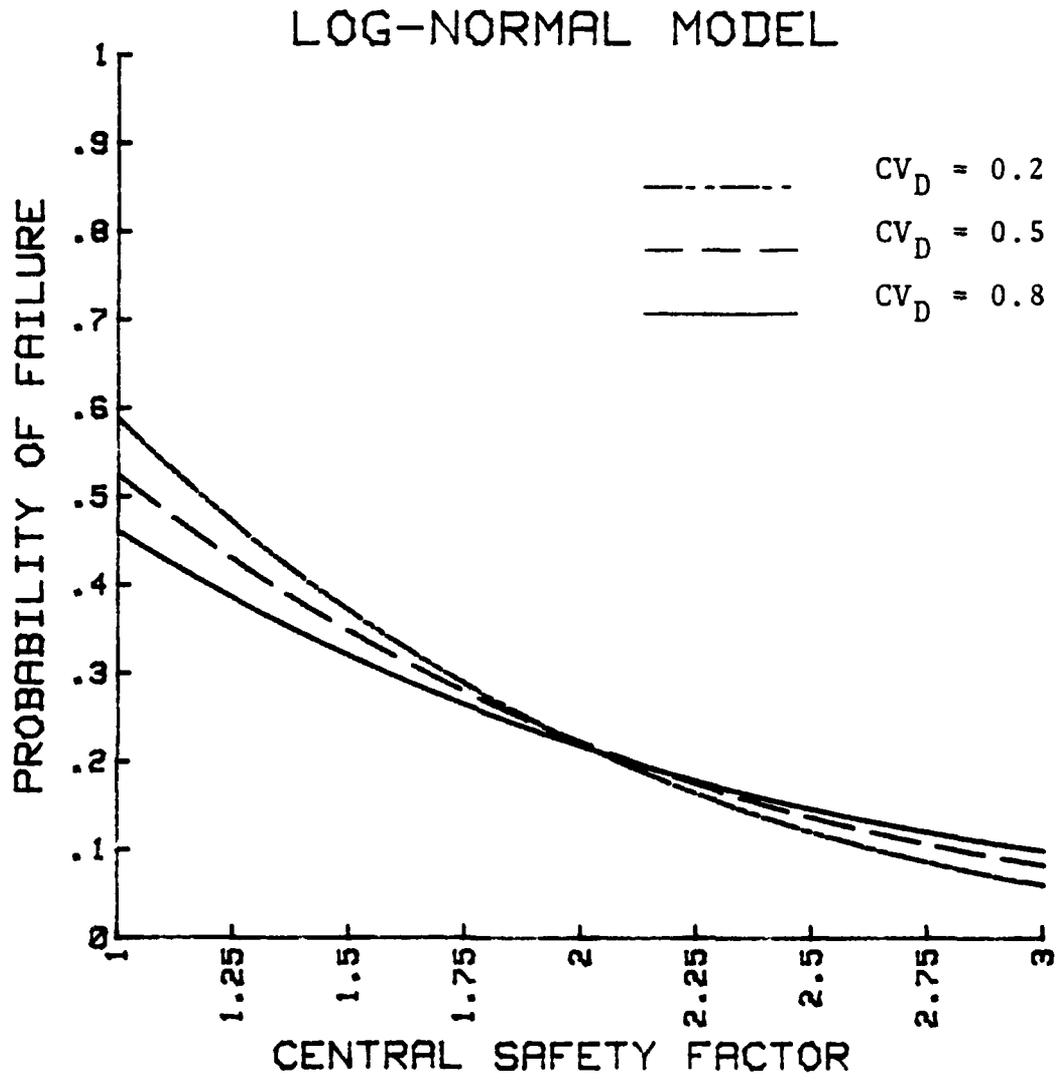


Figure 5.4 LOG-NORMAL MODEL COEFFICIENT OF VARIATION SENSITIVITY

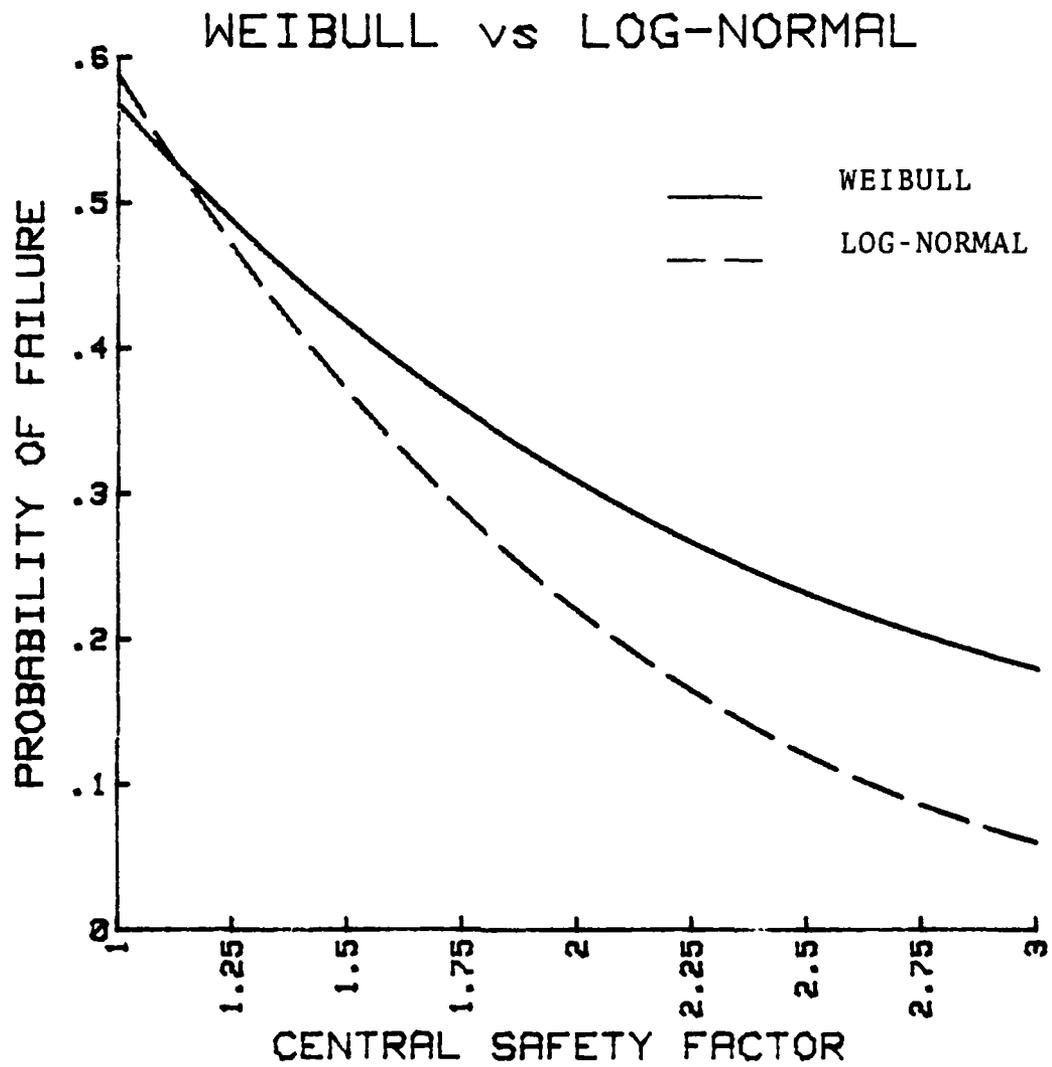


Figure 5.5 WEIBULL VS LOG-NORMAL $CV_D = 0.2$ SENSITIVITY

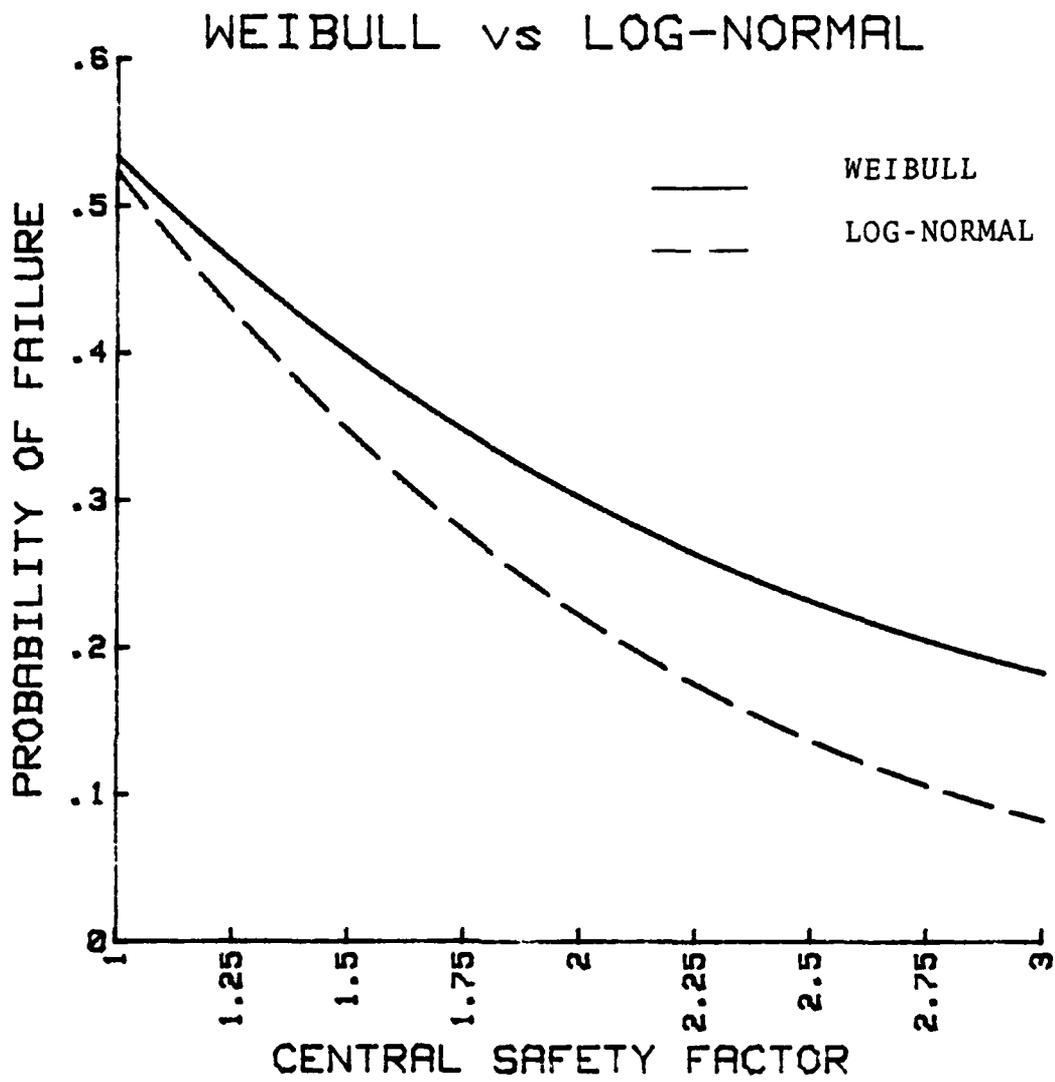


Figure 5.6 WEIBULL VS LOG-NORMAL $CV_D = 0.5$ SENSITIVITY

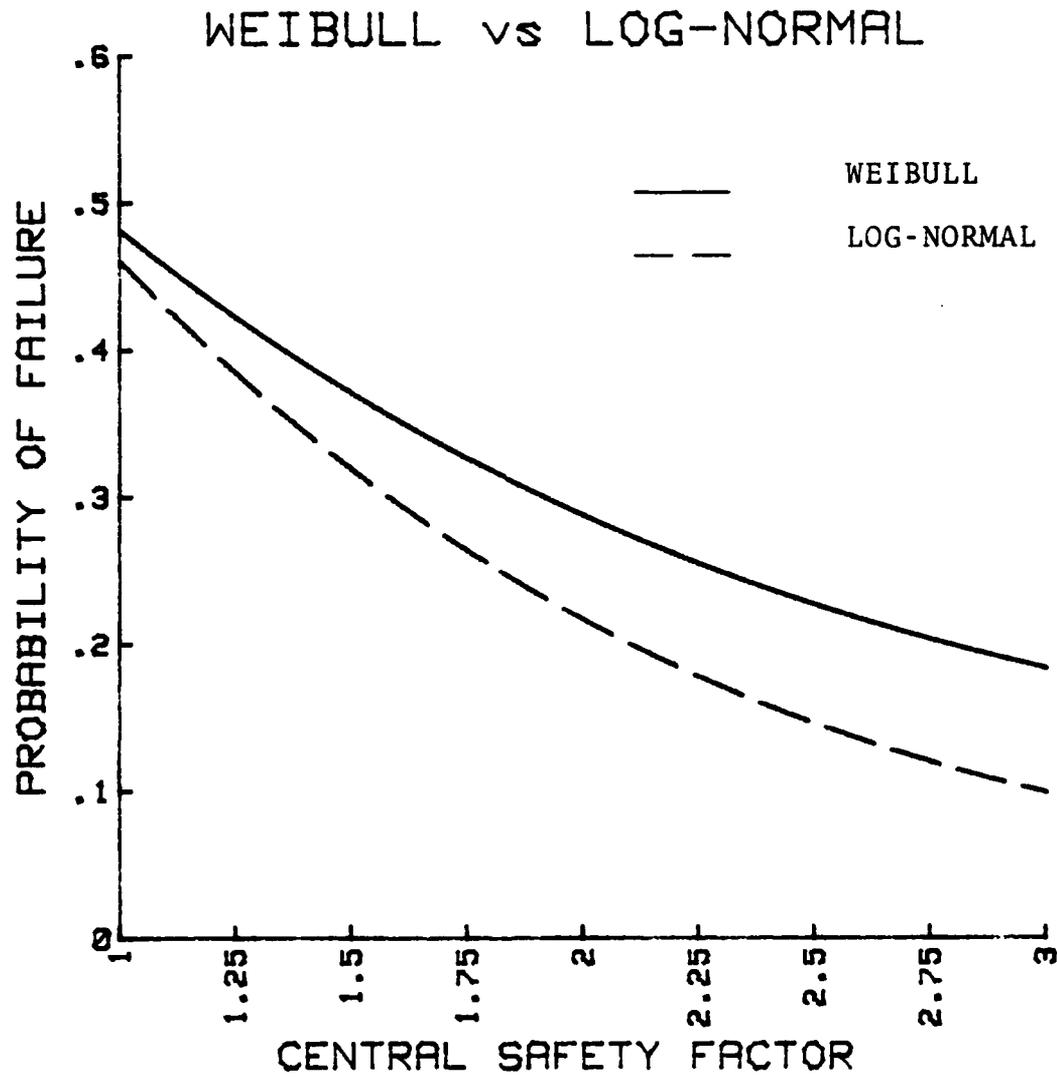


Figure 5.7 WEIBULL VS LOG-NORMAL $CV_D = 0.8$ SENSITIVITY

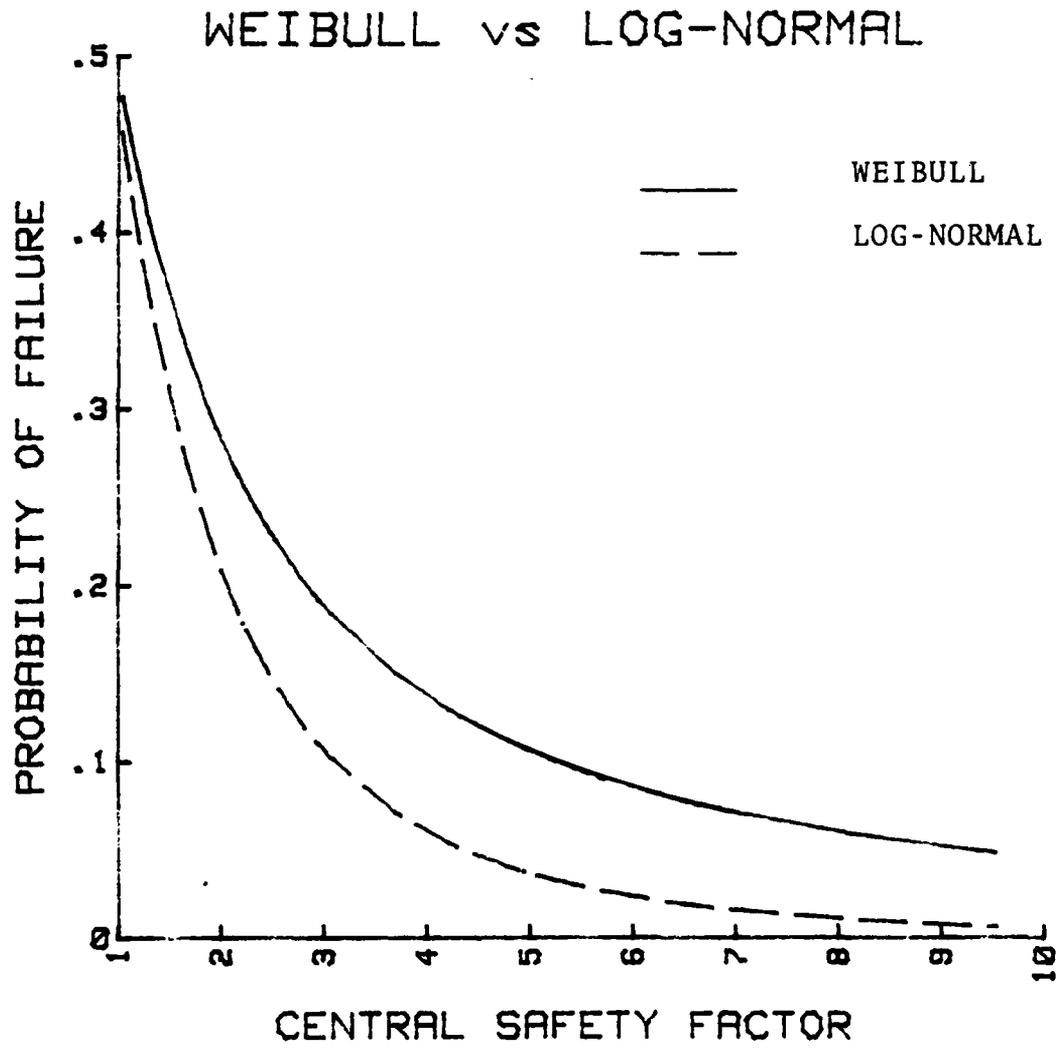


Figure 5.8 WEIBULL VS LOG-NORMAL $CV_D = 0.8$ SENSITIVITY

D. SENSITIVITY TO APPLIED CYCLES: SINGLE MEMBER

Having examined the models sensitivity to the coefficient of variation of the design damage index, testing proceeded, using applied design cycles as the sensitivity variable. Based on the results of the first test phase, the time history data block was fixed at two seconds (i.e., 0.002 second time interval, with 1000 points in the random process simulation). Additionally, the design damage index coefficient of variation was only tested at $CV_D = 0.2$ and 0.8 . In that the Weibull model showed convergence of predicted probabilities of failure for high central safety factors (i.e., low magnitudes of applied cycles), with $CV_D = 0.5$ tracking between $CV_D = 0.8$ and 0.2 in the sensitive range, and although the log-normal model was sensitive to CV_D in all ranges, $CV_D = 0.5$ tracked consistently between the bounds of the other two test values, even after cross over occurred, the second phase testing was conducted without $CV_D = 0.5$ as a test value.

As in the first study, design cyclic life was $1.8 \cdot 10^{10}$ cycles. Beta values of 12, 12.5, 13, 13.5 and 14 were tested for each design damage index coefficient of variation and applied stress cycle value tested. For single member testing, applied cyclic life was varied from 10^8 to 10^{12} cycles.

