GUST RESPONSE OF A LIGHT, SINGLE-ENGINED, HIGH-WING AIRCRAFT

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GUST RESPONSE OF A LIGHT, SINGLE-ENGINED, HIGH-WING AIRCRAFT

by

C.J. LUDOWYK*

SUMMARY

A recently developed Fortran program for calculating rigid-aircraft gust response has been applied to obtain Longitudinal and Lateral transfer functions and output response spectra for a general aviation, high-wing aircraft.

* On attachment to Aerodynamics Division from Structures Division during the course of this work.

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NOTATION

\[ a_t \]
- lift curve slope of horizontal tail

\[ b_t \]
- tail span, 3.96 m (13.0 ft)

\[ C_D \]
- drag coefficient, \( D/\frac{1}{2} PV^2 S \)

\[ C_{D_a} \]
- \( \frac{\partial C_D}{\partial \alpha} \), per radian

\[ C_{D_e} \]
- \( \frac{\partial C_D}{\partial \epsilon} \), per radian

\[ C_{Dv} \]
- \( V \frac{\partial C_D}{\partial V} \)

\[ C_{L} \]
- lift coefficient, \( L/\frac{1}{2} PV^2 S \)

\[ C_{L_a} \]
- lift curve slope \( \frac{\partial C_L}{\partial \alpha} \), per radian

\[ C_{L_b} \]
- \( \frac{\partial C_L}{\partial (\epsilon/2 V)} \), per radian

\[ C_{Ld} \]
- \( \frac{\partial C_L}{\partial (\alpha/2 V)} \), per radian

\[ C_{Lv} \]
- \( V \frac{\partial C_L}{\partial V} \)

\[ C_{Lz} \]
- rolling-moment coefficient, \( \text{moment}/\frac{1}{2} PV^2 S b \)

\[ C_{Lz} \]
- effective - dihedral parameter, \( \frac{\partial C_L}{\partial \theta} \), per radian

\[ C_{Lp} \]
- \( \frac{\partial C_L}{\partial (pb/2 V)} \), per radian

\[ C_{Lx} \]
- \( \frac{\partial C_L}{\partial (rb/2 V)} \), per radian

\[ C_m \]
- pitching-moment coefficient, \( M/\frac{1}{2} PV^2 S \epsilon \)

\[ C_{ma} \]
- pitch stiffness parameter, \( \frac{\partial C_m}{\partial \alpha} \), per radian

\[ C_{ma} \]
- \( \frac{\partial C_m}{\partial (\epsilon/2 V)} \), per radian

\[ C_{me} \]
- elevator effectiveness parameter, \( \frac{\partial C_m}{\partial \epsilon} \), per radian

\[ C_{mq} \]
- \( \frac{\partial C_m}{\partial \epsilon} \), per radian

\[ C_{m} \]
- \( V \frac{\partial C_m}{\partial V} \)

\[ C_n \]
- yawing-moment coefficient, \( \text{moment}/\frac{1}{2} PV^2 S b \)

\[ C_{nb} \]
- directional-stability parameter, \( \frac{\partial C_n}{\partial \beta} \), per radian

\[ C_{np} \]
- \( \frac{\partial C_n}{\partial (pb/2 V)} \), per radian

\( \ldots/\text{cont.} \)
NOTATION (CONT.)

\( C_{n_r} \) \( \frac{\partial C_n}{\partial (rb/2 V^e)} \), per radian

\( C_w \) weight coefficient, \( W/\frac{1}{2} \rho V^2 S \)

\( C_y \) side force coefficient, \( Y/\frac{1}{2} \rho V^2 S \)

\( C_{Y_2} \) \( \frac{\partial C_y}{\partial \alpha} \), per radian

\( C_{Y_P} \) \( \frac{\partial C_y}{\partial P} \), per radian

\( C_{Y_T} \) mean aerodynamic chord, 1.50 m (4.91 ft)

\( C_G \) centre of gravity

\( D \) drag force, N

\( g \) acceleration due to gravity, m/sec\(^2\)

\( h \) distance of C.G. from wing leading edge, expressed as a fraction of \( c \)

\( h' \) altitude, m (2000 ft in this study)

\( I_x \) moment of inertia about X body axis, 1395 kg-m\(^2\) (1025.6 slug-ft\(^2\))

\( I_y \) moment of inertia about Y body axis, 1480 kg-m\(^2\) (1088.1 slug-ft\(^2\))

