COMPOSITE LAMINATE WEIGHT OPTIMIZATION ON THE TIMEX-SINCLAIR 1000 MICROCOMPUTER

GERALD V. FLANAGAN
MECHANICS AND SURFACE INTERACTIONS BRANCH
NONMETALLIC MATERIALS DIVISION

FEBRUARY 1983

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This technical report has been reviewed and is approved for publication.

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Nonmetallic Materials Division

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Nonmetallic Materials Division

FOR THE COMMANDER

FRANKLIN D. CHERRY, Chief
Nonmetallic Materials Division

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In this report, an automated composite laminate sizing technique is presented, which optimizes for minimum weight. The technique can be coded for a microcomputer and a listing is given for the Timex-Sinclair 1000. The program is interactive and easy to use. Ply ratios are optimized for point stress under multiple independent loads.

This program is available on cassette tape and can be obtained by sending a blank 15 or 30-minute tape to AFWAL/MLBM, Wright-Patterson AFB, Ohio 45433 and referencing this report.
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SECTION I

PROGRAM DESCRIPTION

CLASS (Composite Laminate Automated Sizing for Strength) is an interactive optimization program designed to run on a small microcomputer. The listing presented here is for a Sinclair ZX81 or a Timex-Sinclair 1000 microcomputer with a 16K memory expansion. The version of Basic is standard enough that translations to other microcomputers is possible.

The program will find a minimum thickness laminate which will not fail under any of the load conditions entered. Ply orientations are chosen by the user. The program's capability in handling multiple, independent, loads could be useful for loads which change with time or for situations where there is uncertainty in calculating the loads. As the program is currently dimensioned, four independent load combinations and 18-ply orientations can be entered.

Only point stresses are considered, thus the program optimizes the laminate only at one point in the structure. Furthermore, the program assumes in-plane loads only and no out-of-plane deflections. This implies a symmetric laminate, but stacking sequence is not a factor in the program. The layer thicknesses generated by the program are the total and must be divided by 2 to get the halves of a symmetric laminate.

No knowledge of optimization techniques is needed to run the program and very little knowledge of laminate plate theory is needed. In addition, material properties for five common advanced composites are stored in the program, or the computer can ask for new properties through prompts.
SECTION II

GENERAL INSTRUCTIONS

The Timex 1000 or Sinclair ZX81 manual includes tape loading instructions. Because CLASS takes so long to read (approximately 7 minutes), it's a good idea to test tape player volume level with a short one or two line program to see if all is well. Load the tape using "CLASS" as the name. If the tape loads properly, it will automatically begin execution. An example of the video prompts and appropriate responses are given in this report. As a number is entered, it appears at the bottom of the screen. The number can be changed using the delete key (shifted zero). Once ENTER is pressed, there is no way to change entries until the end of the input sequence when the program asks for corrections. If there are mistakes, answering "Y" to this query will restart the sequence. A subroutine is included for making a hardcopy of input and results if a printer is available. At the completion of the routine, after all results have been displayed, the program will restart itself.

Run times can be quite long. They range from a minute for a 2-layer laminate, to an hour for an 18-layer laminate subject to multiple loads.

Much of the information included in this report is intended for those who wish to understand and modify the program and is not needed to run it.
1) T300/5208
2) BORON/5505
3) AS/3501
4) SCOTCHPLY 1002
5) KEVLAR 49/EPOXY
6) AVAILABLE
7) NEW

SELECT MATERIAL
HOW MANY PLY ANGLES
ENTER PLY ORIENTATION (DEGREES)
PLY 1 = 
PLY 2 = 
ENTER NUMBER OF INDEPENDENT LOADING CONDITIONS
ENGLISH OR SI UNITS (E/S)
LOADING CONDITION 1
N1 = 
N2 = 
N6 = 

ENTER ENGINEERING CONSTANTS IN GPA
EX = ?
EY = ?
VX = ?
ES = ?