Figure 5.9 is a presentation of predicted probability of failure versus number of cycles for the Weibull model, for $CV_D = 0.2$. This plot shows that for $\beta = 12$ (the structure with the lowest resistance to fatigue failure), the probability of

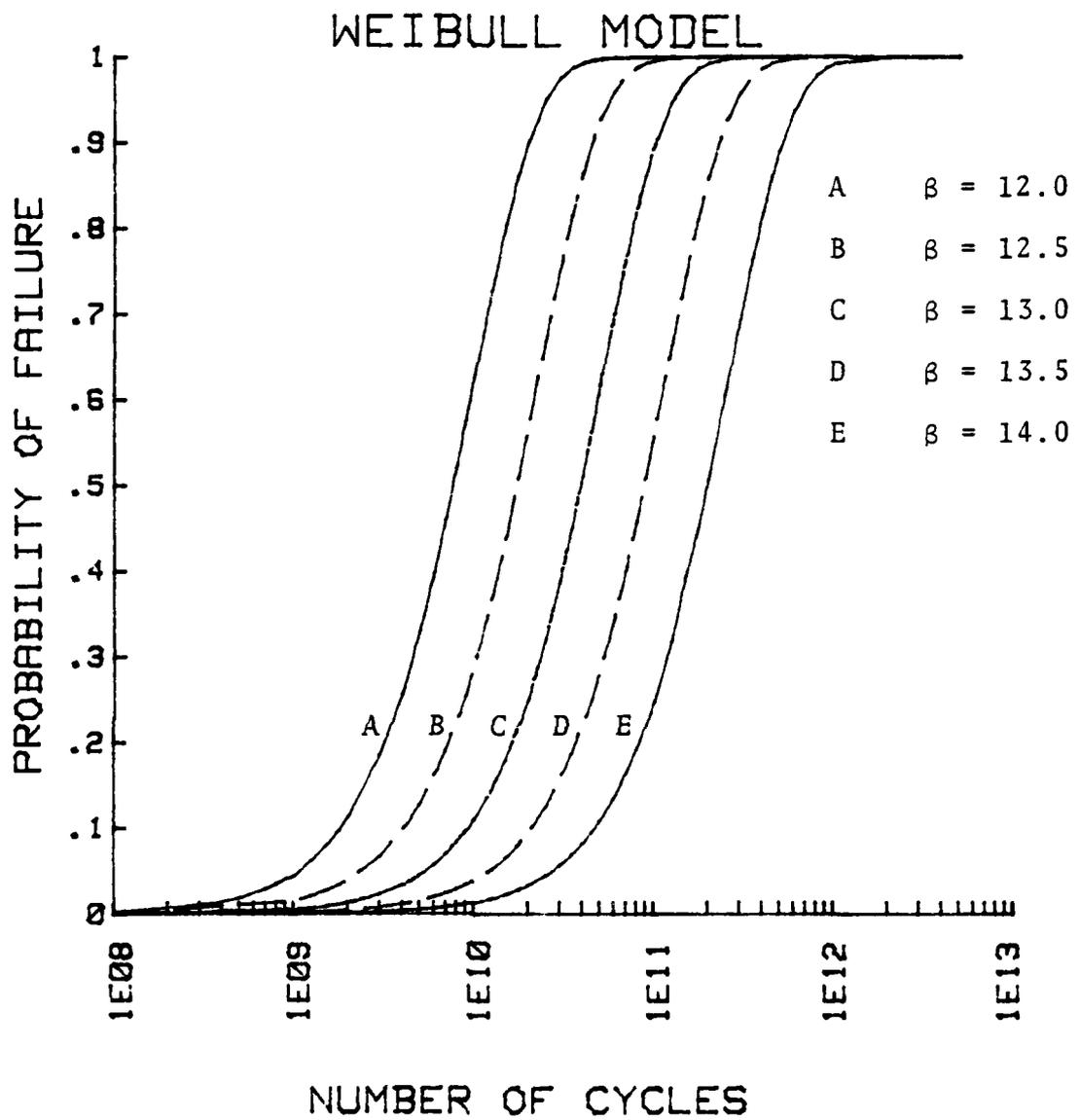


Figure 5.9 WEIBULL MODEL LIFE CYCLE SENSITIVITY $CV_D = 0.2$

failure is about one hundred percent at $1.8 \cdot 10^{10}$ cycles, and as the beta value is increased (i.e., higher resistance to fatigue failure), the components cyclic life is increased. As beta is increased linearly, the design fatigue life increases exponentially. This is amplified in Figure 5.10. When the data of Figure 5.9 is plotted on a log-log scale, as in Figure 5.10, when $\beta = 12$, the probability of failure at $1.8 \cdot 10^{10}$ cycles is one hundred percent, while for $\beta = 14$, the probability of failure is only about two percent.

Figure 5.11 is a presentation of the predicted probability of failure versus number of cycles for the log-normal model, for $CV_D = 0.2$. This plot exhibits the same shape as the Weibull plot, but in the low probability of failure region, the log-normal model predicts an approximate one order of magnitude higher cyclic life before component failure begins, and a slightly lower cyclic life to one hundred percent failure. These differences are amplified in Figure 5.12, which depicts a comparison of the two models. As in the coefficient of variation study, the models are virtually indistinguishable in the central region, and are about one order of magnitude apart in the left tail region of the distribution. As previously discussed, these observations were expected, and felt to be a necessary occurrence in order to establish the source code methodology as valid.

Figure 5.13 depicts the log-normal model on a log-log plot, emphasizing the left tail region performance. Figure 5.14 compares