\( I_z \) moment of inertia about Z body axis, 2563 kg-m\(^2\) (1884.3 slug-ft\(^2\))

\( I_{xz} \) product of inertia, 123 kg-m\(^2\) (90.4 slug-ft\(^2\))

\( \ell_t \) distance from horizontal tail aerodynamic centre to the aircraft C.G., 4.51 m (14.8 ft)

\( M \) pitching moment, N-m

\( m \) aircraft mass, 1318.2 kg (90.06 slugs)

\( N \) yawing moment, N-m

\( n_z \) normal acceleration, \( g \) units

\( p \) roll rate, rad/sec

\( P_d \) dynamic pressure

\( P_g \) lateral gust gradient, \( -(1/V_e) \left( \frac{\partial w}{\partial y} \right) \)

\( q \) pitch rate, rad/sec

\( r \) yaw rate, rad/sec

\( S \) wing area, 16.26 m\(^2\) (175 ft\(^2\))

\( S_t \) horizontal tail area, 3.59 m\(^2\) (38.65 ft\(^2\))
NOTATION (CONT.)

T  thrust, N
T'  thrust coefficient, T/ρV^2 S
V  airspeed, m/sec
V_H  horizontal tail volume ratio, \( \ell_t S_t / S_c \) (0.67)
W  aircraft weight
Y  side force, N
\( \alpha \)  angle of attack, radians
\( \alpha_g \)  angle of attack due to \( \gamma \) gust, \( (\gamma_g / V_e) \)
\( \beta \)  angle between thrust vector and fuselage ref. line
\( \beta_g \)  angle between thrust vector and fuselage ref. line
\( \delta_e \)  elevator deflection, positive when trailing edge is down, deg.
\( \zeta \)  damping ratio
\( \gamma \)  roll angle, deg
\( \gamma \)  climb angle, deg
\( \rho \)  air density, kg/m^3 (0.0022 slug/ft^3 at 2000 ft)
\( \beta \)  pitch angle, deg
\( \delta_{\text{gust}} \)  input gust intensity, m/sec (10 ft/sec in this study)
\( \delta_v \)  standard deviation of aircraft velocity response, ft/sec
\( \delta_{\alpha} \)  standard deviation of angle of attack response, deg.
\( \delta_q \)  standard deviation of pitch rate response, deg/sec
\( \delta_{\theta} \)  standard deviation of pitch angle response, deg
\( \delta_{n_z} \)  standard deviation of normal acceleration response, fractional g
\( \delta_{\phi} \)  standard deviation of aircraft sideslip response, deg
\( \delta_{p} \)  standard deviation of aircraft roll rate response, deg/sec
\( \delta_{r} \)  standard deviation of aircraft yaw rate response, deg/sec
\( \delta_{\phi} \)  standard deviation of aircraft roll angle response, deg
\( \delta_{\theta} \)  standard deviation of aircraft yaw angle response, deg
T  time constant, seconds
\( \gamma \)  yaw angle, deg

../cont.
<table>
<thead>
<tr>
<th>Subscripts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>Parameter or coefficient at trimmed condition</td>
</tr>
<tr>
<td>g</td>
<td>Refers to gust</td>
</tr>
<tr>
<td></td>
<td>A dot over a symbol signifies a derivative with</td>
</tr>
<tr>
<td></td>
<td>respect to time</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

The work reported here is part of a task to determine the response spectra of various aircraft due to gust turbulence. It was done for the Advanced Engineering Laboratory, Salisbury, which requires data for use in the design of inertially stabilised airborne control systems. This work has also served as an introduction to the concepts of aircraft stability and control, to augment the author's instrumentation and control background.

The aircraft configuration which is the subject of this Memo is broadly representative of high wing, general aviation monoplanes. Specifically, the aerodynamic data used in the mathematical models have been obtained from Reference 2 and Reference 3. Estimates for center of gravity, weight and inertias have been made using information in Reference 3 and Reference 4 as a guide.

This Memo outlines the procedure used to calculate the various stability derivatives and tabulates all the input data to the GUSTR program described in Reference 1. Output frequency responses and spectra results are included and a table of calculated responses to an input gust intensity of 10 ft/sec is given.

2. LONGITUDINAL PARAMETERS

2.1 Trim Determination

The parameters for three CG positions of 15%, 25% and 35% of mean aerodynamic chord, \( \xi \), were evaluated. These three positions were chosen on the following basis.

