1.0  1.1  1.25
2.5  2.2  2.0
3.2  2.8  1.8
4.0  1.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A
DETECTION OF SOLAR GRAVITY MODE OSCILLATIONS

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DETECTION OF SOLAR GRAVITY MODE OSCILLATIONS

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ABSTRACT
An analysis of solar velocity data obtained at the Stanford Solar Observatory shows the existence of solar global oscillations in the range 45 to 105 microHz (160 to 370 minutes). These oscillations are interpreted as internal gravity modes of degree $\ell=1$ and $\ell=2$. A good estimate of the order of the modes has also been made.
Observations of the global solar velocity field have been recorded at the Stanford Solar Observatory since 1976 (1). Similar recordings from the Crimean Astrophysical Observatory are available starting from 1974 (2). A spectral analysis of the combined data showing the prominent 160.01 minute peak was recently published (3). A careful examination of the combined Stanford-Crimea spectrum has convinced us that there may be a few other peaks in the low frequency portion of the spectrum which are significantly above the noise level. For that reason, we decided to reanalyze the original Stanford data.

The Data Set and Analysis

The previous report described the observing and data reduction procedures in detail (3). The observations are differential measures of the line-of-sight velocity of the solar surface. They are made by comparing the average doppler shift from the center of the solar disk with the doppler shift from a concentric annulus. Since it appeared that interesting features were present in the low frequency part of the spectrum, we extended the computation down to \( \nu = 0 \). Since only the low frequency part of the spectrum is examined here, the data was averaged into 5-minute intervals and normalized within each day as in the previous analysis. First the Fourier transform of the 4 years of Stanford data
(1977-1980) was computed using a standard fast Fourier transform code (fft). It was found that the largest power is in the range 45 microHz – 105 microHz (about 160 to 370 minutes). The resulting power spectrum shows a number of sets of lines with separations corresponding to day side bands (i.e. 11.57 microHz) which obviously come from the nightly gaps in the data. Comparing the 4-year spectrum with individual yearly spectra, we discovered that the strongest lines were more prominent in 1979 than in the other years. This may be due to the relatively clear skies and the distribution of observing times in 1979. As a first step, we then focus our analysis on that year. Figure 1a shows the 1979 spectrum. The observations were begun on 7 April and continued through 23 July with most of the data collected in late May through July. There were a total of 240 hours of data available. Although the analysis was done using daily normalized data, the results were rescaled using the average normalizing factor to provide an approximate scale in m/s.

An estimate of the number of peaks in the spectrum that are significantly above the noise can be made by examining the cumulative distribution of the spectrum. A power spectrum computed from a normally distributed noise source will have an exponential distribution, thus the logarithm of the
Figure 1. The power spectrum of velocity observations from the Stanford Solar Observatory in 1979. The spectrum in the range 45–105 μHz (360 to 160 min.) is shown. Part (a) shows the original spectrum, part (b) shows the spectrum on the same scale after 14 peaks were identified and the associated sinusoidal waves subtracted from the data. The scale shown has been corrected for the average normalizing factor.
cumulative distribution will decrease linearly with power (e.g. (4)). A departure from a straight line is an indication of the presence of significant spectral features and the slope of the line is a measure of the variance ($\sigma^2$) in the spectrum.

Figure 2 shows this plot for the 45-105 microHz range of the 1979 spectrum. The actual data is represented by large dots. The smaller points are for a spectrum computed the same way from data constructed by taking the original data for each day in reversed order, thus keeping the original window function. Any coherent signal present in the original data will be eliminated by this procedure. The noise statistics for the modified data power spectrum should be unchanged for periods up to the average daily observing time, i.e. for frequencies higher than 40 microHz. It can be seen that there are more peaks with value above 0.20 (m/s)$^2$/freq than in the reversed data. The level at which the actual distribution begins to depart from the noise distribution is around 2.5 $\sigma$. Since the spectrum was computed with a resolution of 0.02 microHz but has a natural resolution of only 0.11 microHz, there will be about five points shown for each significant peak in the spectrum. Also, since the data has gaps at night, the day sidelobe structure introduces 2 to 4 apparently significant artificial peaks.
Figure 2. The cumulative distribution function of the spectrum in Fig. 1a. The log of the number of spectral estimates with size larger than a given value is plotted vs that value. The large dots correspond to the observed spectrum. The small dots are from a similar spectrum computed with the data within each day taken in reversed order. The small dots then refer to a spectrum with the same window and noise characteristics but no coherent oscillations. The variance which is deduced from the slope is $\sigma^2 = (20 \, \text{cm/s})^2$. 
for each true peak. These considerations suggest that the largest 10 or so peaks are most likely not noise.