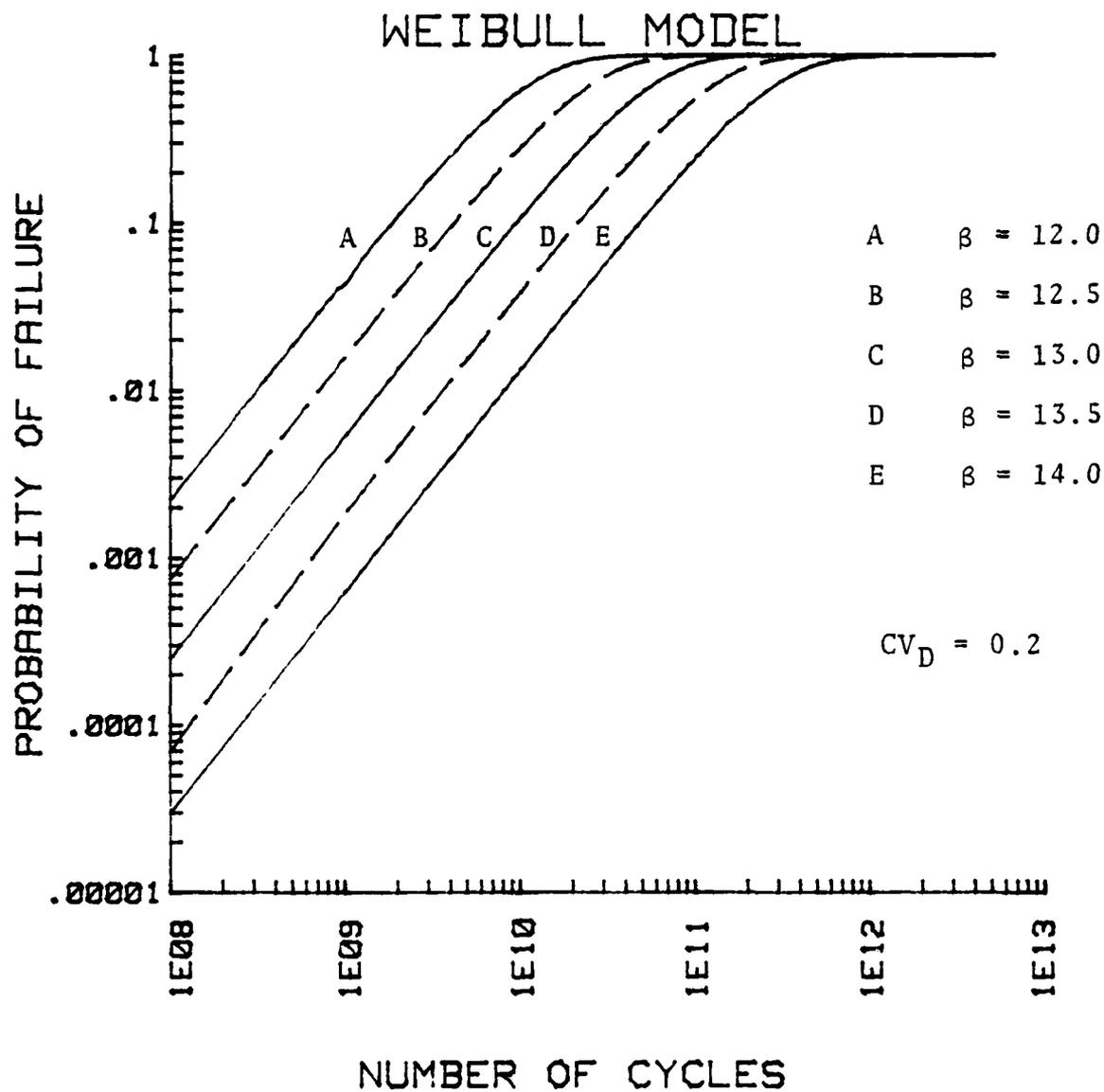


Figure 5.10 WEIBULL MODEL LIFE CYCLE SENSITIVITY: TAIL REGION

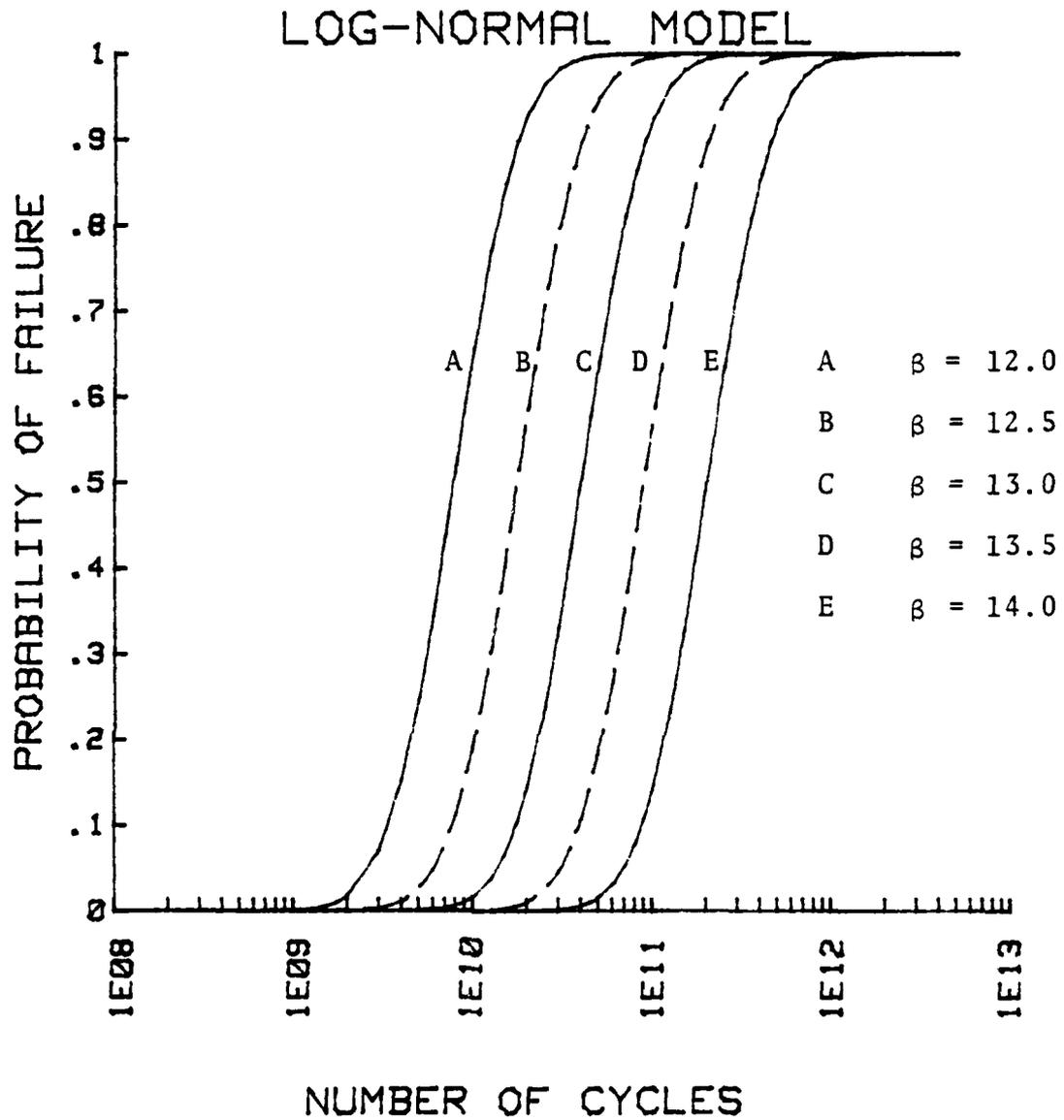


Figure 5.11 LOG-NORMAL MODEL LIFE CYCLE SENSITIVITY $CV_D = 0.2$

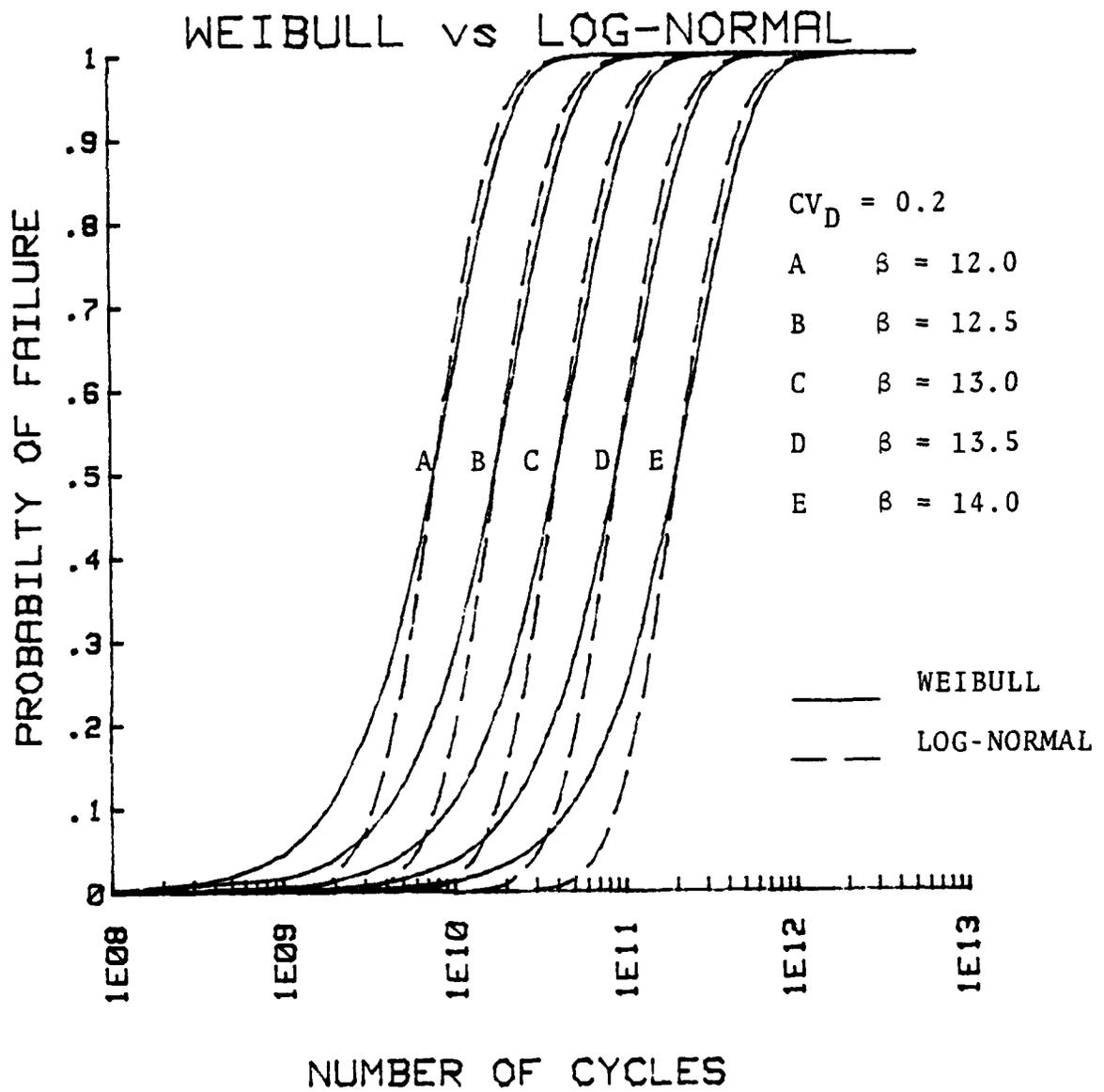


Figure 5.12 WEIBULL VS LOG-NORMAL LIFE CYCLE SENSITIVITY

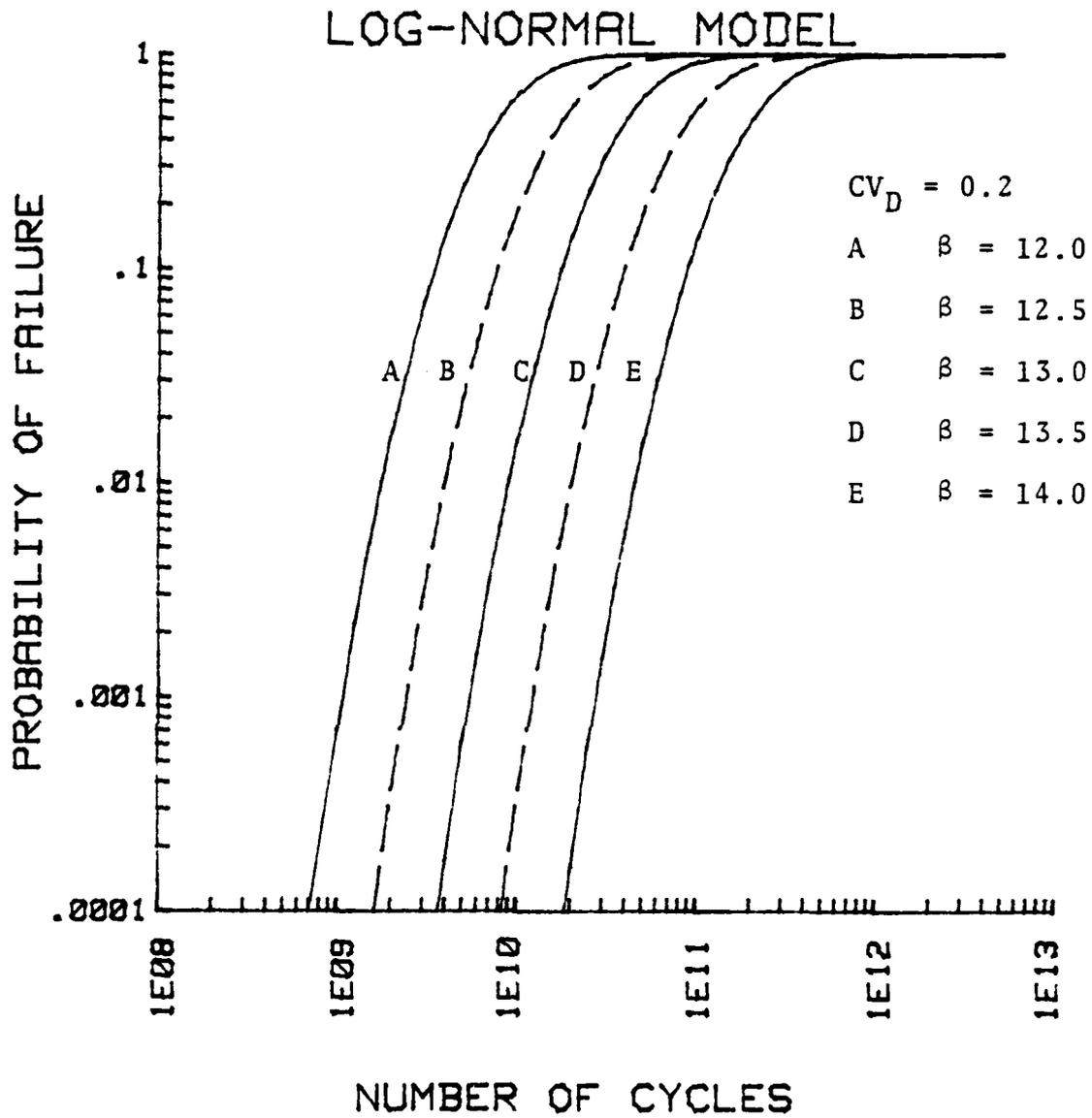


Figure 5.13 LOG-NORMAL MODEL LIFE CYCLE SENSITIVITY: TAIL REGION

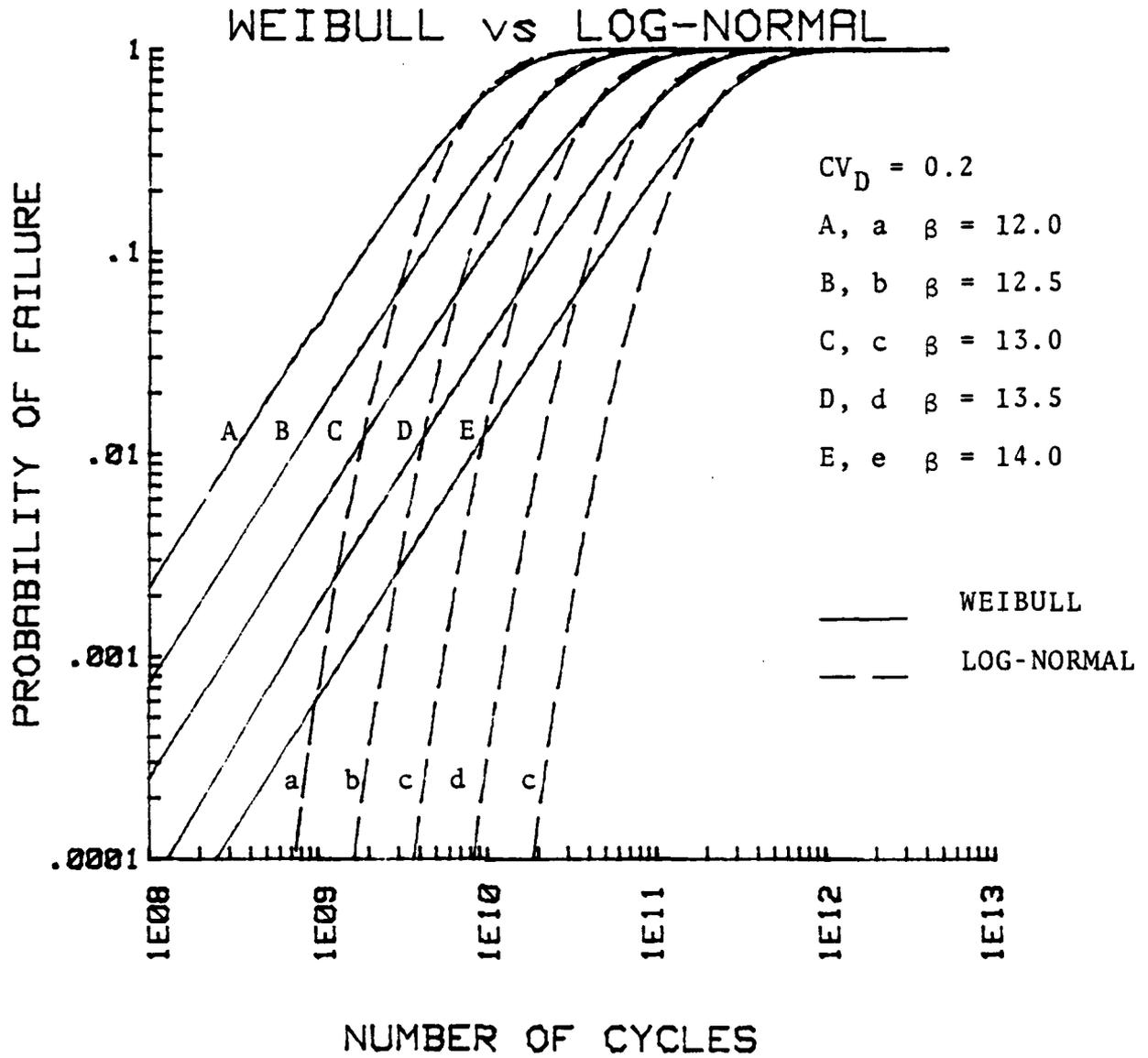


Figure 5.14 WEIBULL VS LOG-NORMAL: TAIL REGION COMPARISON

the performance of the Weibull and log-normal distribution models in the left tail region. Again, the one order of magnitude separation is exhibited in the left tail region, and near identical predictions in the central region of the distribution.

In the log-normal plot, the structure with the lowest resistance to fatigue failure (i.e., $\beta = 12$) reaches one hundred percent predicted failure slightly before $1.8 \cdot 10^{10}$ cycles. This is due to the poor resolution of any log-normal probability model deep in the tail region of a data distribution. This observation is repeated for all beta values. That is, complete failure is first predicted by the log-normal model, and onset of failure is first predicted by the Weibull model.

To verify that the established trends were applicable for other values of the coefficient of variation of the design damage index, the analysis was repeated for $CV_D = 0.8$. Figures 5.15 and 5.16 show the Weibull and log-normal models performance in the same manner as Figures 5.9 and 5.11 did for $CV_D = 0.2$. Figure 5.17 is a comparison of the two models for $CV_D = 0.8$. In all three plots, the shape of the curves and trends of the plots follow the same pattern established by the $CV_D = 0.2$ plots.

Figure 5.18 is a comparison of the coefficient of variation sensitivity as a function of applied cycles for the Weibull model. As was the case in the CV_D sensitivity study, for low cyclic lives, the model is insensitive to CV_D , with negligible

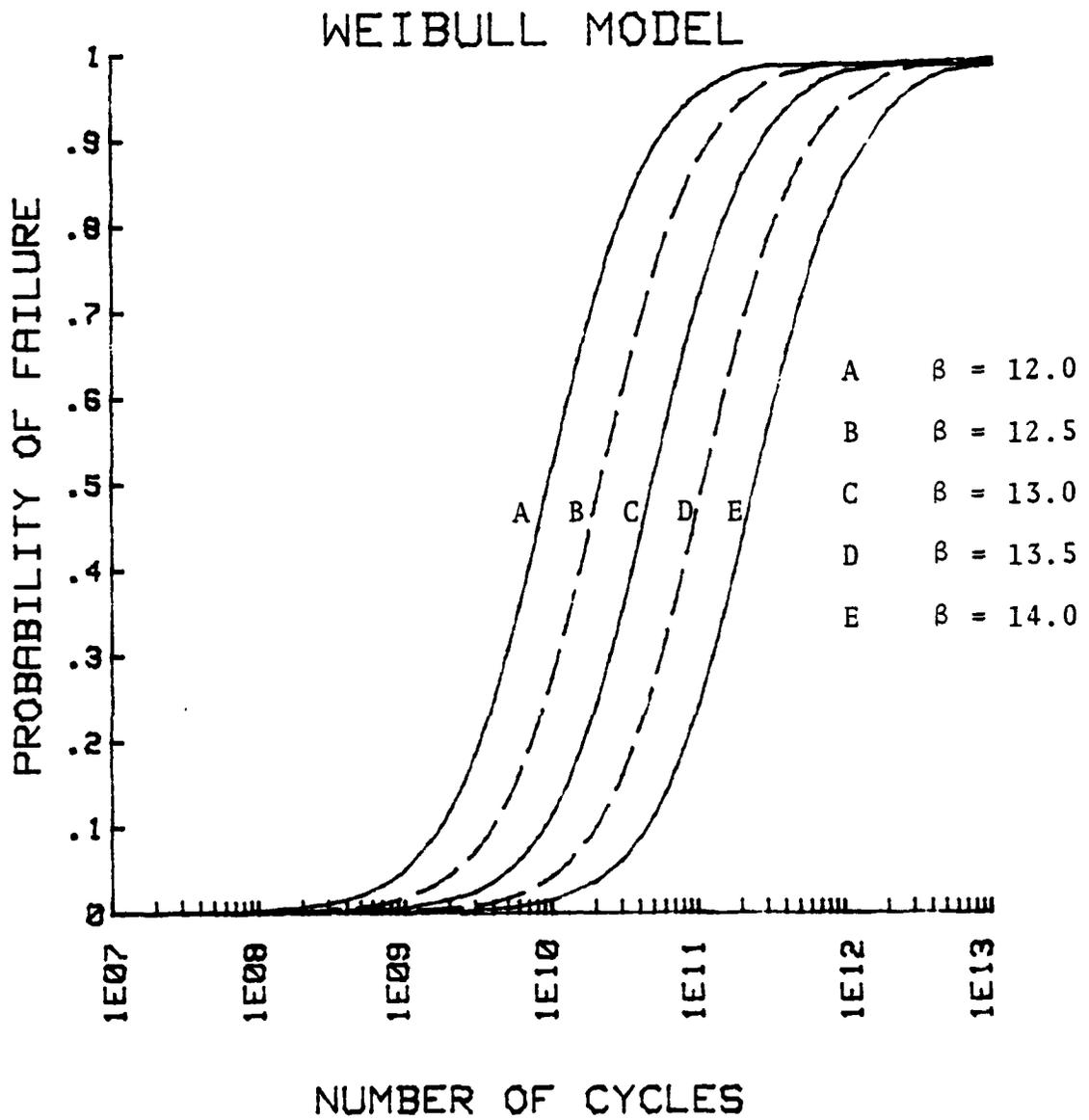


Figure 5.15 WEIBULL MODEL LIFE CYCLE SENSITIVITY $CV_D = 0.8$

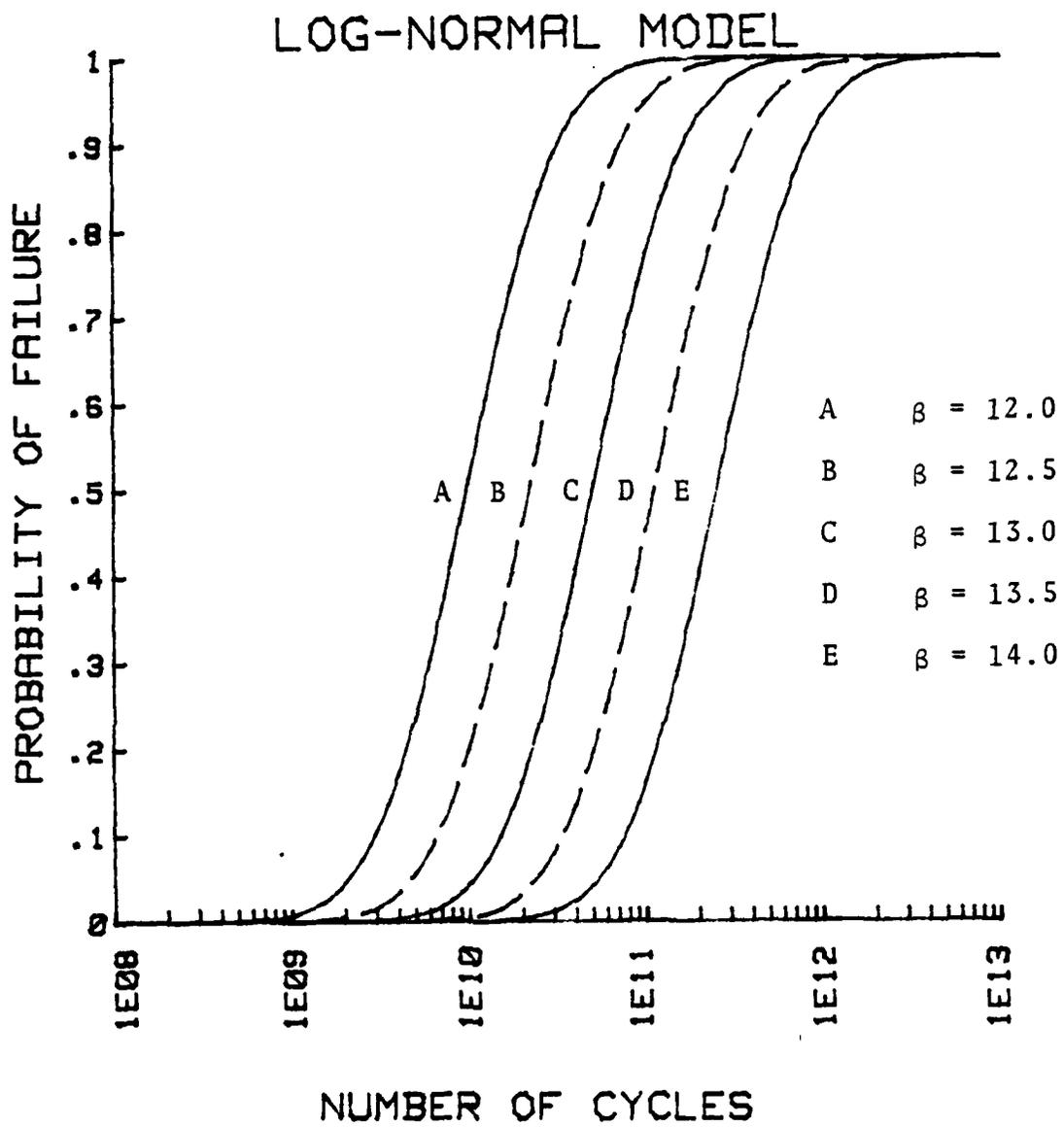


Figure 5.16 LOG-NORMAL MODEL LIFE CYCLE SENSITIVITY $CV_D = 0.8$

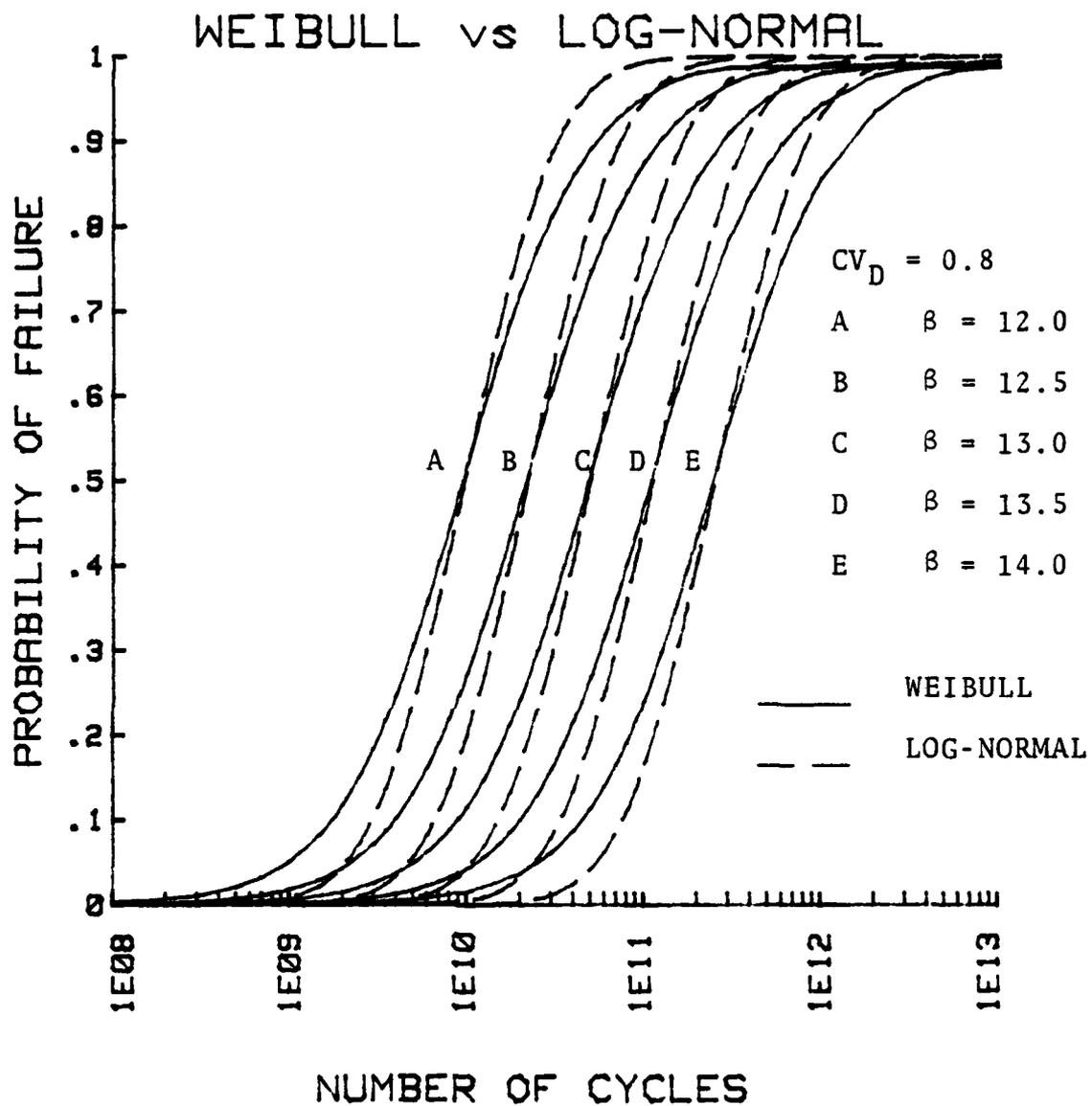


Figure 5.17 WEIBULL VS LOG-NORMAL LIFE CYCLE SENSITIVITY

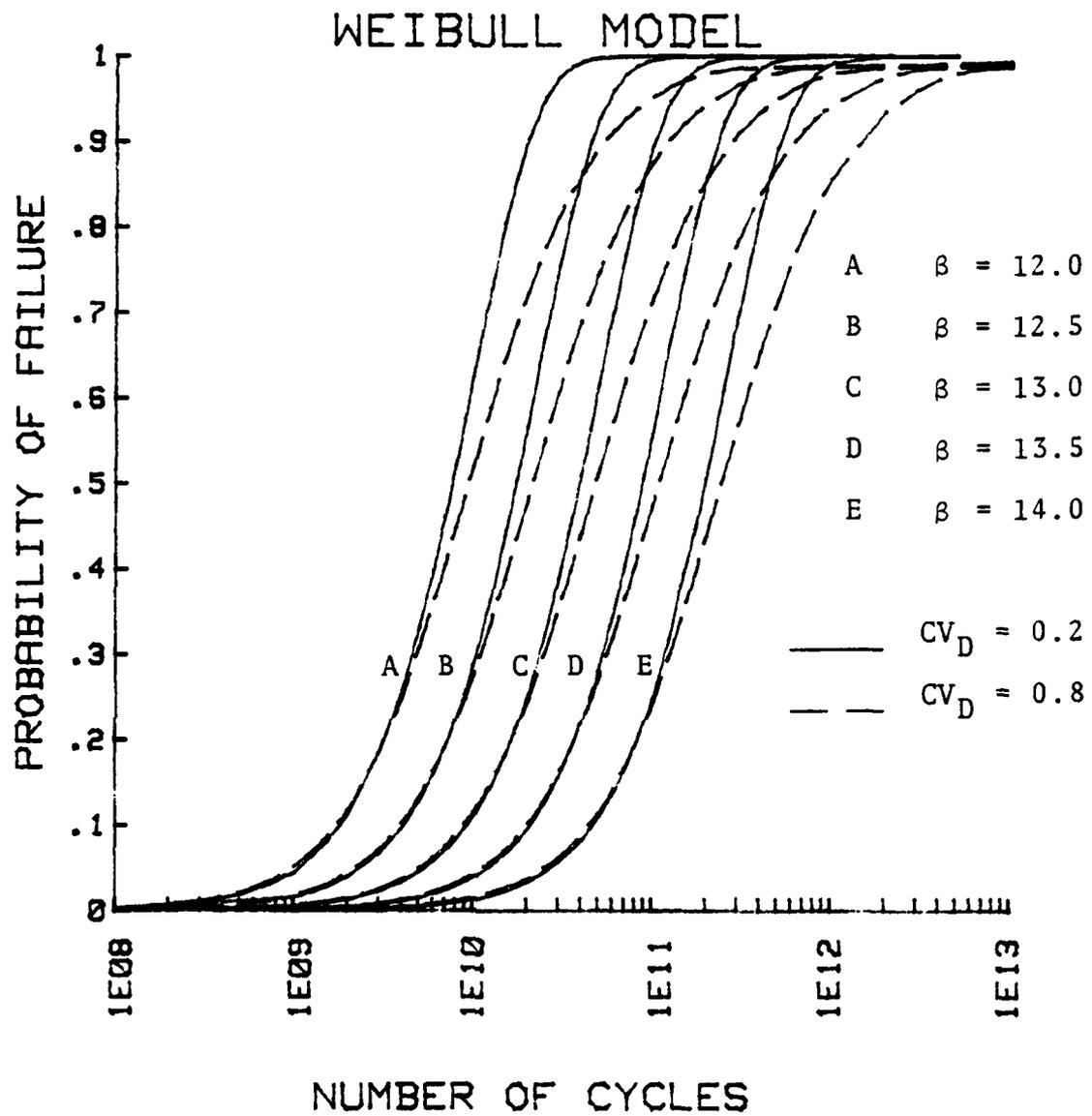


Figure 5.18 WEIBULL MODEL $CV_D = 0.2$ VS $CV_D = 0.8$

separation in predicted failure probabilities. As cyclic life nears design life, there is a separation in predicted values, with the $CV_D = 0.2$ being the more conservative value. From the source code generated data, the $CV_D = 0.8$ plots do eventually reach a predicted probability of failure of one hundred percent, but so far into cyclic life, that it does not show on this plot.