The moment reference for the model in Ref. 2 was 0.2 m from the wing leading edge, or 13.34% c. Ref. 2 also indicated that the model was neutrally stable with the CG as far aft as 40% \( \xi \). Therefore, three CG positions of 15%, 25% and 35% \( \xi \) were chosen for this study, together with two speed cases of 120 knots and 160 knots. This resulted in a total of six sets of trim angle of attack, \( \alpha \), elevator angle, \( \delta_e \) and thrust coefficient, \( T_c \), together with six corresponding sets of longitudinal stability derivatives. An altitude of 2000 ft and zero climb angle have been assumed for all cases.

The trim values of \( \alpha \), \( \delta_e \), and \( T_c \) were obtained by first tabulating the \( C_m \), \( C_{L}' \), \( C_D \) curves in fig. 7(a), (b), (c) of Ref. 2 for

\[
0^{\circ} < \alpha < 10^{\circ} \\
-15^{\circ} < \delta_e < 5^{\circ} \\
0^{\circ} < T_c < 0.3
\]
This file of 540 data points was used in a least squares fitting program (Fig. 1) to provide coefficients for \( C'_L \), \( C'_D \) and \( C'_m \) in the following (not necessarily linear) representation:

\[
C'_L = a_0 + a_1 \alpha + a_2 \dot{\alpha} + a_3 T' \quad \ldots (1)
\]

\[
C'_D = b_0 + b_1 \alpha + b_2 \alpha^2 + b_3 \dot{\alpha} + b_4 \dot{\alpha}^2 + b_5 \alpha \dot{\alpha} + b_6 T' \quad \ldots (2)
\]

\[
C'_m = c_0 + c_1 \alpha + c_2 \alpha^2 + c_3 \dot{\alpha} + c_4 \dot{\alpha}^2 \quad \ldots (3)
\]

Note that the representations of forces and moments, \( C'_L \), \( C'_D \) and \( C'_m \), given in Equations (1) to (3) include both aerodynamic and thrust contributions. Consequently the trim equations are (see Fig. 2)

\[
C'_L - C_w \sin \gamma_e = 0 \quad \ldots (4)
\]

\[
C'_D + C_w \cos \gamma_e = 0 \quad \ldots (5)
\]

\[
C'_m = 0 \quad \ldots (6)
\]

where \( C_w = W/\rho V^2 S \), \( \gamma_e \) is the steady climb angle and \( C'_L \), \( C'_D \) and \( C'_m \) are given as functions of \( \alpha \), \( \dot{\alpha} \) and \( T' \) by Equations (1) to (3) above. For given values of density, \( \rho \), aircraft weight, \( W \), trim speed, \( V \) and climb angle, \( \gamma_e \), Equations (4) to (6) constitute a set of non-linear algebraic equations for trim values of \( \alpha \), \( \dot{\alpha} \) and \( T' \). The solution is obtained in program TRIM (Fig. 1) and the results for the six cases considered are given in Table 1.

When it is necessary to separate out the aerodynamic component of, say, \( C'_D \) from the thrust component, this can be done by noting that Equation (2) can be rewritten (see Fig. 2)

\[
C'_D \cos(\alpha + \alpha'_T) = b_0 + b_{1,1} \alpha + b_{2,2} \alpha^2 + b_{3,3} \dot{\alpha} + b_{4,4} \dot{\alpha}^2 + b_{5,5} \alpha \dot{\alpha} + b_{6,6} T' \quad \ldots (7)
\]

or, for small \((\alpha + \alpha'_T)\)

\[
C'_D = b_0 + b_{1,1} \alpha + b_{2,2} \alpha^2 + b_{3,3} \dot{\alpha} + b_{4,4} \dot{\alpha}^2 + b_{5,5} \alpha \dot{\alpha} + (b_6 + 1) T' \quad \ldots (7)
\]
where \( C_p \) is now the aerodynamic component only. Similarly, in Equation (1) \( C'_c \) can be replaced by \( C_L + T_c \sin(\alpha + \alpha_c) \) which is approximately equal to \( C_L \) (Aerodynamic) for small \( \alpha \).

The coefficients for Equations (1) to (3) obtained from the least squares fitting program are listed in Table 2. A sample check on their accuracy was made by verifying that Equations (4) to (6) were satisfied when the trim values of \( \alpha \), \( \delta_e \) and \( T'_c \) were substituted in these best regression solutions.