Since the spectrum appears to be dominated by the sidelobe structure from the observing times, a procedure must be performed to eliminate these sidelobes. An iterative peak removal technique was used to find and remove the peaks one at a time. First an fft was computed and the largest peak determined. Next that peak was accurately found with a fine resolution simple Fourier transform in the vicinity of the peak. Finally the corresponding sinusoidal signal was subtracted from the original data. This procedure was repeated, producing a list of frequencies free of day sidelobes. The list of the first 22 peaks found in this way includes 14 peaks in the 45-105 microHz range. The list of 14 peaks is given in Table 1.

To see the effect of removing these peaks from the original data, the spectrum of the residual data after subtracting the 22 sinusoids is shown in figure 1b. Figure 1b is shown to the same scale as figure 1a. It is clear that far more than 14 peaks have been removed in the resulting spectrum. This method of peak identification appears to be a powerful tool for analysis of power spectra computed from data with complicated windows. The method finds a table of periods, phases and amplitudes that best clean the spectrum.
Table 1

Frequencies of 17 peaks found in the 45-105 microHz range of the 1979 spectrum. The frequencies are in microHz with the corresponding periods shown in minutes. The power is shown as $(m/s)^2$ per 0.11 microHz. The classification of n and $\ell$ are described in the text. The mode in parentheses is an alternate identification for the peak at 62.30 microHz and its associated side-lobes. Note that the peak at 59.52 microHz could be identified as part of an $\ell=1$ or $\ell=2$ series. The modes marked with an asterisk are also found in the spectrum from the Crimean Astrophysical Observatory(9).

<table>
<thead>
<tr>
<th>freq.</th>
<th>period</th>
<th>power</th>
<th>$\ell$</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>58.25</td>
<td>286.1</td>
<td>0.15</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>59.52</td>
<td>280.0</td>
<td>0.52</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>64.13</td>
<td>259.9</td>
<td>0.17</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>65.30</td>
<td>255.2</td>
<td>0.22</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>73.43</td>
<td>227.0</td>
<td>0.18</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>(73.84)</td>
<td>(225.7)</td>
<td>(0.98)</td>
<td>(1)</td>
<td>(8)</td>
</tr>
<tr>
<td>*83.50</td>
<td>199.6</td>
<td>0.09</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>95.70</td>
<td>174.1</td>
<td>0.23</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>*96.94</td>
<td>171.9</td>
<td>0.31</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>56.33</td>
<td>295.9</td>
<td>0.49</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>59.52</td>
<td>280.0</td>
<td>0.52</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>63.12</td>
<td>264.1</td>
<td>0.42</td>
<td>2</td>
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<tr>
<td>66.65</td>
<td>250.0</td>
<td>0.13</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>46.23</td>
<td>360.5</td>
<td>0.25</td>
<td>day/4</td>
<td>-</td>
</tr>
<tr>
<td>62.29</td>
<td>267.6</td>
<td>0.98</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>*92.21</td>
<td>180.7</td>
<td>0.10</td>
<td>day/8</td>
<td>-</td>
</tr>
</tbody>
</table>
From experiments with artificial data, we have concluded that while the correct peak is usually found, a side lobe introduced by the window is occasionally selected. This is a weakness of the procedure that requires care in examining the resulting peaks. We have also found that the amplitudes determined from data with a complicated window are not to be relied upon.

The method described above relies upon the single assumption that the examined signals consist of coherent sine waves. It is then not surprising that for each frequency removed, a complete day-sidelobes set of lines disappears from the spectrum. A further check is necessary to determine whether or not the signal is coherent for the full span of the observations. For that purpose, the data with the tabulated frequencies removed was divided into two parts, 7 April through 15 June and 16 June through 23 July. The spectra of each part separately did not show any of the removed frequencies. If any of the frequencies removed were not present in both halves of the data, they would show in the spectrum of one or both halves of the cleaned data.