Figure 5.19 is a comparison of the coefficient of variation sensitivity for the log-normal model. Again, the predicted trends are sensitive to the CV_D throughout the range of study, with a noticeable cross over point. Early in cyclic life, $CV_D = 0.8$ is the more conservative value, and near design cyclic life, $CV_D = 0.2$ is the more conservative value.

E. SENSITIVITY TO APPLIED CYCLES: MULTIMEMBER

Fatigue life of a weakest link structure with fifty members was evaluated. Again, beta values of 12, 12.5, 13, 13.5 and 14, in conjunction with design damage index coefficients of variation of 0.2 and 0.8, with a fixed time history data block duration of two seconds, were tested. All members were assumed to follow the same probability model for the damage index, and all members were specified to have the same probability of failure for a given loading spectrum and elapsed stress cycles.

The upper bound for failure of the structure was determined and plotted in terms of total number of applied cycles, using

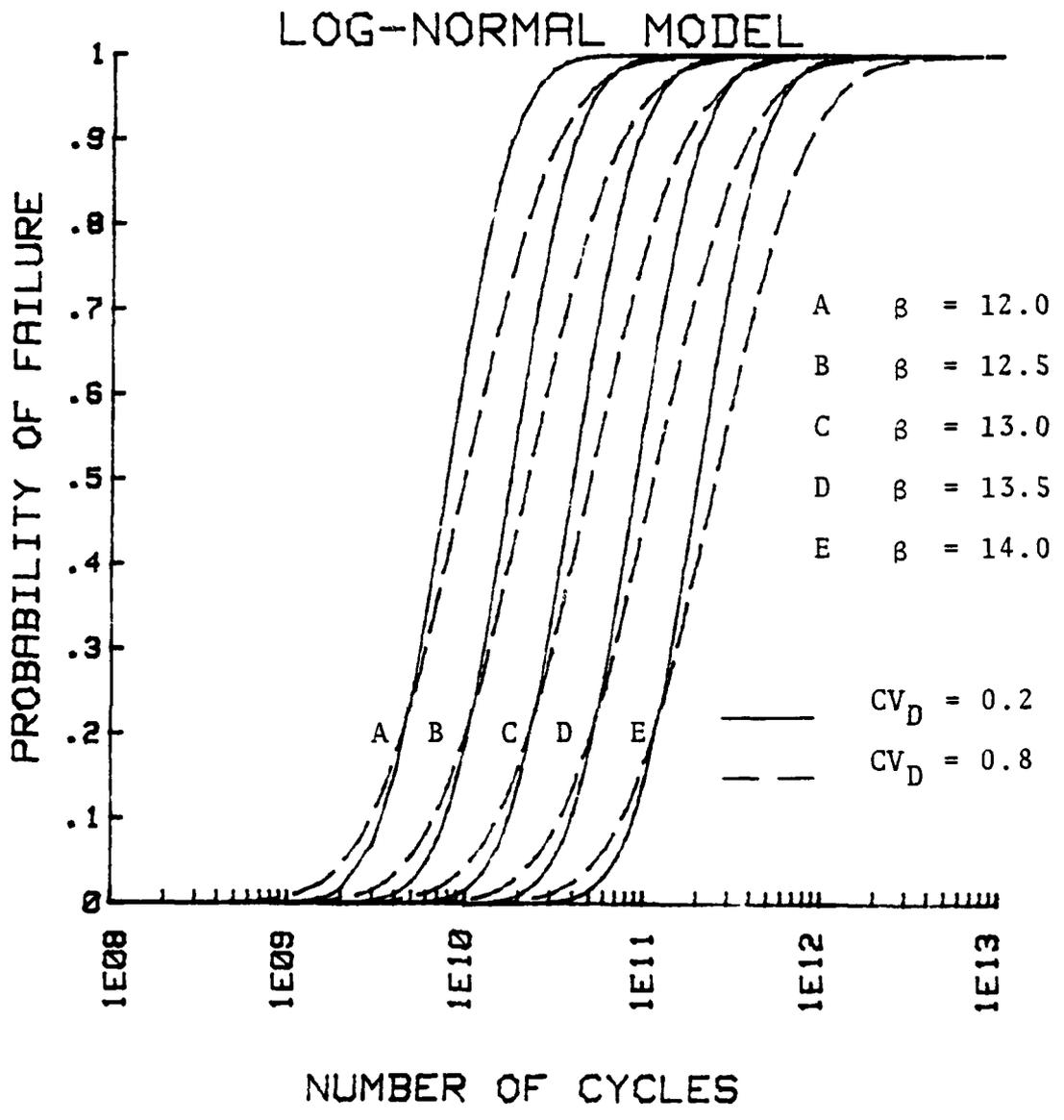


Figure 5.19 LOG-NORMAL MODEL $CV_D = 0.2$ VS $CV_D = 0.8$

the procedure and equations outlined in Chapter III, section G. For a design damage index coefficient of variation of 0.2, Figure 5.20 presents the results of the Weibull model, and Figure 5.21 presents the results of the log-normal model. Figure 5.22 compares the two models, with the Weibull model being significantly more conservative than the log-normal model.

As Figure 5.22 demonstrates, very few additional cycles can be placed on a log-normal modeled system before predicted failure transits from near zero to one hundred percent. The Weibull model, on the other hand, does have an obvious transition range. The presence of this transition range is very beneficial if maintenance and inspection procedures are incorporated into fatigue life prediction modeling.

Figures 5.23 through 5.25 present the same information for $CV_D = 0.8$. Figure 5.26 compares the Weibull model for the two CV_D 's. As expected, there is a negligible separation in the curves for each beta value, due to insensitivity to CV_D at low cyclic life. Figure 5.27 compares the log-normal model for the two CV_D 's studied. This plot does show a separation in predicted value for each beta, for each CV_D , which is due to the log-normal model being sensitive to the CV_D throughout the range of analysis.

Figure 5.28 is a comparison of the Weibull and log-normal models for a combination of single and multimember predicted results. In that for the entire range of beta's studied, the plots all have the same shape, a single beta value is presented.