2.2 Aircraft Weight and Inertias

The CG envelope in Reference 4 gives a ratio of

\[
\text{Airplane Weight (Most forward CG position)} = 0.81
\]

Max. Take off weight

Using the same factor on the aircraft of reference 2, with maximum take-off weight = 3600 lbf gives

\[ W(\text{fwd CG}) = 2916 \text{ lbf.} \]

Therefore a figure of 2900 lbf for aircraft weight was used throughout this study, being the smallest maximum allowable weight within the CG envelope considered.

The inertias \( I_x \), \( I_y \), \( I_z \) and \( I_{xy} \) were taken from Reference 3 and were not corrected for the different CG positions of this study. These values, together with those of the pertinent geometric characteristics used in this study, are listed in the notation.

2.3 Determination of Longitudinal Stability Derivatives

The combination of wind tunnel results (Ref. 2) and flight measurements (Refs. 3 and 4) was the source of some derivatives, while others not directly measured, were derived using the formulae of Reference 5. Note that the static derivatives, e.g. \( C_L \), \( C_{nq} \) (in the lateral case) etc. were obtainable from both wind tunnel and flight tests. Dynamic derivatives, e.g. \( C_{nq} \), \( C_{m_p} \) (in the lateral case) etc. were only obtainable from the flight tests. Also, \( C_{nq} \) and \( C_{m_p} \) could not be separated from each other in the flight test results, therefore this was an instance where \( C_{nq} \) was first calculated and \( C_{m_p} \) then followed from the subtraction of \( C_{nq} \) from the combined flight test result of \( C_{nq} + C_{m_p} \).
2.3.1 Static derivatives

Parameters $C_L$, $C_{p\alpha}$, $C_{\alpha}$, $C_D$, $C_{p\phi}$, $C_{\phi}$, $C_m$, $C_L$, $C_{\phi}$ were evaluated directly from the least squares representations of Eqns. (1), (3) and (7). E.g. From Eqn. (7)

$$C_{D_{\alpha}} = \frac{\partial C_D}{\partial \alpha} = b_1 + 2b_2 \cdot \alpha_{\text{Trim}}$$

2.3.2 The $V$ derivatives

The aircraft has a constant speed propeller, therefore (Ref. 5)

$$C_{T_v} = -3C_{D_e}$$

The derivative $C_L$ has an expression typical of the $V$ derivatives, where (Ref. 5)

$$C_{L_v} = M \frac{\partial C_L}{\partial M} + C_{e\phi} e_v \frac{\partial C_L}{\partial \phi_d} + C_{T_v} \frac{\partial C_L}{\partial T_c}$$

$\frac{\partial C_L}{\partial M} = 0$ below 0.6M (negligible compressibility effects)

$\frac{\partial C_L}{\partial \phi_d} = 0$ (negligible aeroelastic effects)

and $\frac{\partial C_L}{\partial T_c} = 0$ (from the Least Squares representation of $C_L$).

Thus $C_{L_v} = 0$. 
Similarly, the first two terms disappear in the expression for $C_{D_v}$ and $C_{m_v}$, leaving

$$C_{D_v} = C_{D_T} \cdot C_{D_T'} \cdot C_{D_T''}$$

$$C_{m_v} = C_{m_T} \cdot C_{m_T'} \cdot C_{m_T''}$$

2.3.3 The $q$ derivatives

$$C_{L_q} = -2C_{L_d} (h - h_o) + (2a_t V_H) \quad (\text{Ref. 5})$$

where

- $h_o = 0.75$ for subsonic flow
- $V_H = \frac{i}{S_t} = 0.67$
- $\ell_t = 14.8 \text{ ft}$ (scaled from fig. 2, Ref. 2)
- $S_t = 38.65 \text{ ft}^2$ \quad (Ref. 7)
- $a_t = 3.72/\text{rad}$ \quad (Calculated from Ref. 8, knowing the tail aspect ratio)

Therefore for Case 1, with CG at 15% $c$

$$h = 0.15.$$ 

Thus

$$C_{L_q} = 6.40 + 4.99$$

$$C_{L_q} = 11.39/\text{rad}.$$ 

Also

$$C_{m_q} = C_{m_q} - 2C_{L_d} (h - \bar{h})^2 - 2a_t V_H \ell_t \quad (\text{Ref. 5})$$

For subsonic flow, $C_{m_q} = 0$

$$\bar{h} = 0.5$$
Therefore for Case 1

\[ C_{m_q} = 0 - 1.31 - 15.04 \]
\[ C_{q} = -16.35/\text{rad}. \]

2.3.4 The \( \delta \) derivatives

Ref. 3 gives \( C_{m_o} + C_{n_o} = -18.5 \) at \( C_L = 0.36 \)

Hence \( C_{m_o} = -2.15/\text{rad} \) for Case 1.

and \( C_{L_o} = -\frac{\delta}{L} \), \( C_{m_o} = 0.713/\text{rad/sec} \) for Case 1.