Two of the fourteen peaks are apparently day aliases at 1/4 and 1/8 of a day. (With only 3 months of data the previously reported peak at 160.01 minutes can not be distinguished from 1/9 of a day and is masked by the sidelobes
of the lower day-harmonics). This leaves 12 peaks for further analysis. There is some ambiguity in the true identification of the largest of the remaining peaks. The first 1/day sidelobe is of the same amplitude as the main peak. This could be due to two true peaks with separation 1/day or 2/day. By careful examination of the peak shapes, we have identified the peak as a large peak at 62.3 microHz with a smaller peak at 96.9 microHz, although the identification could have been made as a true peak at 73.8 microHz with smaller peaks at 50.7 and 96.9 microHz. The alternate identifications are shown in parentheses in Table 1.

**Interpretation**

In order to interpret the peaks found in this spectrum, we must seek guidance from theory. Estimates for the spacing of g-mode oscillations made from the full asymptotic approximation (5) and calculated from complete solar models (6) suggest that the g-mode oscillations of the same degree \( \ell \) should be about equally spaced in period for order large enough \( n > 6 \). Therefore we checked the list of prominent peaks for equal spacing in period. Three of the four largest peaks are separated in period by 15.5 minutes. This is very close to the spacing of the standard solar model for g-modes with degree two and order greater than 10.
To aid in further identification of the modes we can use the asymptotic approximation (5) which shows that for sufficiently large order $n$ the period $T$ is:

$$T = T_0 \left( n + \frac{\ell}{2} - \frac{1}{4} \right) \sqrt[4]{\ell(\ell + 1)}$$

If we assume that the three peaks with 15.5 minute separation are part of an $\ell=2$ series, we can calculate $T_0$ and then the probable spacing for the $\ell=1$ and $\ell=3$ series. Doing this we see that most of the remaining peaks are likely to be part of the $\ell=1$ series. If we have an estimate for $\ell$, we can plot the observed periods as $T \sqrt[4]{\ell(\ell+1)}$ vs $(k + \ell/2 - 1/4)$ where $k$ is an integer increasing with period. If the peaks are consistent with the model and we have assigned the correct degree $\ell$, the points will all lie on a straight line. From the intercept of this line we can determine both the best value for $n$ and check the $-1/4$ term. This has been done in Figure 3.

The three largest peaks were used to define the $\ell=2$ series and thus to find $T_0$. The filled circles in Figure 3 represent the three largest peaks which determine the $\ell=2$ series. The small dots represent all the other modes identified in table 1. The line was found from the three large $\ell=2$ peaks only. It can be seen that the other peaks are organized in the expected g-mode structure. Note that in several cases two peaks have been assigned to one order $n$ in
Figure 3. The observed modes are plotted within the structure of the asymptotic formula described in the text. The line is determined from the three largest peaks that have been assigned.
the $\ell=1$ series. These peaks have an average separation of 1.22 microHz and could be evidence of rotationally split modes. The $T_0$ implied by the identified modes is $38.6 \pm 0.5$ minutes. The intercept of the best fit line with the abscissa is expected to be 0.25 and is found to be $0.2 \pm 0.1$ thus the order $n$ is most likely correctly determined.

Now we must ask if the Stanford instrument is sensitive to degree $\ell=2$ $g$-modes oscillations. For the previously reported acoustic mode oscillations in the 5-minute range, the Stanford velocity differencing scheme is most sensitive to modes of degree $\ell$. For low frequency $g$-modes, Gough and Cristensen-Dalsgaard (8) have shown that for periods around 160 minutes the Stanford instrument is most sensitive to modes with degree $\ell=5$ to $\ell=7$. For longer periods however, the instrument begins to be relatively more sensitive to modes with lower degree. This variation with period, unlike the case for the $p$-modes, is due to the importance of the horizontal motions induced by the internal gravity modes.

The largest peak at 62.29 microHz appears not to be consistent with the asymptotic formula with the $T_0$ we have found. If the alternate choice of the 62.20 microHz-73.84 microHz pair were chosen, the peak at 73.84 microHz which would be identified as $\ell=1$, $n=8$. In this case a peak at 50.7 microHz shows up in the peak finding procedure and
would be identified as $\ell=2$, $n=20$. We also note that the frequency of this peak is almost identical with the frequency difference between 5-minute modes with order and degree $(n,\ell=1)$, $(n,\ell=0)$ and close to the difference between $(n,\ell=2)$, $(n,\ell=1)$. This beat frequency could show up in the Stanford observations through a non-linear coupling in the Sun. We do not, however, find any peak near 136 microHz which would correspond to beats between acoustic modes of the same degree but with order differing by one.

The relative peak sizes and simplicity of the analysis leads us to the present mode identification. However, alternative identifications are quite possible, but they can only be tested with more data from other years or from other observatories.
Acknowledgements.

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