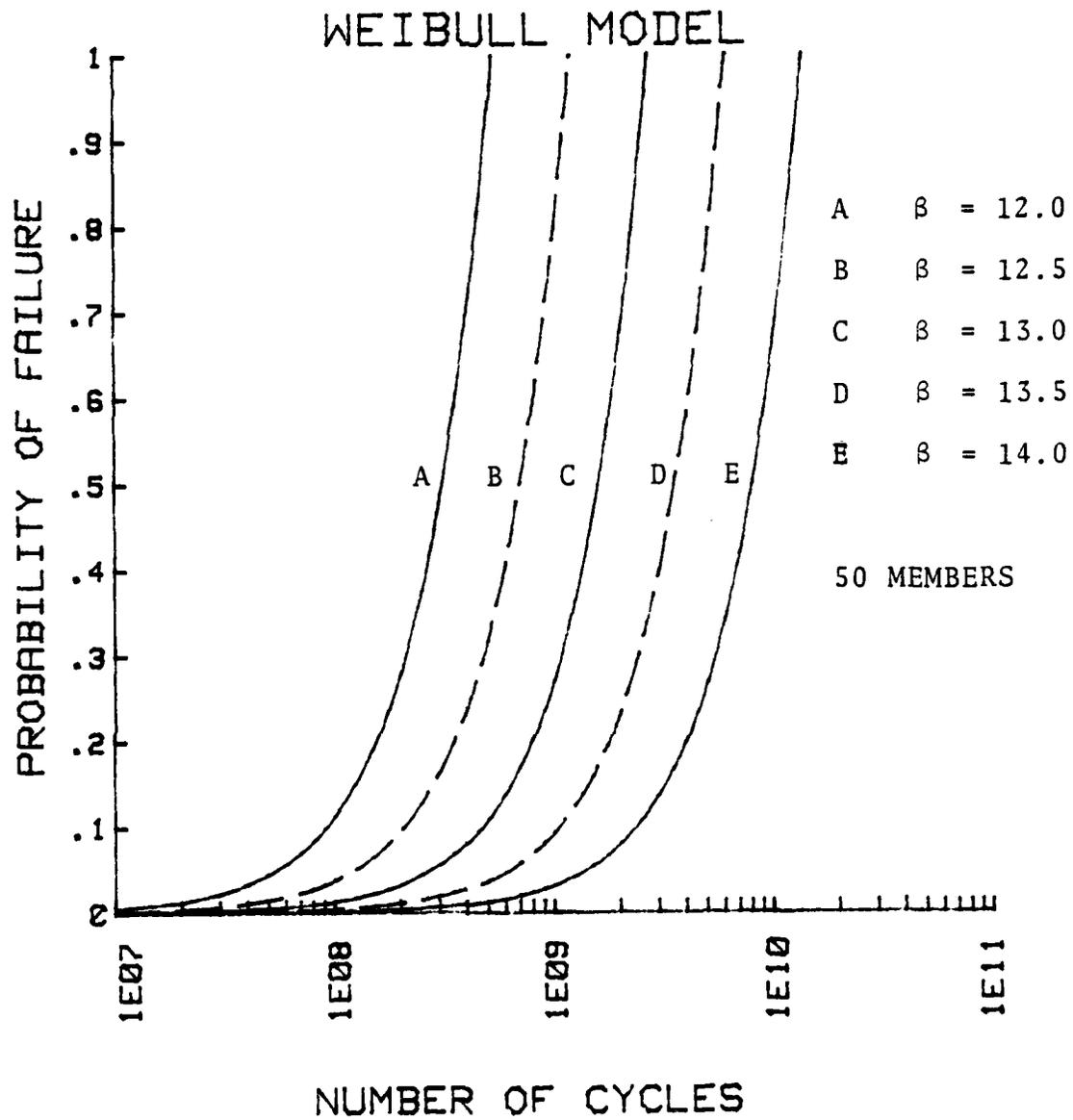


Figure 5.20 WEIBULL MULTIMEMBER LIFE CYCLE SENSITIVITY $CV_D = 0.2$

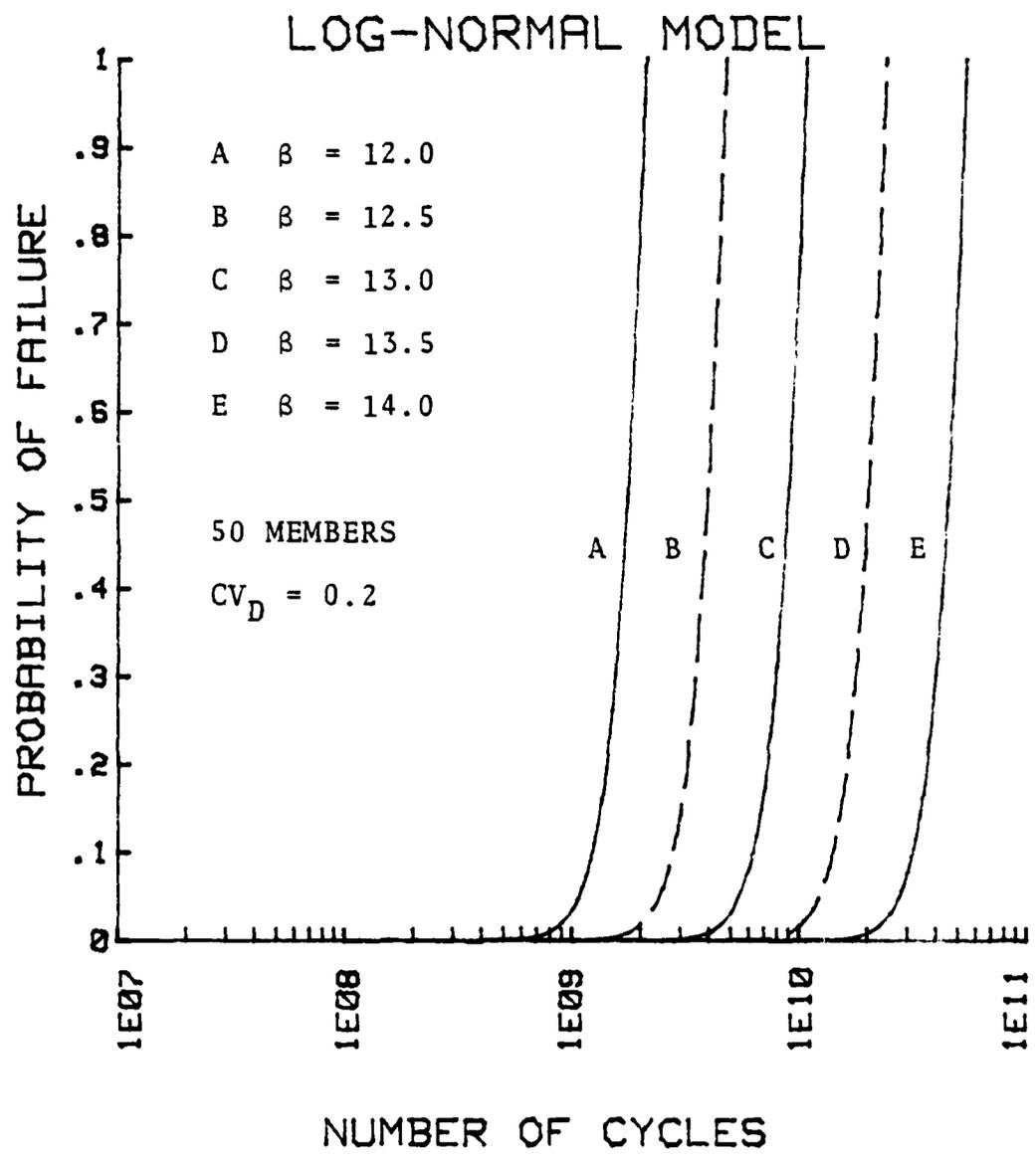


Figure 5.21 LOG-NORMAL MULTIMEMBER LIFE CYCLE SENSITIVITY

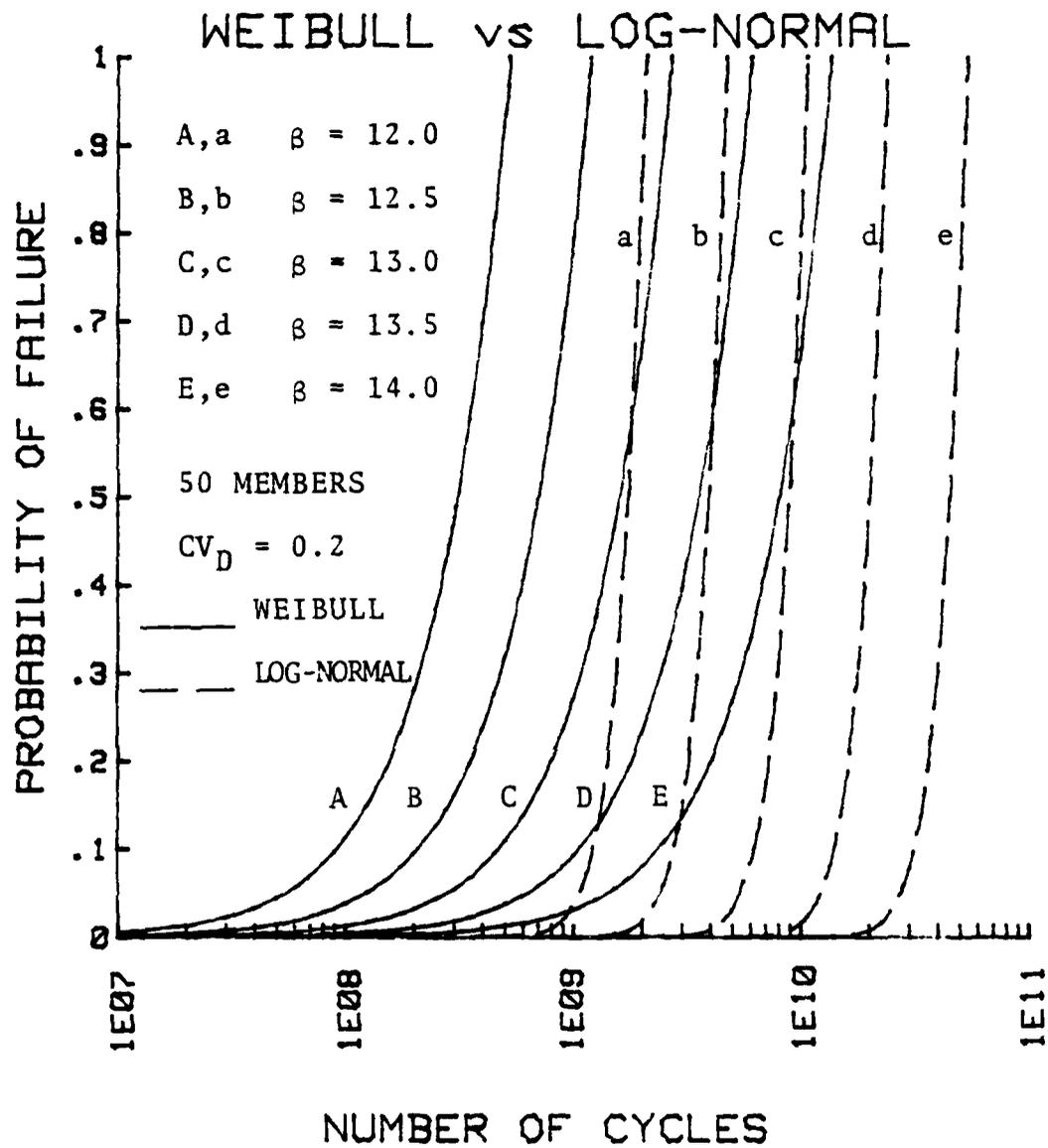


Figure 5.22 WEIBULL VS LOG-NORMAL MULTIMEMBER COMPARISON

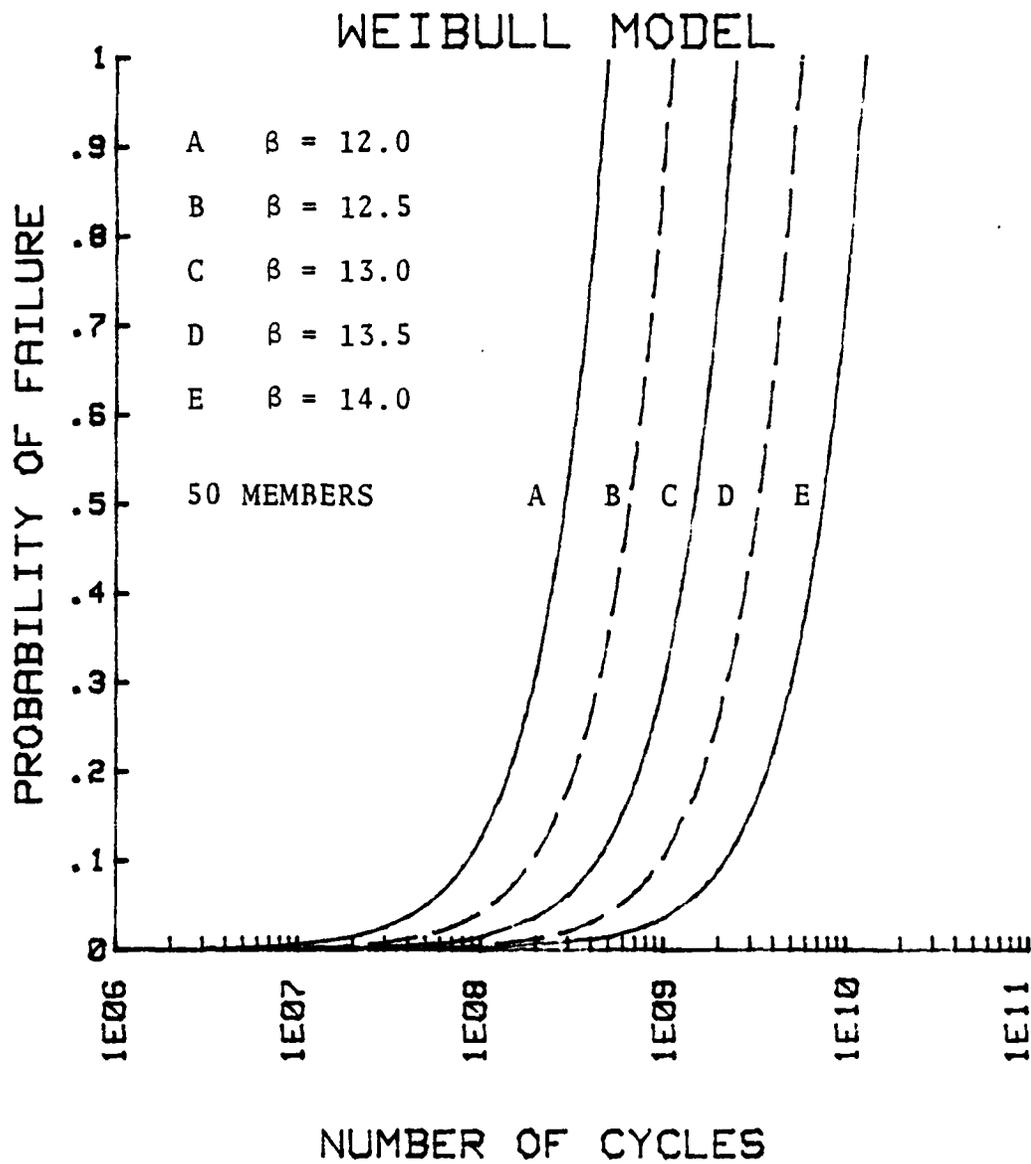


Figure 5.23 WEIBULL MULTIMEMBER LIFE CYCLE SENSITIVITY $CV_D = 0.8$

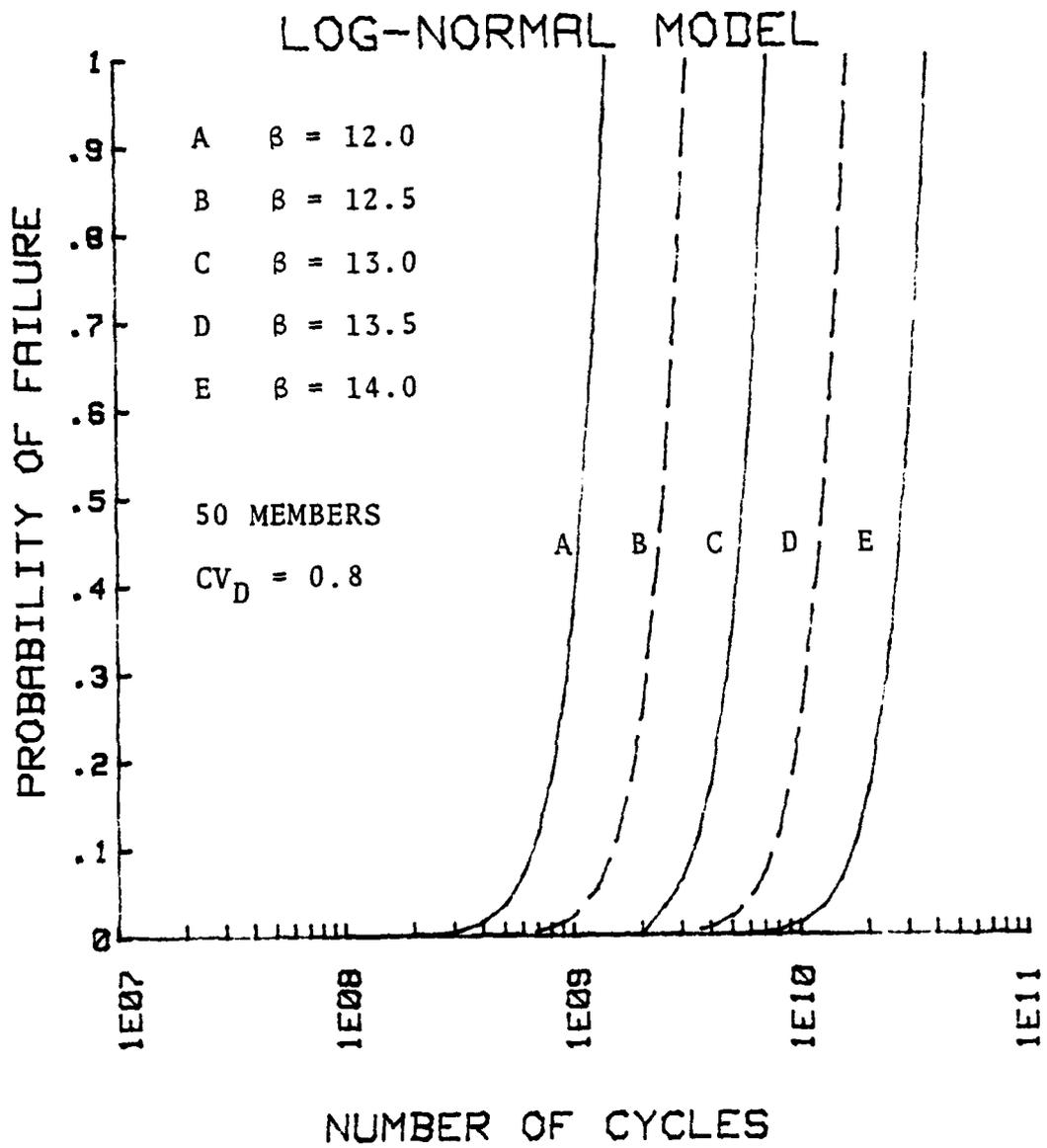


Figure 5.24 LOG-NORMAL MULTIMEMBER LIFE CYCLE SENSITIVITY

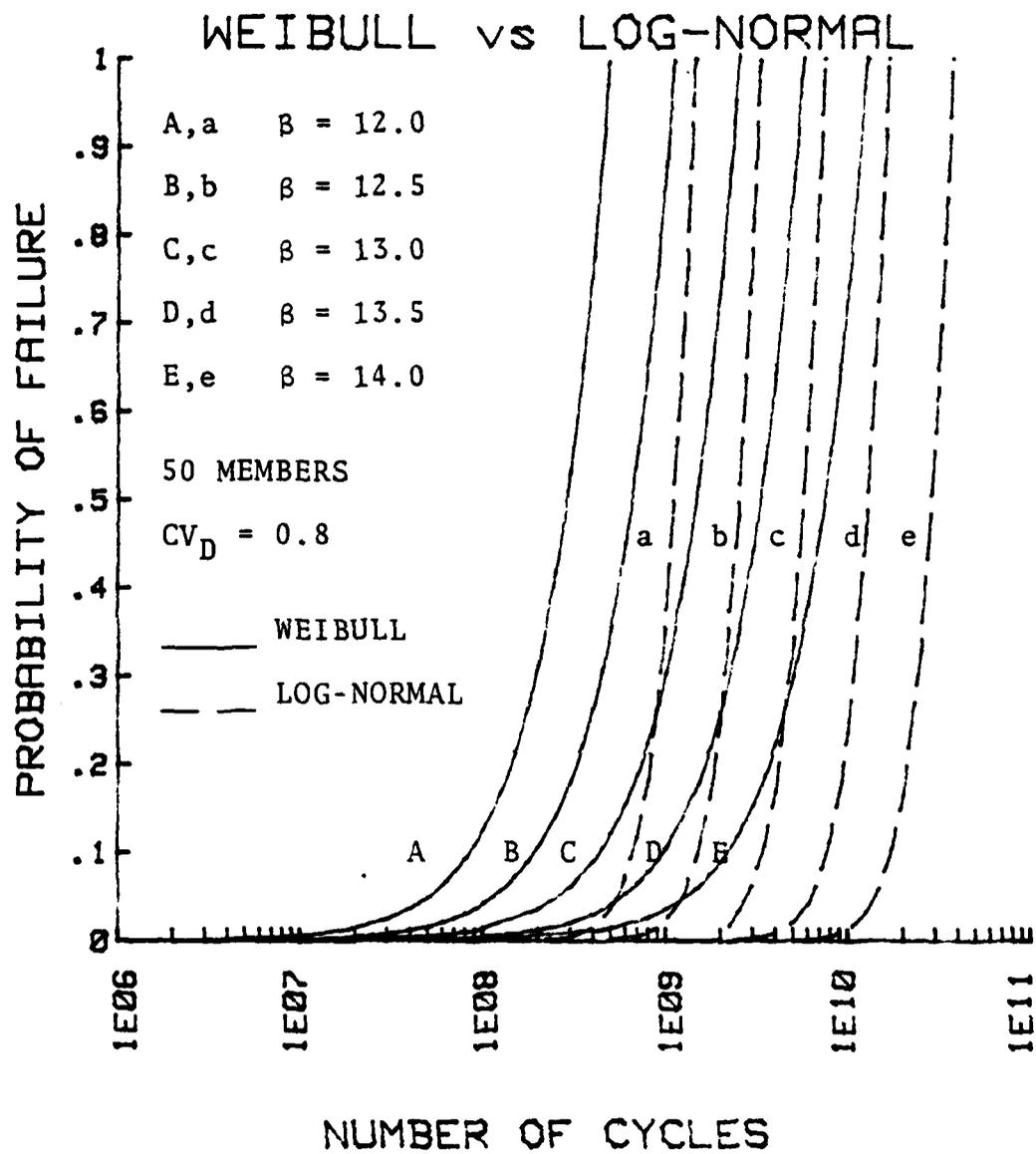


Figure 5.25 WEIBULL VS LOG-NORMAL MULTIMEMBER COMPARISON

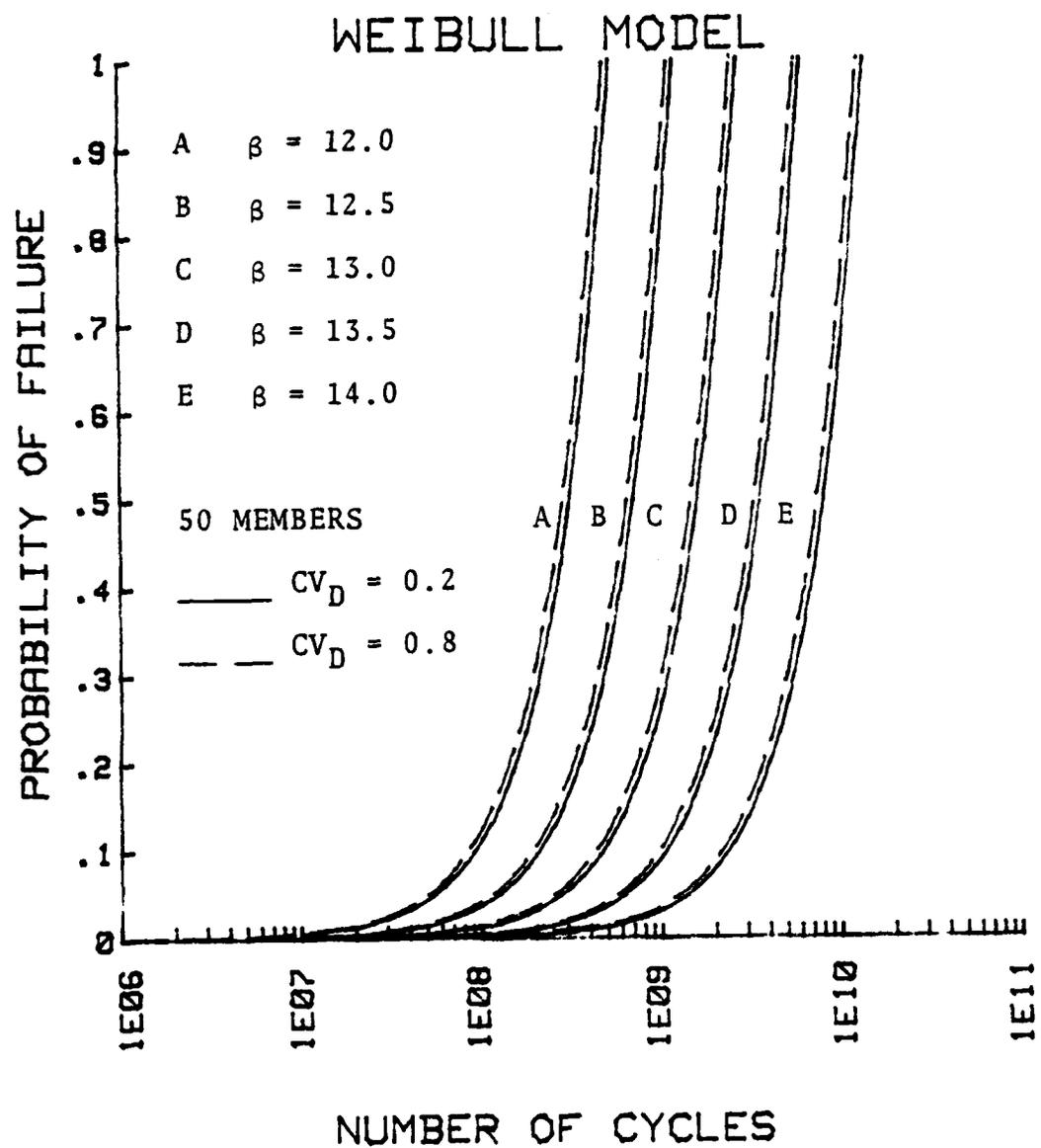


Figure 5.26 WEIBULL MULTIMEMBER $CV_D = 0.2$ VS $CV_D = 0.8$

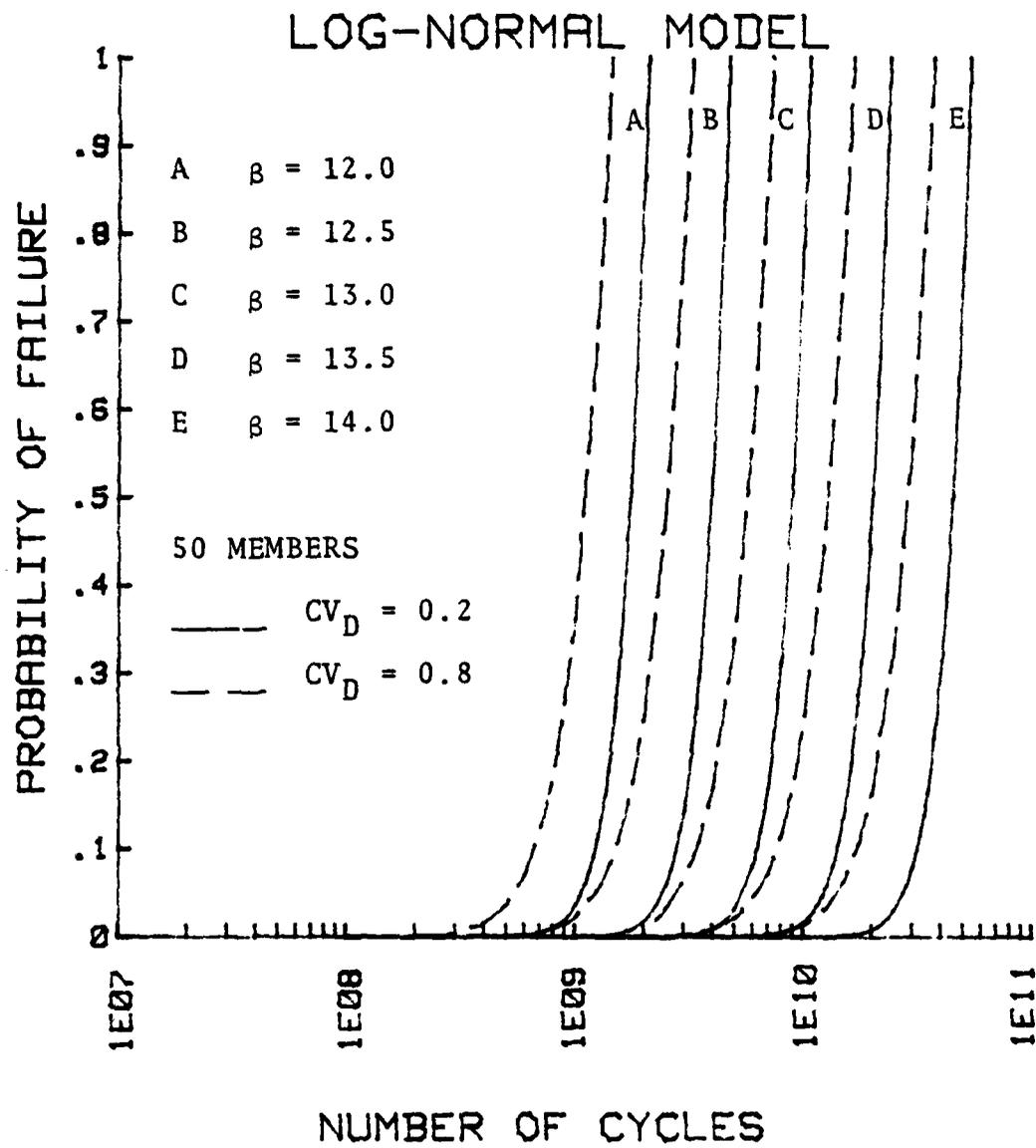


Figure 5.27 LOG-NORMAL MULTIMEMBER $CV_D = 0.2$ VS $CV_D = 0.8$

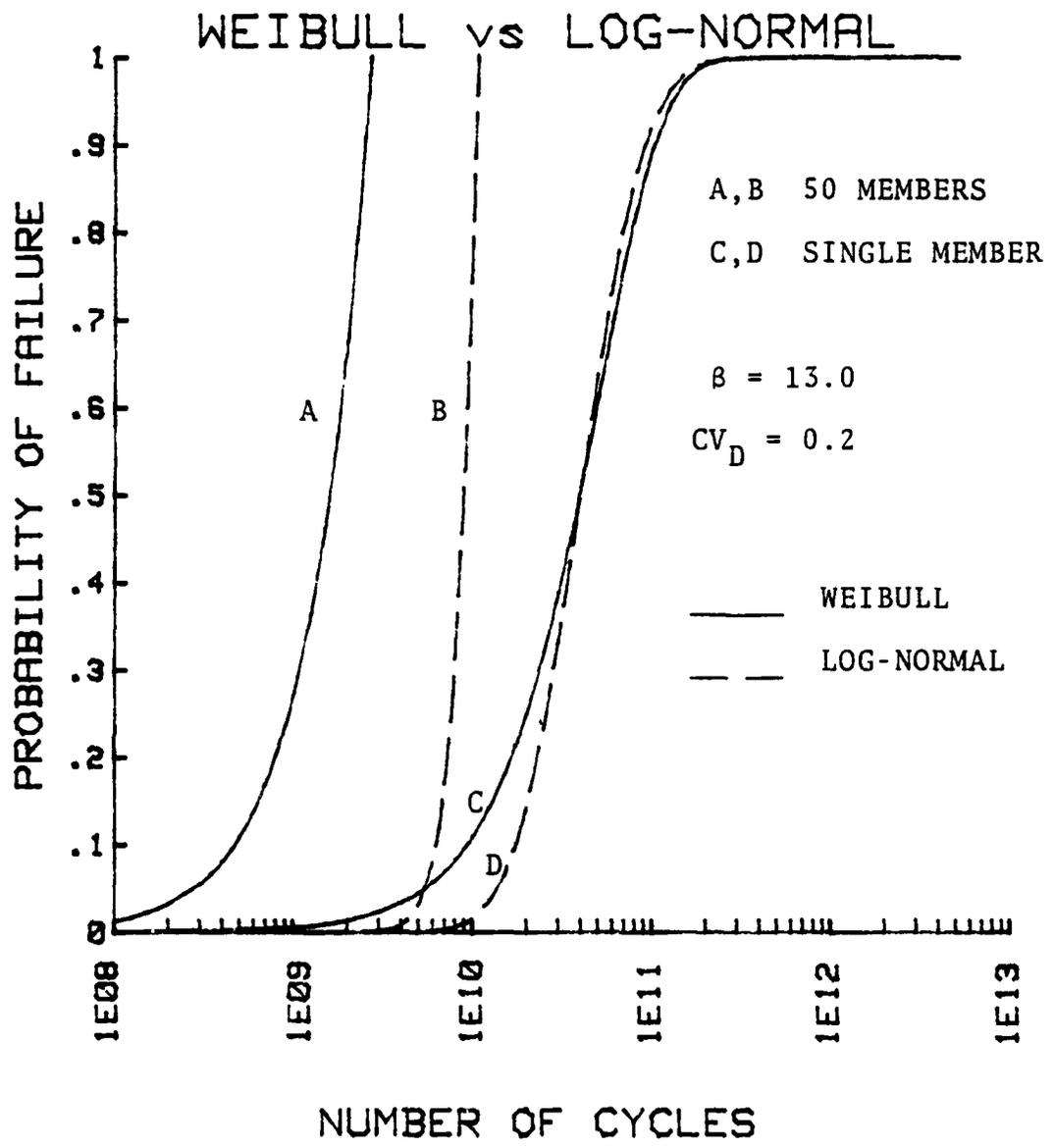


Figure 5.28 WEIBULL VS LOG-NORMAL: SINGLE AND MULTIMEMBER

In this case, beta equals 13 was chosen to represent all beta values. A review of Figure 5.28 shows a significant separation in single member and multimember predicted life for the Weibull model. The observed separation in predicted life is somewhat more than a complete order of magnitude in cyclic life. For the log-normal model, the separation in predicted life is only about one-half of an order of magnitude. For a system in which the probability of failure is the same for all components of the system, at least a one order of magnitude difference in a single components life and the system life would be expected. Therefore, the Weibull model is more representative of a systems extrapolated fatigue life. This is due primarily, to the response of the Weibull model in the left tail region of the distribution.

This adaptability of the Weibull model to system life extrapolation was expected, in that the Weibull distribution has been extensively studied in its application to system lives [Ref. 31]. Other probabilistic distributions have been adapted to the weakest link analogy, but the Weibull distribution is still considered the best for this purpose. Many investigators argue that the Weibull model is too conservative in the left tail region of a fatigue data distribution (i.e., Wirsching, et. al.), but proponents of either distribution model do concede that the Weibull model is best for system life extrapolation [Ref. 30].

VI. SUMMARY AND CONCLUSIONS

A. SUMMARY

A probability based approach to evaluate the high-cycle fatigue life of single component and multiple member, weakest link structures, in random stress environments, was described. This approach utilizes the Palmgren-Miner linear damage rule, with compensation for mean stress effects and the statistical variability of the critical damage index. The cumulative design damage index D was assumed to be a random variable, following the same probability distribution as the critical damage index Δ , for the particular probability distribution being analyzed (i.e., Weibull or log-normal). Cyclic stress history was assumed to be representable by a finite, repeating, time history data block.

Statistical reduction and best fit analysis of selected existing experimental data for Δ was accomplished. The subsequent application of order statistics and probability paper plot of the data produced a best fit Weibull distribution, with a mean of 0.90 and a coefficient of variation of 0.67, vice the proposed model [Ref. 13], of an assumed log-normal distribution with a mean of 1.00 and a coefficient of variation of 0.65. The calculated Weibull distribution shape parameters b_{Δ} and c_{Δ} were 0.9791 and 1.3289 respectively.

Log-normal models can be formulated as a function of the mean and coefficient of variation of the data distribution, which are the same for either log-normal or Weibull distributions. Therefore, the developed Weibull model was tested by comparing it to the values of fatigue life obtained by assuming the data did fit a log-normal distribution, and solving the closed form mathematical representation of the log-normal distribution.

Sensitivity studies of the predicted probability of failure as dependent on resistance to fatigue failure (i.e., negative reciprocal of the fatigue strength exponent), coefficient of variation of the cumulative design damage index, central safety factor, and applied stress cycles were conducted. For both probability distribution models, as beta increases linearly, fatigue life increases exponentially. The log-normal model is sensitive to the coefficient of variation of the design damage index throughout the entire range of analysis (central safety factor or applied stress cycles), but the more conservative value of CV_D is dependent on elapsed cyclic life, with a pronounced cross-over. The Weibull model is insensitive to CV_D in early cyclic life (i.e., elapsed life less than about eighty percent of design life). During that portion of life in which the Weibull model is sensitive to CV_D , the lower the value of CV_D , the more conservative the predicted fatigue life (i.e., the higher the predicted probability of failure for a given value of the sensitivity variable).

For all selected methods of graphical presentation of results, the predicted probability of failure for the two models yield comparative observations that agree with the published behavior [Refs. 10, 13, 30, and 32] of the two distributions. That is, in the middle region of the data distribution, the two models are virtually indistinguishable. In the left tail region of the data distribution, the Weibull model is significantly more conservative than the log-normal model, with up to a one order of magnitude difference in predicted fatigue life.

When the weakest link analogy was used to extrapolate an upper bound of a multiple member structures fatigue life, the Weibull model was considerably more conservative in predicted fatigue life. The separation in the two models ranged from about one order of magnitude at end of life to two orders of magnitude in the left tail region.

B. CONCLUSIONS

Extensive investigation of, and efforts to develop probabilistic fatigue life prediction models has occurred. Early models were developed using cyclic life as the basis of analysis, due to the availability of cyclic test data. In low cycle fatigue regimes, test data can be produced in a relatively short time span. In the high cycle fatigue regime, extended periods of testing are required, but test data can be obtained. In the ultra-high cycle fatigue regime, it can

take years to cycle a single sample the required number of times to represent the components life. To get a representative data base, testing costs can exceed component life-cycle costs, thus prediction models that can use a much more limited data base are desirable.

From the above requirement, fatigue life prediction models using the Palmgren-Miner linear damage law and known statistical distributions of fatigue data are being developed. A FORTRAN computer code which utilizes this method was developed and tested, using both the Weibull and log-normal distribution models. Testing of the code demonstrates that the generated data complies with the published trends of prediction models formulated on cyclic life for these two distributions.

New structural designs entail significant modeling and/or testing. Components designed to operate in the ultra-high cycle fatigue regime can entail cyclic testing up to 10^{12} cycles for a single sample, and then repeating the test on "identical" samples a sufficient number of times to get a representative data base. This model can be used to selectively reject material compositions which appear to be feasible, but probabilistic analysis imply will fail earlier than design requirements allow. Having used the model as a first cut analysis of potential design components, testing expense can be reserved for viable compositions, not spent on compositions that might work.

An extensive literature search has demonstrated that there are as many proponents for using the Weibull model as for using

the log-normal model. The log-normal model is easy to formulate, and has extensive library routines providing closed form solution techniques. Its simplicity of implementation makes it a very popular model to use, and more importantly, many existing sets of fatigue data fit the log-normal distribution. The Weibull model is not as easily implemented. The Weibull shape parameters must be calculated, and then the mathematical formulation must be numerically integrated. The major advantage of the Weibull model is its adaptability to multiple member structures fatigue life extrapolation from single component predicted fatigue life values.

The intention of this study was not to prove the Weibull model superior to the log-normal model, or vice versa. The selected experimental data fit the Weibull distribution better than it fit the log-normal distribution, so it was known that the Weibull model would be superior in this case. Many data sets can be found in which the log-normal distribution fits better, thus the log-normal distribution would be superior. The purpose for using both distributions was to have a comparison method for analyzing the developed Weibull models predictions. The comparative testing produced observations consistent with all comparisons located in the literature, thus implying the formulation of the models was correctly accomplished.

Based on the interrelationships of the two models, and the compliance with published behavior of existing fatigue

life prediction models, it is felt that the developed source code is an acceptable and viable fatigue life prediction tool.

VII. RECOMMENDATIONS FOR FUTURE WORK

A. GENERAL RECOMMENDATIONS

In that the developed source code is new and was not drafted by a programmer, some modifications to the code which enhance efficiency of computer cost factors are possible. A significant cost factor is the numerical integration routine used. There may be more efficient and/or more accurate library routines available for numerical integration. Additionally, as submitted in Appendix B, the source code contains numerous write and associated format statements which were beneficial in code implementation, but may be deemed superfluous information in general use.

The code was tested using only one set of fatigue data, and more importantly, associated control parameters were not consistent with the material properties of the source data. It is recommended that additional data be obtained, and a rigid adherence of consistent material property parameters be applied to the new data to confirm predicted probabilistic fatigue life complies with the data sets life. Also, it may be a beneficial exercise to select a data set that fits the log-normal distribution better than it does the Weibull distribution, to analyze it and compare the results of that study with this study to evaluate model performance based on statistical fit of the data.