A summary of the Longitudinal derivatives is given in Table 3.

3. LATERAL DERIVATIVES

The derivatives \( C_{p}, C_{\ell}, C_{n}, C_{\phi}, C_{r}, C_p, C_{\alpha} \) were all derived from the flight test results of Reference 3. The 'maximum likelihood' results were chosen and interpolated values of the derivatives for \( C_{\alpha} = 0.36 \), corresponding to 120 knots at 2000 ft, were obtained. The \( \delta \) derivatives, which were also available from the wind tunnel results of Reference 2, were used as a cross-check. \( C_{\alpha} \) agreed within 1%. \( C_{n_p} \), after suitable correction for CG position, \( n_p \)

was found to be 10% higher and \( C_{\phi} \) was found to be 7.5% higher in the wind tunnel results. A listing of the lateral derivatives used is given in Table 4.

The lateral gust response was computed for \( V_e = 120 \) knots at 2000 feet with CG located at 0.15 \( \bar{z} \). Other cases can be run as required.
The method of correcting $C_{n_B}^*$, given at a particular CG location (e.g. 0.28 $\bar{c}$ in Ref. 3), for a different CG location, (e.g. 0.15 $\bar{c}$ in the lateral case computed), is as follows:-

$$C_{n_B}^*(0.15 \bar{c}) = C_{n_B}^*(0.28 \bar{c}) - (0.26 - 0.15) \bar{c} \cdot C_{Y_B}$$

E.g. for $C_{n_B}^*(0.28 \bar{c}) = 0.050$/rad and $C_{Y_B} = -0.59$/rad

$$C_{n_B}^*(0.15 \bar{c}) = 0.060$/rad.

4. RESULTS

The results of all longitudinal and lateral cases run are tabulated in Tables 5 and 6. The computed frequency responses and output spectra of the GUSTR program (Ref. 1) are shown in Figs. 3 to 22. The von Karman gust spectrum was used, with an $\sigma_g$ input to the longitudinal model and both $\bar{g}_c$ and $\bar{g}_w$ inputs to the lateral model (Ref. 1). All output variance calculations in Table 5 were based on an input gust intensity of $\sigma_g = 10$ ft/sec. and the formula used was (Ref. 1):

$$\frac{2}{2} \frac{\text{response}}{\sigma_g} = 4.6 \times \text{(area under curve)} \times \text{(scale factor)}$$

Note that the computer program uses radian measure and f.p.s. units in the output plots.

Areas under the output spectra curves were measured with a planimeter in the range

- $4 \leq \log \Omega^1 \leq -1$
corresponding to a frequency range of

0.0032 Hz - 3.2 Hz for the 120 knot cases
and 0.0043 Hz - 4.3 Hz for the 160 knot cases.

The theoretical range of validity is at $\xi = 0.2$

i.e. at $\log \xi = -1.39$.

However, for comparing the relative trends between the various cases, extending the cut-off limit to $\log \xi = -1.00$ (as done in this exercise) should not introduce significant error.

5. DISCUSSION

The results are generally as expected, especially the variation of natural frequencies and damping ratios of the different modes with speed, $V_e$, and CG location. It is worth noting that at both rearward CG locations of 0.35 $c$ (Cases 3 and 6), the Short Period Mode has real roots.

The output spectra most likely to be of relevance to the designer of inertia stabilized airborne control systems are the attitude and rate spectra, viz. $\phi$, $\psi$, $\theta$, $\omega_x$, $\omega_y$, $\omega_z$, $\phi$, $\psi$, $\theta$, $\omega_x$, $\omega_y$, $\omega_z$ in the longitudinal cases and $\phi$, $\psi$, $\theta$, $\omega_x$, $\omega_y$, $\omega_z$ in the lateral cases. As shown in Table 5, the deviation of pitch rate response, $\omega_\phi$, decreases strongly as the CG shifts rearwards while $\omega_\theta$ also decreases somewhat, particularly at 160 knots. There is, at the same time, only a slight increase in variance of the normal acceleration response, $c_n$. Therefore from the inertial platform designer's standpoint, a CG location of 0.35 $c$ would be desirable. However, a more practical figure of CG at 0.30 $c$ may be a better compromise between acceptable handling qualities and inertial platform design requirements.

Of the lateral responses in Table 6, the dominant contribution of $p_0$ to the roll response, $\omega_\omega$, is due to the large value of the roll damping derivative, $C_{p\omega}$. For all other response, the $g$ gust is dominant.

All the results presented here are for the aircraft 'stick fixed' response. Reference 4 suggests that the 'stick free' response of these aircraft would not be significantly different due to control linkage stiction and friction.
6. CONCLUSION

This Memo has outlined the steps taken to obtain the longitudinal and lateral rigid-aircraft gust response of a light single-engined high-wing aircraft configuration for a few specific cases of speed, CG location, aircraft weight and altitude. Estimates of all required static and dynamic derivatives are provided so the results for any other combination of flight conditions or CG location can be readily obtained.

7. ACKNOWLEDGEMENTS

The author wishes to thank Mr. D.A. Secomb and Mr. E.S. Moody for facilitating his attachment to Aerodynamics Division and is especially grateful to Mr. R.A. Peik under whose supervision this work was performed, for his valuable guidance and patience shown to a non-aerodynamicist.
REFERENCES


TABLE 1 - TRIM CONDITIONS

In all cases, altitude, \( h = 2000 \text{ ft} \); climb angle, \( \psi_e = 0^\circ \).

<table>
<thead>
<tr>
<th>Identifier</th>
<th>( V_e ) (Knots)</th>
<th>CG Position (% ( c ))</th>
<th>( \gamma_{\text{trim}} ) (deg.)</th>
<th>( \delta_{\text{trim}} ) (deg.)</th>
<th>( T_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>120</td>
<td>15</td>
<td>1.87</td>
<td>0.90</td>
<td>0.060</td>
</tr>
<tr>
<td>Case 2</td>
<td>120</td>
<td>25</td>
<td>1.70</td>
<td>1.98</td>
<td>0.060</td>
</tr>
<tr>
<td>Case 3</td>
<td>120</td>
<td>35</td>
<td>1.56</td>
<td>2.96</td>
<td>0.060</td>
</tr>
<tr>
<td>Case 4</td>
<td>160</td>
<td>15</td>
<td>-0.09</td>
<td>2.67</td>
<td>0.060</td>
</tr>
<tr>
<td>Case 5</td>
<td>160</td>
<td>25</td>
<td>-0.11</td>
<td>2.83</td>
<td>0.060</td>
</tr>
<tr>
<td>Case 6</td>
<td>160</td>
<td>35</td>
<td>-0.15</td>
<td>3.07</td>
<td>0.060</td>
</tr>
</tbody>
</table>

TABLE 2 - BEST REGRESSION SOLUTIONS FOR \( C_L \), \( C_D \), \( C_m \)