Most probabilistic fatigue life prediction models utilize the two parameter Weibull or log-normal distribution. In an effort to enhance the precision of cyclic life models, recent studies into the use of the three parameter distribution has been conducted [Refs. 10,32]. In these studies, the third parameter is called the least life parameter, i.e., minimum cycles to failure. In terms of the critical damage index, a least life parameter would be the value of the cumulative damage index at which first failure occurs (assumed to be zero in the two parameter model). For initial implementation, the least life parameter could be assigned the value of Δ in the selected data set with the lowest magnitude. The selected data set would have provided a least life Δ of about 0.01, but from a review of the literature [Ref. 12], the least life value is more generally around 0.18 for log-normal distributions.

Using a least life parameter may reduce the left tail region separation of the two models. Additionally, incorporation of a least life parameter could be beneficial, not only in enhancing accuracy of predicted life compared to actual life from test data, but may ease the implementation of maintenance and inspection procedure effects analysis on the model.

B. MAINTENANCE AND INSPECTION

Reliability analysis of structures subjected to scheduled maintenance and inspection procedures has been investigated, primarily by engineers in the aircraft industry. Efforts to

incorporate the probabilistic analysis of crack detection, and subsequent repair or replacement of the faulty component, effects on structural fatigue life are being pursued [Refs. 33,34]. When a crack is initiated, the components fatigue strength progressively decreases as the crack propagates, increasing fatigue failure rate with time. If the faulty component is detected by inspection and repaired or replaced, the static and fatigue strength of the structure are renewed. This regeneration of component fatigue strength enhances the structure's fatigue resistance and life.

One possible way to account for the effects of inspection and maintenance procedures would be to adjust the least life parameter upward, commensurate with the reliability of the maintenance and inspection procedure. In that detection, thus repair of a crack is a probabilistic function of flaw size and resolution of the inspection procedure used (i.e., visual inspection, liquid penetrant, ultrasonic, magnetic particle, magnetic field perturbation, or radio-graphic), some cracks are not detected, so fatigue life does not restart after inspection. However most cracks that would cause early failure, represented by the left tail region, would be detected, and eliminated [Ref. 34]. The predicted fatigue life can then be recalculated, rejecting those sets of Δ deep in the left tail. Or, if the least life parameter were used, the tail region could be retained, with the least life parameter magnitude adjusted upward. This allows use of the entire statistical distribution, and

accounts for the increased probability of survival. Rejecting data elements in the tail region requires recalculation of the mean and coefficient of variation of the "new" distribution. If the Weibull distribution were used, recalculation of the shape parameters is also required.

Structures other than aircraft undergo scheduled maintenance and inspection procedures (i.e., offshore structures, bridges, buildings etc.). Thus, incorporation of accountability for these procedures would enhance the usefulness of any probabilistic fatigue life prediction model. Therefore, investigation into incorporation of maintenance and inspection procedures accountability in the developed fatigue life prediction model is the primary recommendation for future work. It is believed that modifying the source code to use the three parameter vice the two parameter distributions would ease this implementation.

C. MULTIAXIAL FATIGUE

A literature search was unproductive in locating material covering probabilistic fatigue life prediction for components subjected to multiaxial fatigue. Extensive research is in progress to evaluate multiaxial fatigue in the deterministic sense, but no consensus yet exists as to one method of analysis. Numerous proposals for high cycle fatigue analysis have been generated. Garud [Ref. 35] presented an excellent state-of-the-art survey of multiaxial fatigue and discusses most of the proposed models.

Each proposed model appears to be limited to a specific application, i.e., proportional cyclic loading in all axial directions, in-phase loading or out-of-phase loading, etc. Until an accepted correlation is developed, complex multi-axial fatigue analysis of predicted fatigue life must be deferred well into the future. On the other hand, simple cases of biaxial fatigue may be solvable now. In the high cycle fatigue regime, equivalent stress, determined by the Tresca or Von-Mises criterion can be used [Refs. 10,35,36]. However, recent studies indicate this procedure only applies for in-phase loading with fixed principle axes [Ref. 35].

For this limited case, investigation into using a representative PSD with the rainflow counting method to generate a mean stress for each axis, then application of either the Tresca or Von-Mises criterion to generate an equivalent stress can be accomplished. This equivalent stress would then be used in evaluating $E[S^8]$, with the remaining analysis proceeding as outlined in Figure 4.1. The predicted failure (i.e., elapsed cycles to fifty percent failure) could then be compared with the cycles to failure determined via the classical deterministic approach of S-N curve analysis (outlined in reference 36).

An additional study could be made by developing a PSD representative of the equivalent stress from the Tresca or Von-Mises criterion, then using this PSD with the rainflow cycle counting method as if the study were initially uniaxial.

Then, comparison of application of equivalent stress before cyclic stress simulation and application of equivalent stress after cyclic stress simulation could be conducted.

D. SUMMARY

The growth potential of the developed source code is unlimited. Some general improvements have been discussed, and two in-depth expansions have been proposed. Based on background material available, incorporation of maintenance and inspection procedure effects on prolonging fatigue life appears to the subject area with the most promising, immediate, applicability.

APPENDIX A
COMPUTER CODE USERS GUIDE

A. INTRODUCTION

The computer code "FATIGUE FORTRAN" is designed to calculate the deterministic fatigue life, Weibull model probability of failure and the log-normal model probability of failure for a single structural component. Any one, or a selected combination of the three models can be chosen. Additionally, calculations accounting for, or neglecting, mean stress effects are possible.

B. COMPUTER CODE DESCRIPTION

Representative time history data blocks, applied power spectral density functions, material fatigue strength exponents, Weibull model shape parameters for the critical damage index, coefficient of variation for the design damage index and design cycle life to be analyzed are inserted as variables. The program then generates a random version spectral density function and converts the time history data block to relative stress amplitudes for the specified time sequence. The simulated stress history is then changed to a peak and trough representation for Rainflow cycle counting and subsequent calculation of the mean and standard deviation of the simulated cyclic stress history. The deterministic fatigue life is then calculated for each beta value to be analyzed.

The program then calculates the Weibull shape parameters for the design damage index from the mean and standard deviation of the stress history simulations, as well as the mean, standard deviation and coefficient of variation of the critical damage index from the critical damage index shape parameters provided as input values. From these values, the Weibull model distribution and density functions are calculated and inserted into a numerical integration routine for determination of the probability of failure. The integration routine used is DQSF, a double precision routine using a combination of Simpson's rule and Newton's three-eighths rule.

The Weibull model formulation implies integration to infinity, which is not realistic. To alleviate this problem, the calling program is established so that the first iteration of the integral is evaluated from zero to the calculated mean of the damage index, (both the design and the critical damage index are used). Subsequent iterations are conducted using the damage index with one standard deviation of the damage index added for each iteration. From probabilistic theory, it is accepted that the mean plus three standard deviations of the mean for a given distribution will include 99.7% of all elements of that distribution. Rather than limit the integration to this level, the iteration is conducted until the difference between successive iterations is less than one percent, or ten standard deviations have been added. This typically

results in five standard deviations being added, with resultant inclusion of all but $6E-07$ percent of the area under the distribution curve.

The Weibull model is evaluated using both the critical damage index and the design damage index with their associated standard deviations as the upper limit of integration. The two methods yield essentially the same predicted probability of failure. For parametric studies of the critical damage index, the user can chose to comment out the design damage index analysis section.

The program then proceeds to the log-normal model, with its first step being the calculation of the log-normal shape factor. This shape factor is referred to as the "central safety factor," and is the ratio of the means of the two damage indexes. The parameters of the standardized argument for normal distributions are then evaluated and the standardized argument is then calculated and passed to a routine called MDNOR. This routine then returns the probability of survival for the specified standardized argument. The probability of failure is then calculated as one minus the probability of survival.

C. COMPUTER CODE SUBROUTINES

Three library routines are used, and are imbedded in the attached program listing. The MDNOR and GGUBS routines are from the IMSLDP library and are precompiled, so imbedding is not required. The DQSF routine is from the NONIMSL library

and is also precompiled, thus imbedding is not necessary. The remaining imbedded subroutines are non-library routines and are tailored to this particular program. Table II provides a summary of subroutine names and purpose.

The only subroutines with internal write statements are SPECT and WEIBR. In both cases, for parametric studies, these write statements and formats can be commented out if desired. SPECT generates the relative spectral density parameters, and these values do not change for a given PSD and time history. WEIBR generates the Weibull shape factors for the design damage index. These values do change during parametric studies if the coefficient of variation of the design damage index is one of the varied parameters. However, the calculated values are returned to the main program for use in the Weibull model section, thus writing these values is not necessary.