\[
C_L = 0.1738 + 0.0930 \alpha + 0.0139 \delta_e
\]
\[
(C_D - T_c') = 0.0513 + 0.0005 \alpha^2 + 0.0010 \delta_e - 0.9436 T_c'
\]
\[
C_m = 0.0956 - 0.0354 \alpha - 0.0391 \delta_e + 0.0972 T_c'
\]
\[..\text{[CG @ 0.15 \( c \}]\]
\[
C_m = 0.0991 - 0.0159 \alpha - 0.0010 \alpha^2 - 0.0377 \delta_e + 0.0992 T_c'
\]
\[..\text{[CG @ 0.25 \( c \}]\]
\[
C_m = 0.1056 - 0.0016 \alpha^2 - 0.0363 \delta_e + 0.1013 T_c'
\]
\[..\text{[CG @ 0.35 \( c \}]\]
<table>
<thead>
<tr>
<th></th>
<th>CASE 1, 4</th>
<th>CASE 2, 5</th>
<th>CASE 3, 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{D_e} = C_{D_T} )</td>
<td>0.057</td>
<td>0.057</td>
<td>0.057</td>
</tr>
<tr>
<td>( C_{L_\alpha} ) (/rad)</td>
<td>5.330</td>
<td>5.330</td>
<td>5.330</td>
</tr>
<tr>
<td>( C_{D_\alpha} ) (/rad)</td>
<td>0.118</td>
<td>0.107</td>
<td>0.098</td>
</tr>
<tr>
<td>( C_{m_\alpha} ) (/rad)</td>
<td>-2.029</td>
<td>-1.110</td>
<td>-0.292</td>
</tr>
<tr>
<td>( C_{L_0 e} ) (/rad)</td>
<td>0.797</td>
<td>0.797</td>
<td>0.797</td>
</tr>
<tr>
<td>( C_{m_0 e} ) (/rad)</td>
<td>-2.239</td>
<td>-2.160</td>
<td>-2.080</td>
</tr>
<tr>
<td>( C_{D_0 e} ) (/rad)</td>
<td>0.059</td>
<td>0.059</td>
<td>0.059</td>
</tr>
<tr>
<td>( C_{m_T} ) (/rad)</td>
<td>0.097</td>
<td>0.099</td>
<td>0.101</td>
</tr>
<tr>
<td>( C_{T_v} ) (-)</td>
<td>-0.162</td>
<td>-0.162</td>
<td>-0.162</td>
</tr>
<tr>
<td>( C_{D_v} ) (-)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( C_{m_v} ) (-)</td>
<td>-0.009</td>
<td>-0.009</td>
<td>-0.009</td>
</tr>
<tr>
<td>( C_{L_q} ) (/rad/sec)</td>
<td>11.390</td>
<td>10.320</td>
<td>9.250</td>
</tr>
<tr>
<td>( C_{m_q} ) (/rad/sec)</td>
<td>-16.350</td>
<td>-15.710</td>
<td>-15.280</td>
</tr>
<tr>
<td>( C_{m_\alpha} ) (/rad/sec)</td>
<td>-2.150</td>
<td>-2.790</td>
<td>-3.220</td>
</tr>
<tr>
<td>( C_{L_\alpha} ) (/rad/sec)</td>
<td>0.713</td>
<td>0.926</td>
<td>1.068</td>
</tr>
</tbody>
</table>

\[ C_{L_e} = C_{m_e} = 0.36 \] for cases 1, 2, 3

\[ C_{e} = 0.20 \] for cases 4, 5, 6
TABLE 4 - LATERAL STABILITY DERIVATIVES

CG at 15% \( \bar{c} \); \( W = 2900 \) lbf; \( V_e = 120 \) knots; \( h = 2000 \) ft \( (C_{Le} = 0.36) \)
\( \gamma_e = 0^\circ \).

<table>
<thead>
<tr>
<th>Derivative</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\ell} )</td>
<td>(-0.076 / \text{rad})</td>
</tr>
<tr>
<td>( C_{n} )</td>
<td>(0.060 / \text{rad})</td>
</tr>
<tr>
<td>( C_{y} )</td>
<td>(-0.590 / \text{rad})</td>
</tr>
<tr>
<td>( C_{\ell} )</td>
<td>(-0.467 / \text{rad/\text{sec}})</td>
</tr>
<tr>
<td>( C_{n} )</td>
<td>(-0.055 / \text{rad/\text{sec}})</td>
</tr>
<tr>
<td>( C_{y} )</td>
<td>(-0.037 / \text{rad/\text{sec}})</td>
</tr>
<tr>
<td>( C_{\ell} )</td>
<td>(0.075 / \text{rad/\text{sec}})</td>
</tr>
<tr>
<td>( C_{n} )</td>
<td>(-0.102 / \text{rad/\text{sec}})</td>
</tr>
<tr>
<td>( C_{y} )</td>
<td>(0)</td>
</tr>
</tbody>
</table>

**Note:** \( C_{n} \) is the corrected value for CG at 0.15 \( \bar{c} \).
### TABLE 5 - LONGITUDINAL RESPONSE

<table>
<thead>
<tr>
<th>RUN IDENTIFIER</th>
<th>CG POSITION (% C)</th>
<th>PHUGOID MODE</th>
<th>SHORT PERIOD MODE</th>
<th>INPUT $\dot{\alpha}_{\text{qust}}$ = 10 ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Period (sec)</td>
<td>$\zeta$</td>
<td>Time to damp to $\frac{1}{4}$ ampl. (sec.)</td>
</tr>
<tr>
<td>CASE 1</td>
<td>15</td>
<td>30.3</td>
<td>0.15</td>
<td>21.6</td>
</tr>
<tr>
<td>CASE 2</td>
<td>25</td>
<td>32.5</td>
<td>0.17</td>
<td>21.1</td>
</tr>
<tr>
<td>CASE 3</td>
<td>35</td>
<td>43.7</td>
<td>0.24</td>
<td>20.1</td>
</tr>
</tbody>
</table>

| CASE 4         | 15                | 40.3          | 0.28    | 15.9            | 0.51          | 0.56    | 0.10            | 1.59  | 2.03  | 3.53  | 2.07  | 0.18  |
| CASE 5         | 25                | 43.3          | 0.30    | 15.9            | 0.65          | 0.70    | 0.10            | 2.00  | 1.97  | 2.36  | 1.90  | 0.19  |
| CASE 6         | 35                | 58.0          | 0.40    | 16.0            | $r_1=0.18$    | -       | -               | 4.01  | 1.90  | 0.98  | 1.43  | 0.22  |