TABLE II

COMPUTER CODE SUBROUTINES

RANSPC	generates random spectral density function
GGUBS	basic uniform random number generator
STAT	evaluates mean and standard deviation
RNFLW	rainflow cycle counting, trough to peak
RNDRP	rainflow cycle counting, peak to trough
SPECT	computes standard deviation, irregularity factor, expected rate of zero crossings and expected rate of peaks
WEIBR	calculate Weibull parameters b and c from the mean and standard deviation
DQSF	numerical integration routine using Simpson's rule and Newton's three-eighths rule
MDNOR	Normal or Gaussian probability distribution function evaluator

D. COMPUTER CODE INPUT PARAMETERS

A sample input file is presented as Table III for demonstration of format specified location and argument type, i.e., character string, real or integer value. A description of each variable as encountered in the input file follows. Each parameter is referred to as it is named in the program.

First Row

- 1) HED: Character string title of specific test case.

Second Row

- 1) NP: Number of input frequencies of stress response power spectral density (PSD) function.
- 2) IN: Number of points in random time history simulation.
- 3) NE: Number of BETA values (negative reciprocal of fatigue strength exponent) to be considered.
- 4) DT: Time interval in random time history simulation (sec).
- 5) DF: Average frequency interval to characterize the power spectral density function (Hz).
- 6) THM: Mean of time history relative stress response.
- 7) SU: Ultimate strength of component.
- 8) IFLAG1: Integer flag value to specify type of analysis.
 - =0 Deterministic Fatigue Life analysis
 - =1 Probabilistic Fatigue analysis
 - =2 both

Third Row

1-5) EXPX(NE): BETA array, values of BETA to be analyzed.

Fourth Row

1,3,5) WF(NP): Frequencies of input PSD.

2,4,6) SP(NP): Amplitude of PSD at specified frequency.

Fifth Row

1) IWL: Integer flag value to specify type of probabilistic analysis.

=0 Weibull model

=1 log-normal model

=2 both

2) CL: Lower limit of variability search for use in determining Weibull shape parameters for the design damage index.

3) CU: Upper limit of variability search.

4) BDEL: Weibull shape parameter for critical damage index (intercept of Weibull critical damage index distribution).

5) CDEL: Weibull shape parameter for critical damage index (slope of Weibull critical damage index distribution).

6) CVWM: Coefficient of variation of the design damage index.

7) CYCL: Design life cycles to be analyzed.

TABLE III
 SAMPLE COMPUTER CODE INPUT FILE

COLUMN ALIGNMENT AS SPECIFIED BY FORMAT STATEMENTS

	101	201	301	401	501	601	701
•SK							
SAMPLE TEST CASE 8							
3	1000	5	0.002	1.0	0.5	60.0	2
	12.0	12.5	13.0	13.5	14.0		
	30.0	0.00	50.0	0.16	80.0	0.00	
2	1.0	10.0	0.9791	1.3289	0.80	1.000E+10	

E. VARIABLE USAGE

A PSD, time history data block, and material (ultimate strength and fatigue strength exponents) are chosen and fixed. Then parametric studies are accomplished by varying the design damage index coefficient of variation and cyclic life. Depending on the chosen coefficient of variation, the upper and lower limits of variability search may have to be changed. For $0.1 < CVWM < 1.0$, the resident values of CL and CU are adequate.

F. COMPUTER CODE USAGE

The program is tailored for usage on the IBM 3033, with the FORTHX compiler. Unit five is used for input, and unit six is used for output. In addition, unit seven is used for creating a data file for subsequent graphical presentation of a time history plot, and unit eight is used for creating a data file for subsequent graphical presentation of a simulated power spectral density plot. The data from unit seven can be placed directly into a plot routine, such as PLOTG, and submitted to the versatec plotter. The data on the unit eight file must be used as input to a Fast Fourier Transform routine, which generates the data points for use in a plot routine.

For parametric studies, it is recommended that after the initial data run, the write statements and associated format statements for the unit seven and unit eight data be commented out. Additionally, a significant number of the unit six write statements and associated format statements can be commented out, leaving only the required information for the particular study.

Program array dimension sizes are listed on the first page of the program. Recommended values, as functions of the input variables are provided.


```

C-----CHANGING TO POINT PROCESS OF PEAKS AND TROUGHS
IN=IN-1
J=0
DO 70 I=1,IN
  V(I)=0.0
DO 80 I=2,IN
  IF ((X(I+1)-X(I))/(X(I)-X(I-1)).GT.0.) GO TO 80
  J=J+1
  Y(J)=X(I)
CONTINUE
C-----APPLYING RANDOM CYCLE COUNTING METHOD
J=J-1
IF ((Y(I)-Y(2)).LT.0.) GO TO 100
DO 90 I=1,J
  Y(I)=Y(I+1)
DO 100 I=1,J-2
  V((I+1)/2)=Y(I)
DO 120 I=2,J,2
  P(I/2)=Y(I)
J=J/2
J1=J
C-----STRESS/STRAIN RANGE MEASURED FROM VALLEY TO PEAK
CALL RNFLW (J,V,P,Y)
DO 130 I=1,J
  XM(I)=V(I)+(Y(I)/2)
IF (XM(I).LT.0.) XM(I)=0.
CONTINUE
WRITE (6,490)
DO 140 I=1,J
  WRITE (6,500) I,V(I),P(I),Y(I),XM(I)
CONTINUE
CALL STAT (Y,J,XMN1,XSTD1)
C-----STRESS/STRAIN RANGE MEASURED FROM PEAK TO VALLEY
JJ=J-1
JJ1=JJ
DO 150 I=1,JJ
  VV(I)=V(I+1)
CALL RNDRP (JJ,VV,P,YY)
DO 160 I=1,JJ
  XMD(I)=P(I)-(YY(I)/2)
IF (XMD(I).LT.0.0) XMD(I)=0.0
CONTINUE
WRITE (6,510)
DO 170 I=1,JJ
  WRITE (6,500) I,P(I),VV(I),YY(I),XMD(I)
CONTINUE
CALL STAT (YY,JJ,XMN2,XSTD2)
XMEAN=(XMN1+XMN2)/2.0

```

```

A 570
A 980
A 990
A1000
A1010
A1020
A1030
A1040
A1050
A1060
A1070
A1080
A1090
A1100
A1110
A1120
A1130
A1140
A1150
A1160
A1170
A1180
A1190
A1200
A1210
A1220
A1230
A1240
A1250
A1260
A1270
A1280
A1290
A1300
A1310
A1320
A1330
A1340
A1350
A1360
A1370
A1380
A1390
A1400
A1410
A1420
A1430
A1440

```

```

C
XSTD= (XSTD1 +XSTD2) /2.0
WRITE (6,52 C)
CALL SPECI (NP ,G,DY,W,ALP,FG,XMO,SO)
WRITE (6,53 C)
WRITE (6,54 C) YMEAN,STD
WRITE (6,55 C)
WRITE (6,56 C) J,XMN1,XSID1,JJ,XMN2,XSID2,XMEAN,XSTD
C-----ESTIMATE E (S**M) WITH MEAN AND WITHOUT MEAN
WRITE (6,58 C)
DO 200 I=1,NE
E=EXPX(I)
YI=0.0
YYI=0.0
DO 180 L=1,J/(1,-XM(L)/SU)**E
YI=YI+(Y(L)/(1+Y(L)**E)
YYI=YYI+(Y(L)/(1,-XMD(L)/SU)**E
180
190
YYI=YYI/(1+JJ)
YYI=YYI/(1+JJ)
WRITE (6,57 C) E,YI,YYI
MM(I)=YI
WOM(I)=YYI
CONTINUE
C-----ESTIMATE FATIGUE LIFE USING DETERMINISTIC APPROACH
IF (IFLAG1.NE.1) WRITE (6,590)
DO 210 I=1,NE
E=EXPX(I)
WMS=SU**E/(XMD*WMI)
WOMS=SU**E/(XMD*WOM(I))
WMD=WMS/86400.0
WMD=WOMS/86400.0
WMY=WMD/365.0
WOMY=WMD/365.0
PHWM(I)=WMS*XMO
PHWO(I)=WOMS*XMG
IF (IFLAG1.EQ.1) GO TO 220
WRITE (6,60 C) E,WMS,WMD,WMY,WOMS,WOMD,WOMY
210
220
CONTINUE
IF (IFLAG1.EQ.0) GO TO 360
C-----**PRCBABALISTIC APPROACH*****
WRITE (6,64 C)
WRITE (6,65 C) ((EXPX(I),PHWM(I),PHWO(I)),I=1,NE)

```

```

A1 450
A1 460
A1 470
A1 480
A1 490
A1 500
A1 510
A1 520
A1 530
A1 540
A1 550
A1 560
A1 570
A1 580
A1 590
A1 600
A1 610
A1 620
A1 630
A1 640
A1 650
A1 660
A1 670
A1 680
A1 690
A1 700
A1 710
A1 720
A1 730
A1 740
A1 750
A1 760
A1 770
A1 780
A1 790
A1 800
A1 810
A1 820
A1 830
A1 840
A1 850
A1 860
A1 870
A1 880
A1 890
A1 900
A1 910
A1 920

```

```

C-----ESTIMATE FATIGUE LIFE IN CYCLES
WRITE (9,660) CYCL
DO 230 I=1,NE
E=EXPX(I)
PHDWM=PHWM(I)
PHDWM=PHWM(I)
DMIDWM=CYCL/PHDWM
DMIDOM=CYCL/PHDWM
WRITE (6,670) E,DMIDWM,DMIDOM
DIWM(I)=DMIDWM
SDDIWM(I)=CVM*DMIDWM
SDDIWO(I)=CVM*DMIDOM
CONTINUE
IF (IWL.EQ.1) GO TO 250
WRITE (6,370)
C-----GENERATE WEIBULL PARAMETERS
DO 240 I=1,NE
E=EXPX(I)
WRITE (6,680) E
DM=DIWM(I)
DMSTD=SDDIWM(I)
WRITE (6,690)
CALL WEIBR (DM,DMSTD,CL,CU,B,C)
BWM(I)=B
CWM(I)=C
DMO=DIWM(I)
DMSTDO=SDDIWO(I)
WRITE (6,700)
CALL WEIBR (DMO,DMSTDO,CL,CU,B,C)
BWO(I)=B
CWO(I)=C
CONTINUE
C-----CALCULATE MEAN, STANDARD DEVIATION AND COEFFICIENT OF VARIATION
XMEANW=BDEL*DGAMMA(1,+(1./CDEL))
XSTDDW=BDEL*(DGAMMA(1,+(2./CDEL))-((DGAMMA(1,+(1./CDEL)))**2))
XCVDW=DSQR(((DGAMMA(1,+(2./CDEL)))/(DGAMMA(1,+(1./CDEL))))-1.)
WRITE (6,710) XMEANW,XSTDDW,XCVDW
DELME=XMEANW
DLSTDW=XSTDDW
IF (IWL.EQ.1) GO TO 340
WRITE (6,720)
DO 260 I=1,NE
E=EXPX(I)
DM=DIWM(I)
CSFW=DELME/DM
WRITE (6,730) E,CSFW

```

```

A1 550
A1 540
A1 550
A1 560
A1 570
A1 580
A1 590
A2 000
A2 010
A2 020
A2 030
A2 040
A2 050
A2 060
A2 070
A2 080
A2 C90
A2 100
A2 110
A2 120
A2 130
A2 140
A2 150
A2 160
A2 170
A2 180
A2 190
A2 200
A2 210
A2 220
A2 230
A2 240
A2 250
A2 260
A2 270
A2 280
A2 290
A2 300
A2 310
A2 320
A2 330
A2 340
A2 350
A2 360
A2 370
A2 380
A2 430
A2 440

```

```

260 CONTINUE
C-----WRITE (6,370)
CALCULATE PROBABILITY OF FAILURE BASED ON WEIBJLL DISTRIBUTION
WRITE (6,740)
NDIM=50
DO 330 L=1,NE
E=EXPX(L,75C) E
WRITE (6,76C)
ZU=DELM
DINC=DLSDM
DO 320 M=1,2
IF (M.EG.2) WRITE (6,770)
LL=0
DO 300 J=1,10
HM=ZU/FLOAT(NDIM)
DINT=0.0
DO 270 K=1,NDIM
AW=(DINT/BDEL)*CDEL
ALW=CWM(L)/EMM(L)
A2W=(DINT/BWM(L))*CWM(L)-1.0)
A3W=(DINT/BWM(L))*CWM(L)
FDELW=1.-CDEXP(-AW)
FDESW=ALW*A2W*DEXP(-A3W)
UH=FDELW*FDESW
QW(K)=UH
DINT=DINT+HM
CONTINUE
CALL DQSF (HM,QW,QWT,NDIM)
QQ(J)=QWT(NDIM)
IF (J.EQ.1) GO TO 280
IF (QQ(J).GT.QQ(J-1)) GO TO 280
QQ(J)=QQ(J-1)
GO TO 310
CONTINUE
LL=LL+1
ZU=ZU+DINC
IF (J.EQ.1) GO TO 290
QS=(QQ(J)-Q(J-1))/QQ(J-1)
QS TOP=DABS(CS)
IF (QSTOP.LE.0.01) GO TO 310
CONTINUE
CONTINUE
CONTINUE
ZU=DIHM(L)
DINC=SDDIHM(L)
CONTINUE
CONTINUE
320
330

```

```

A245U
A2460
A2470
A2480
A2490
A2500
A2510
A2520
A2530
A2540
A2550
A2560
A2570
A2580
A2590
A2600
A2610
A2620
A2630
A2640
A2650
A2660
A2670
A2680
A2690
A2700
A2710
A2720
A2730
A2740
A2750
A2760
A2770
A2780
A2790
A2800
A2810
A2820
A2830
A2840
A2850
A2860
A2870
A2880
A2930
A2940
A2950
A2960

```

```

340 CONTINUE
IF (IML.EQ.C) GO TO 360
WRITE (6,37C)
C-----ESTIMATE FATIGUE LIFE USING LOG-MEAN DISTRIBUTION
C-----
WRITE (6,790)
CALCULATE
DO 350 I=1,NE
E=EXPX(I)
DMM=DMM(I)
WRITE (6,81C) DMM,DELM
CSF=DELM/DMM
WRITE (6,82C) CSF
WRITE (6,83C) CVM, XCVWD
C-----CALCULATE STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION
ACVDDI=1.0+XCVWD**2
ACVCDI=1.0+XCVWD**2
ALCVP=DSQRT(ACVDDI/ACVCDI)
ASLCP=DSQRT(ALCVP)
ALCV=DLOG(ACV)
ALFCT=(ACSF*ALCV)/ASLCP
WRITE (6,84C) ALFCT
CALL MENDR (ALFCT,PRFLN)
PRBFLN=1.0-PRFLN
WRITE (6,85C) PRBFLN
CONTINUE
CONTINUE
350
360
C
370 STOP
380 FORMAT (1H1,F10.5)
390 FORMAT ((15E12.4))
400 FORMAT ((2E15.8))
410 FORMAT (13F10.0)
420 FORMAT (16F10.0)
430
440
1 N S I,1,12X,8HFREQ(HZ),7X,13HDENSITY FACTOR,15X,34HGENERATED DATA POINTS=,15,/,16H D E
2 F U R A T A L,10X,5H
1,5X,20A4,/,15X,39HNUMBER OF POINTS IN RANDOM PROCESS(IN)=,F12.7/,15X,48H A
2 E Q U E N C I E S (N P I)=,15,/,15X,39HNUMBER OF POINTS IN RANDOM PROCESS(IN)=,F12.7/,15X,48H A
3,15,/,15X,36H TIME INTERVAL IN RANDOM PROCESS(DT)=,F12.7/,15X,48H A
4 V E R A G E F R E Q . I N T E R V A L T O C H A R A C T E R I Z E S P E C T R A L,15H D E N S I T Y I N H Z (D
5 F I)=,F15.5,/,15X,26HMEAN OF RANDOM PROCESS(B)=,F15.5,/,15X,22HULTIM

```

```

A2570
A2580
A2990
A3000
A3010
A3020
A3030
A3040
A3050
A3060
A3070
A3080
A3090
A3100
A3110
A3120
A3130
A3140
A3150
A3160
A3170
A3180
A3190
A3200
A3210
A3220
A3230
A3240
A3250
A3260
A3270
A3280
A3290
A3300
A3310
A3320
A3330
A3340
A3350
A3360
A3370
A3380
A3390
A3400
A3410
A3420
A3430
A3440

```

```

450 GATE STRENGTH(SU)=F15.5//,15X,65H ANALYSIS TYPE(IFLAG);(O=DETERMINI
7 STIC; I=PROBABILITY,2=BOTH)=,15,///
18 FREQ(HZ)=14X,15,2F20.5)
460 FFORMAT ((10X,15,3F20.5)
470 FFORMAT ((15F10.0)
480 FFORMAT ((1H,1//,15X,35H STRESS RANGES MEASURED FROM VALLEYS,36H TO P
490 FFORMAT ((1H,1//,15X,35H STRESS RANGES MEASURED FROM VALLEYS,36H TO P
500 LEAKS AND MEAN STRESS RANGE,9X,11H MEAN STRESS)
510 2HP EAK 18X,12H STRESS RANGE,9X,11H MEAN STRESS)
520 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
530 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
540 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
550 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
560 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
570 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
580 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
590 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
600 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
610 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
620 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
630 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
640 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
650 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
660 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL
670 FFORMAT ((1H,1//,15X,36H STRESS RANGES MEASURED FROM PEAKS TO,35H VAL

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2 WITHOUT MEAN STRESS=1E15.8 //)
FORMAT (1X,10H FOR BETA=,F10.3,25H WEIBULL PARAMETERS ARE: )
1 DAMAGE INDEX WITH MEAN. //,11X,17H STRESS CONSIDERED. //)
FORMAT (//,5X,167H ***CALCULATED WEIBULL PARAMETERS FOR DESIGN D
1 DAMAGE INDEX WITHOUT //,11X,22H MEAN STRESS CONSIDERED //)
110X,41H EXPECTED VALUE OF CRITICAL DAMAGE INDEX=,F12.8 //,10X,45H $
2 STANDARD DEVIATION OF CRITICAL DAMAGE INDEX=,F12.8 //,10X,51H COEFFI
3 CIENT OF VARIATION OF CRITICAL DAMAGE INDEX=,F12.8 //)
1 RIBUTION. //)
FORMAT (//,5X,57H CALCULATED CENTRAL SAFETY FACTOR FOR WEIBULL DIST
1 NG WEIBULL MODEL. //)
FORMAT (//,1X,10H FOR BETA=,F10.3,18H ANALYSIS YIELDS: //)
1 DAMAGE INDEX. //)
FORMAT (5X,64H CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL
1 DAMAGE INDEX. //)
FORMAT (5X,62H CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DA
1 GE INDEX. //)
FORMAT (5X,33H PREDICTED PROBABILITY OF FAILURE OF DESIGN DA
1 MAGE INDEX. //)
FORMAT (//,1X,10H FOR BETA=,F10.3,18H ANALYSIS YIELDS: //)
2 OF CRITICAL DAMAGE INDEX=,F10.6 //,15X,65H LOG-NORMAL DISTRIBUTION EXPECTED VALUE
FORMAT (5X,23H CENTRAL SAFETY FACTOR=,F10.6 //)
110.3 //,5X,52H COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX=,
FORMAT (5X,56H STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCT
1 ION=,F10.6 //)
END
SUBROUTINE STAT (U,K,XM,STO)
IMPLICIT REAL*8 (A-H,O-Z)
C*** EVALUATE MEAN AND STANDARD DEVIATION OF THE PROCESS U
C
DIMENSION U(1)
XK=K
XM=0.
DO 10 I=1,K
XM=XM+U(I)
CONTINUE
XM=XM/XK
10

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```

A3 930
A3 940
A3 950
A3 960
A3 970
A3 980
A3 990
A4 000
A4 010
A4 020
A4 030
A4 040
A4 050
A4 060
A4 070
A4 080
A4 090
A4 100
A4 110
A4 120
A4 130
A4 140
A4 150
A4 160
A4 170
A4 180
A4 190
A4 200
A4 210
A4 220
A4 230
A4 240
A4 250
A4 260
A4 270
A4 280-
B 10
B 20
B 30
B 40
B 50
B 60
B 70
B 80
B 90
B 100
B 110
B 120

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140
 150
 160
 170
 180
 190
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 210
 220
 230
 240
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 360
 370
 380
 390
 400
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 90
 100
 110
 120
 130
 140
 150
 160
 170
 180
 190
 200
 210

```

K=0
IF (PL(I)-P(I)) 40,40,50
CONTINUE
C(I)=PL(I)-V(I)
GO TO 20
CONTINUE
PM=P(I)
CONTINUE
IF (I+I+K-N) 70,70,90
CONTINUE
IF (V(I+I+K)-V(I)) 80,80,100
CONTINUE
C(I)=PM-V(I)
GO TO 20
IF (P(I+I+K)-PM) 140,110,110
C(I)=P(I+I+K)-V(I) 120,120,130
PL(I+I+K)=P(I+K)
GO TO 20
PL(I+I+K)=PM
PM=P(I+I+K)
GO TO 150
PL(I+I+K)=PM
K=K+1
GO TO 60
RETURN
END
SUBROUTINE RNDRP (N,V,P,C)
IMPLICIT REAL*8 (A-H,O-Z)
C**** RAINFLOW COUNTING (STRESS RANGE FROM PEAK TO VALLEY)
C
C
DIMENSION V(1),P(1),C(1),VL(1250)
DO 10 I=1,N
VL(I)=-10**6
I=0
CONTINUE
IF (I-N) 30,30,160
CONTINUE
K=0
IF (VL(I)-V(I)) 50,40,40
CONTINUE
C(I)=P(I)-V(I)
GO TO 20
CONTINUE
VM=V(I)
CONTINUE
  
```

40
 50
 60
 70
 80
 90
 100
 110
 120
 130
 140
 150
 160
 C
 C
 10
 20
 30
 40
 50
 60

```

70 IF (I+1+K-N) 70,70,90
CONTINUE
IF (P(I+1+K)-P(I)) 100,80,80
80 CONTINUE
90 C(I)=P(I)-VM
GO TO 20
100 IF (V(I+1+K)-VM) 110,110,140
110 IF (VL(I+1+K)-V(I+1+K)) 130,120,120
120 C(I)=P(I)-VL(I)
VL(I+1+K)=V(I+K)
GO TO 20
130 VL(I+1+K)=VM
VM=V(I+1+K)
GO TO 150
140 VL(I+1+K)=VM
K=K+1
150 GO TO 60
160 RETURN
END
SUBROUTINE FANSPC (NP,DF,W,SP,w,G,DY)
IMPLICIT REAL*8 (A-H,O-Z)
C***
C*** GENERATING FANNDOM SPCTRAL FREQUENCIES AND THE VALJE OF
C THE SPCTRAL DENSITY FUNCTION
C
C DIMENSION WF(1),SP(1),W(1),G(1),DY(1),R(1)
DOUBLE PRECISION DSEED
DSEED=5259.0D0
ISTOP=0
DEND=WF(1)
I=0
IU=1
IL=1
CONTINUE
I=I+1
CALL GGUBS (DSEED,I,R)
Q=R(I)*(0.5+C)
DD=DF*(0.5+C)
DCTR=DEND+0.5*DD
DEND=DEND+DC
C IF (DEND.GT.WF(NP)) GO TO 40
DE=WF(NP)-DEND
DP=DF*0.25
IF (DE.LE.DF) GO TO 50
CONTINUE
K=0

```

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220
230
240
250
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280
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300
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320
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340
350
360
370
380
390
400-
10
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30
40
50
60
70
80
90
100
110
120
130
140
150
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170
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190
200
210
220
230
240
250
260
270
280
290

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EEEEEEEEEEEEEEEEEEEEEE
FFFFFFFFFFF

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C          CT(3)=(CU+CL)/2.0
C          DU 20 I=1,3
C          A=1.+(1./CT(I))
C          B=1.+(2./CT(I))
C          GA=DGAMMA(A)
C          GB=DGAMMA(B)
C          V(I)=DSCRT(GB/(GA*GA)-1.)
C          CONTINUE
C          ERROR CRITERION
C          ERR=DABS((CV-V(3))/CV)
C          IF (ITER.LE.4) GO TO 70
C          IF (ITER.GE.15) GO TO 30
C          ITERATION SCHEME
C          IF ((CV-V(3))) 30,70,40
C          CONTINUE
C          CU=CT(2)
C          CL=CT(3)
C          ITER=ITER+1
C          GO TO 10
C          CONTINUE
C          CU=CT(3)
C          CL=CT(1)
C          ITER=ITER+1
C          GO TO 10
C          CONTINUE
C          CL=CL+10.0
C          CU=CU+10.0
C          IF (CU.GE.100.0) 30 TO 60
C          ITER=0
C          GO TO 10
C          CONTINUE
C          WRITE (6,80) ITER,CV,V(3)
C          RETURN
C          C=CT(3)
C          B=XM/DGAMMA(1.+1./C)
C          WRITE (6,90) ITER,CV,XM,RMS,B,C
C          RETURN
C          (5X,10H*** ITER=15,6H CV=F10.5,11H CAL. CV=F10.5)
C          FORMAT (//,10X,27H NUMBER OF ITERATIONS USED=.15,/,10X,26H COEFFI
C          CIENT OF VARIATION=F10.5/,10X,26H MEAN DESIGN DAMAGE INDEX=,F10.
C          25,/,10X,43H STANDARD DEVIATION OF DESIGN DAMAGE IN DEX=,F10.5/,
C          3,38H CALCULATED VALUE OF PARAMETER "B" IS: ,E15.8,/,10X,38H CAL

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230
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250
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270
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290
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340
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390
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410
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440
450
460
470
480
490
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510
520
530
540
550
560
570
580
590
600
610
620
630
640
650
660
670
680
690
700

```


H 950
H 960
H 970
H 980
H 990
H1 000
H1 010
H1 020
H1 030
H1 040
H1 050
H1 060
H1 070
H1 080
H1 090
H1 100
H1 110
H1 120
H1 130
H1 140
H1 150
H1 160-

```

SUM1=HT*(Y(1)+SUM1+Y(3))
Z(1)=0.00
AUX1=Y(3)+Y(3)
AUX1=AUX1+AUX1
Z(2)=SUM2-HT*(Y(2)+AUX1+Y(4))
IF (NDIM-5) 100,90,90
AUX1=Y(4)+Y(4)
AUX1=AUX1+AUX1
Z(5)=SUM1+HT*(Y(3)+AUX1+Y(5))
Z(3)=SUM1
Z(4)=SUM2
RETURN
NDIM IS EQUAL TC 3
SUM1=HT*(1.2500*Y(1)+Y(2)-.2500*Y(3))
SUM2=Y(2)+Y(2)
SUM2=SUM2+SUM2
Z(1)=0.00
Z(2)=SUM1
RETURN
END

```

90
100
C
110
120

APPENDIX C

SAMPLE COMPUTER CODE OUTPUT FILE

N P S F A T I G U E A N A L Y S I S

SAMPLE TEST CASE 8

I N P U T C A T A

NUMBER OF INPUT FREQUENCIES(NPI)= 3
 NUMBER OF POINTS IN RANDOM PROCESS(INI)= 1000
 TIME INTERVAL IN RANDOM PROCESS(DT)= 0.0020000
 AVERAGE FREQUENCY INTERVAL TO CHARACTERIZE
 SPECTRAL DENSITY IN HZ (DF)= 1.00
 MEAN OF RANDOM PROCESS(BI)= 0.50000
 ULTIMATE STRENGTH(SU)= 60.00000
 ANALYSIS TYPE(TFLAG)= 2
 (0=DETERMINISTIC, 1=PROBABALISTIC, 2=BOTH)

2

PROBABILITY DISTRIBUTION MODEL FOR DAMAGE INDEX(IWL)=
 (0=WEIBULL, 1=LOG-NORMAL)
 LOWER LIMIT OF VARIABILITY SEARCH(CU)= 1.000
 UPPER LIMIT OF VARIABILITY SEARCH(CU)= 10.000
 WEIBULL DISTRIBUTION DAMAGE INDEX(BDEL)= 0.97710
 WEIBULL DISTRIBUTION DAMAGE INDEX(CDEL)= 1.32890
 DESIGN INDEX COEFFICIENT OF VARIATION(CVMM)= 0.800
 LIFE IN CYCLES TO FAILURE= 0.100D+11

S P E C T R A L D E N S I T Y

I	FREQ(HZ)	SPECTRAL	DENSITY
1	30.00000	0.0	0.0
2	50.00000	0.0	0.16000
3	80.00000	0.0	0.0

FROM THE SIMULATION

MEAN = 0.5079814

STANDARD DEVIATION = 2.0518550

STATISTICS ON STRESS CYCLES

NUMBER OF REVERSALS; VALLEY TO PEAK = 108
 MEAN = 4.7853503 2.6115879

NUMBER OF REVERSALS (PEAK TO VALLEY) = 107
 MEAN = 4.7475504 2.5879209

OVERALL MEAN VALUE = 4.7664504
 OVERALL STD DEVIATION = 2.5997544

EXPECTED VALUE OF S**M

BETA = 12.00000 E(S**M) WITH MEAN INCLUDED = 0.2138600D+12
 E(S**M) WITH MEAN IGNORED = 0.2084378D+12

BETA = 12.50000 E(S**M) WITH MEAN INCLUDED = 0.7307062D+12
 E(S**M) WITH MEAN IGNORED = 0.7133135D+12

BETA = 13.00000 E(S**M) WITH MEAN INCLUDED = 0.2509870D+13
 E(S**M) WITH MEAN IGNORED = 0.2451112D+13

BETA = 13.50000 E(S**M) WITH MEAN INCLUDED = 0.8632479D+13
 E(S**M) WITH MEAN IGNORED = 0.84553626D+13

BETA = 14.00000 E(S**M) WITH MEAN INCLUDED = 0.2984030D+14
 E(S**M) WITH MEAN IGNORED = 0.2925200D+14

***** DETERMINISTIC APPROACH *****

ESTIMATED FATIGUE LIFE

BETA = 12.000
 FATIGUE LIFE WITH MEAN INCLUDED = 0.17513195D+09 SEC.
 = 2026.990 DAYS
 = 5.553 YEARS

FATIGUE LIFE WITH MEAN IGNORED = 0.17969285D+09
 = 2079.778
 = 5.698
 SEC.
 DAYS
 YEARS

BETA= 12.500
 FATIGUE LIFE WITH MEAN INCLUDED = 0.39704527D+09
 = 4595.431
 = 12.590
 = 0.40672641D+09
 = 4707.482
 = 12.897
 SEC.
 DAYS
 YEARS
 SEC.
 DAYS
 YEARS

BETA= 13.000
 FATIGUE LIFE WITH MEAN INCLUDED = 0.89645123D+09
 = 10375.593
 = 28.426
 = 0.91684384D+09
 = 10611.618
 = 29.073
 SEC.
 DAYS
 YEARS
 SEC.
 DAYS
 YEARS

BETA= 13.500
 FATIGUE LIFE WITH MEAN INCLUDED = 0.20165016D+10
 = 23339.139
 = 63.943
 = 0.20591646D+10
 = 23832.924
 = 65.296
 SEC.
 DAYS
 YEARS
 SEC.
 DAYS
 YEARS

BETA= 14.000
 FATIGUE LIFE WITH MEAN INCLUDED = 0.45207894D+10
 = 52523.951
 = 143.353
 = 0.46095038D+10
 = 53350.739
 = 146.166
 SEC.
 DAYS
 YEARS
 SEC.
 DAYS
 YEARS

***** P R O B A B I L I S T I C A P P R O A C H *****

E(S**BETA)/(SU**BETA)

BETA	WITH MEAN	WITHOUT MEAN
12.000	0.10178252D+11	0.10473320D+11
13.000	0.23075324D+11	0.23637969D+11
13.500	0.52095608D+11	0.53284778D+11
13.500	0.11719426D+12	0.11967374D+12
14.000	0.26273750D+12	0.26789337D+12

CALCULATED NUMBER OF DESIGN CYCLES IS: 0.1000D+11

BETA= 12.000
 DESIGN CUMULATIVE DAMAGE INDEX WITH MEAN STRESS= 0.98248701D+00
 DESIGN CUMULATIVE DAMAGE INDEX WITHOUT MEAN STRESS= 0.95754991D+00

BETA= 12.500
 DESIGN CUMULATIVE DAMAGE INDEX WITH MEAN STRESS= 0.43336336D+00
 DESIGN CUMULATIVE DAMAGE INDEX WITHOUT MEAN STRESS= 0.42304819D+00

BETA= 13.000
 DESIGN CUMULATIVE DAMAGE INDEX WITH MEAN STRESS= 0.19194002D+00
 DESIGN CUMULATIVE DAMAGE INDEX WITHOUT MEAN STRESS= 0.18757086D+00

BETA= 13.500
 DESIGN CUMULATIVE DAMAGE INDEX WITH MEAN STRESS= 0.85328408D-01
 DESIGN CUMULATIVE DAMAGE INDEX WITHOUT MEAN STRESS= 0.83550521D-01

BETA= 14.000
 DESIGN CUMULATIVE DAMAGE INDEX WITH MEAN STRESS= 0.38060802D-01
 DESIGN CUMULATIVE DAMAGE INDEX WITHOUT MEAN STRESS= 0.37328284D-01

CALCULATION OF EXPECTED PROBABILITY OF FAILURE USING WEIBULL MODEL

FOR BETA= 12.000 ,ANALYSIS YIELDS:
CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL DAMAGE INDEX
DAMAGE INDEX PLUS 10 STANDARD DEVIATIONS OF DAMAGE INDEX= 5.081434
PREDICTED PROBABILITY OF FAILURE= 0.518595
CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DAMAGE INDEX
DAMAGE INDEX PLUS 6 STANDARD DEVIATIONS OF DAMAGE INDEX= 5.698425
PREDICTED PROBABILITY OF FAILURE= 0.518079

FOR BETA= 12.500 ,ANALYSIS YIELDS:
CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL DAMAGE INDEX
DAMAGE INDEX PLUS 5 STANDARD DEVIATIONS OF DAMAGE INDEX= 3.290924
PREDICTED PROBABILITY OF FAILURE= 0.272411
CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DAMAGE INDEX
DAMAGE INDEX PLUS 10 STANDARD DEVIATIONS OF DAMAGE INDEX= 3.900270
PREDICTED PROBABILITY OF FAILURE= 0.272529

FOR BETA= 13.000 ,ANALYSIS YIELDS:
CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL DAMAGE INDEX
DAMAGE INDEX PLUS 3 STANDARD DEVIATIONS OF DAMAGE INDEX= 2.334720
PREDICTED PROBABILITY OF FAILURE= 0.114002
CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DAMAGE INDEX
DAMAGE INDEX PLUS 9 STANDARD DEVIATIONS OF DAMAGE INDEX= 1.073908
PREDICTED PROBABILITY OF FAILURE= 0.113990

FOR BETA= 13.500 ,ANALYSIS YIELDS:
CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL DAMAGE INDEX
DAMAGE INDEX PLUS 2 STANDARD DEVIATIONS OF DAMAGE INDEX= 1.8556618
PREDICTED PROBABILITY OF FAILURE= 0.042219
CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DAMAGE INDEX
DAMAGE INDEX PLUS 8 STANDARD DEVIATIONS OF DAMAGE INDEX= 0.631430
PREDICTED PROBABILITY OF FAILURE= 0.042177

FOR BETA= 14.000 ,ANALYSIS YIELDS:
CALCULATED PROBABILITY OF FAILURE BASED ON CRITICAL DAMAGE INDEX
DAMAGE INDEX PLUS 5 STANDARD DEVIATIONS OF DAMAGE INDEX= 3.290924
PREDICTED PROBABILITY OF FAILURE= 0.015745
CALCULATED PROBABILITY OF FAILURE BASED ON DESIGN DAMAGE INDEX
DAMAGE INDEX PLUS 7 STANDARD DEVIATIONS OF DAMAGE INDEX= 0.251201
PREDICTED PROBABILITY OF FAILURE= 0.014855

CALCULATION OF EXPECTED PROBABILITY OF FAILURE USING LOG-NORMAL MODEL

FOR BETA= 12.000 ,ANALYSIS YIELDS:
LOG-NORMAL DIST: EXPECTED VALUE OF DESIGN DAMAGE INDEX= 0.982487
LOG-NORMAL DIST: EXPECTED VALUE OF CRITICAL DAMAGE INDEX= 0.900414
CENTRAL SAFETY FACTOR= 0.916464
COEFFICIENT OF VARIATION FOR DESIGN DAMAGE INDEX= 0.800
COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX= 0.671
STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCTION= -0.027798
PREDICTED PROBABILITY OF FAILURE= 0.511088

FOR BETA= 12.500 ,ANALYSIS YIELDS:
LOG-NORMAL DIST. EXPECTED VALUE OF DESIGN DAMAGE INDEX= 0.433363
LOG-NORMAL DIST. EXPECTED VALUE OF CRITICAL DAMAGE INDEX= 0.9000414
CENTRAL SAFETY FACTOR= 2.077734
COEFFICIENT OF VARIATION FOR DESIGN DAMAGE INDEX= 0.800
COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX= 0.671
STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCTION= 0.851415
PREDICTED PROBABILITY OF FAILURE= 0.197269

FOR BETA= 13.000 ,ANALYSIS YIELDS:
LOG-NORMAL DIST. EXPECTED VALUE OF DESIGN DAMAGE INDEX= 0.191940
LOG-NORMAL DIST. EXPECTED VALUE OF CRITICAL DAMAGE INDEX= 0.9000414
CENTRAL SAFETY FACTOR= 4.691120
COEFFICIENT OF VARIATION FOR DESIGN DAMAGE INDEX= 0.800
COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX= 0.671
STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCTION= 1.726206
PREDICTED PROBABILITY OF FAILURE= 0.042155

FOR BETA= 13.500 ,ANALYSIS YIELDS:
LOG-NORMAL DIST. EXPECTED VALUE OF DESIGN DAMAGE INDEX= 0.085328
LOG-NORMAL DIST. EXPECTED VALUE OF CRITICAL DAMAGE INDEX= 0.9000414
CENTRAL SAFETY FACTOR= 10.552331
COEFFICIENT OF VARIATION FOR DESIGN DAMAGE INDEX= 0.800
COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX= 0.671
STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCTION= 2.597003
PREDICTED PROBABILITY OF FAILURE= 0.004702

FOR BETA= 14.000 ,ANALYSIS YIELDS:
LOG-NORMAL DIST: EXPECTED VALUE OF DESIGN DAMAGE INDEX= 0.038061
LOG-NORMAL DIST: EXPECTED VALUE OF CRITICAL DAMAGE INDEX= 0.900414
CENTRAL SAFETY FACTOR= 23.657243
COEFFICIENT OF VARIATION FOR DESIGN DAMAGE INDEX= 0.800
COEFFICIENT OF VARIATION FOR CRITICAL DAMAGE INDEX= 0.671
STANDARDIZED ARGUMENT FOR NORMAL DISTRIBUTION FUNCTION= 3.464198
PREDICTED PROBABILITY OF FAILURE= 0.000266

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