$h' = 2000$ ft; $W = 2900$ lbf; $\gamma_v = 0^\circ$; Vertical Gust input $(\alpha_q)$ in all cases.
### Table 6 - Lateral Response

<table>
<thead>
<tr>
<th>Gust Input</th>
<th>Dutch Roll Mode</th>
<th>ROLLING Mode</th>
<th>Spiral Mode</th>
<th>Input $\sigma_{\text{gust}} = 10$ ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period (sec)</td>
<td>Time to damp</td>
<td>$1$ (sec.)</td>
<td>$1$ (sec.)</td>
</tr>
<tr>
<td>Lateral gust $\tilde{g}$</td>
<td>1.89</td>
<td>0.23</td>
<td>0.9</td>
<td>0.08</td>
</tr>
<tr>
<td>Gust gradient $p_{\tilde{g}}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>RESULTANT</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$W_t = 2900$ lb; $CG \approx 15\% c$; $V_e = 120$ knots; $h^* = 2000$ ft; $\gamma_e = 0^\circ$. 
FIG. 1 PROCEDURE USED FOR EVALUATING TRIM CONDITIONS.
FIG. 2 LONGITUDINAL FORCES AND MOMENT FOR STEADY SYMMETRIC FLIGHT

For equilibrium:

\[ C = L + T \sin (\alpha + \gamma) - mg \cos \theta \]
\[ C = D - T \cos (\alpha + \gamma) + mg \sin \theta \]
\[ C = M \]
FIG. 3 SPEED RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 120$ KNOTS
FIG. 4 ANGLE OF ATTACK RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 120$ KNOTS
FIG. 5  PITCH RATE RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 120$ KNOTS
FIG. 6 PITCH ATTITUDE RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 120$ KNOTS
FIG. 7 NORMAL ACCELERATION RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 120$ KNOTS
FIG. 8 AIRSPEED RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 160$ KNOTS
FIG. 9 ANGLE OF ATTACK RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 160$ KNOTS
FIG. 10 PITCH RATE RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 160$ KNOTS
FIG. 11 PITCH ATTITUDE RESPONSE TO VERTICAL GUST, \( w_g' \)
FOR \( V = 160 \) KNOTS
FIG. 12 NORMAL ACCELERATION RESPONSE TO VERTICAL GUST, $w_g$, FOR $V = 160$ KNOTS
FIG. 13 SIDESLIP ANGLE RESPONSE TO LATERAL GUST, $v_g$, FOR $V = 120$ KNOTS
FIG. 14 ROLL RATE RESPONSE TO LATERAL GUST, $v_g$, FOR $V = 120$ KNOTS
FIG. 15 YAW RATE RESPONSE TO LATERAL GUST, \(v_g\), FOR \(V = 120\) KNOTS
FIG. 16 ROLL ATTITUDE RESPONSE TO LATERAL GUST, $v_g$, FOR $V = 120$ KNOTS
FIG. 17 YAW ATTITUDE RESPONSE TO LATERAL GUST, $v_g$, FOR $V = 120$ KNOTS
FIG. 18 SIDESLIP ANGLE RESPONSE TO LATERAL GUST GRADIENT, $p_g$,
FOR $V = 120$ KNOTS
FIG. 19 ROLL RATE RESPONSE TO LATERAL GUST GRADIENT, $p_g$, FOR $V = 120$ KNOTS
FIG. 20  YAW RATE RESPONSE TO LATERAL GUST GRADIENT, $P_g'$,
FOR $V = 120$ KNOTS
FIG. 21 ROLL ATTITUDE RESPONSE TO LATERAL GUST GRADIENT, $\Phi_p$, FOR $V = 120$ KNOTS
FIG. 22 YAW ATTITUDE RESPONSE TO LATERAL GUST GRADIENT, $p_g'$, FOR $V = 120$ KNOTS
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