(Tapes will automatically start after loading)
(6 is an available slot for a user defined material. Entering 7 allows new properties to be placed in the slot. These properties can be used in subsequent runs by entering 6)
7 ENTER
2 ENTER
0 ENTER
90 ENTER
1 ENTER
E ENTER
4000 ENTER
1E3 ENTER
° ENTER
(either scientific or explicit notation may be used to enter numbers)
181 ENTER
10.3 ENTER
.28 ENTER
7.17 ENTER
ENTER STRENGTHS IN MPA.
X (TENS.) = ?
X (COMP.) = ?
Y (TENS.) = ?
Y (COMP.) = ?
SHEAR = ?
ENTER MATERIAL NAME (< 15 CHARC)

CORRECTIONS (Y/N)

(blank screen for 100 seconds)
TOTAL LAMINATE THICKNESS
.029706293 IN
2 ACTIVE CONSTRAINTS AFTER 2 ITERATIONS
PRESS ANY KEY TO CONTINUE

ANGLE  RATIO  NO. PLIES
0 0.7542  4.55
90 0.2458  1.48
PRESS ANY KEY TO CONTINUE

STRENGTH RATIOS
1 = ULTIMATE STRAIN: > 1 = SAFE
PLY   LOAD1
0 1
90 1.03
Press any key to continue

HARDCOPY (Y/N)?

BAR GRAPH (Y/N)?

B

(useful if printer is attached)

N ENTER

(useful if many ply angles are used. Shows relative thickness of layers)

N ENTER

(program automatically restarts at beginning. Note that user defined material is now number 6 and can be used without re-entering the properties)

1) T300/5208
2) BORON/5505
3) AS/3501
4) SCOTCHPLY 1002
5) KEVLAR 49/EPOXY
6) T300/EPOXY(2)
7) NEW

SELECT MATERIAL
SECTION III

METHOD

The goal is to minimize the total thickness of a composite laminate subject to failure constraints under static loads. Specifically

\[
\sum_{k=1}^{L} h_k = \min.
\]

subject to \( h_k \geq 0 \)

and

\[
G_{ij}^{(0)} e_i e_j + G_i^{(0)} e_i - 1 \leq 0
\]

where \( h_k \) is the total thickness of all the plies at the kth orientation (which will be referred to as a "layer" in this report). The failure criteria is a first ply failure based on the Tsai-Wu tensor criteria in strain space. The G's are transformed to the laminate axis from the k'th layer's orientation. The strains are associated with the N'th loading combination. This distinction is made since more than one independent loading may be considered. For the definition of the G's in terms of experimental strength data, see reference 1.

Stacking sequence is not included in this formulation, and the laminate is assumed not to bend or warp. Therefore, strains and loads are related by

\[
\hat{N} = |A| \hat{E}
\]

The optimization method applied is a modification of the method of feasible directions. The method can be demonstrated graphically with 2-dimensions, i.e. two layers. In Figure 1 the two equalities

\[
G_{ij}^{(0)} e_i e_j + G_i^{(0)} e_i - 1 = 0
\]

\[
G_{ij}^{(90)} e_i e_j + G_i^{(90)} e_i - 1 = 0
\]
have been plotted as functions of $h^{[0]}$ and $h^{[90]}$ for the single loading condition shown. Any point above and to the right of these two curves is feasible, that is, failure will not occur. Points to the left and below the curves are infeasible. Because our objective function (the sum of the layer thicknesses) is linear, the optimum point will lie on one of these curves or the intersection of multiple curves.

The program starts by finding an initial feasible point (A) which lies on a constraint curve farthest from the origin on the line $h^{[90]}=h^{[0]}$. The distance from the origin is calculated using a strain ratio method. Along any vector which passes through the origin

$$h_{k}^{i+1} = h_{k}^{i} \cdot S/S_{0}$$

where $S$ is a scalar distance and

$$S_{0} = \left[ \sum_{k=1}^{k} (h_{k}^{i})^2 \right]^{1/2}$$

along this vector, strain can be found using

$$\varepsilon_{i} = \frac{\varepsilon_{i}^0 \cdot S_{0}}{S}$$

where $\varepsilon_{i}^0$ is a component of laminate strain evaluated at $S_{0}$. Substituting into the failure criteria we have

$$\frac{G_{ij}^{(c)}}{\varepsilon_{ij}^0} \cdot \frac{\varepsilon_{ij}^0 \cdot S_{0}^2}{S^2} + \frac{G_{ij}^{(c)}}{S_{0}} \cdot \frac{\varepsilon_{ij}^0 \cdot S_{0}}{S} - 1 = 0$$

To ensure the calculated point lies slightly in the feasible region despite any numerical error, the program sets this function equal to the negative of a small number ($E_1$) rather than zero. Solving this equation for positive $S$ we have

$$S = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$
where
\[
A = 1 - E_1
\]
\[
B = \sum_{i=1}^{3} - G_{ij}(\Theta k) e_i e_j S_0
\]
\[
C = \sum_{i=1}^{3} \sum_{j=1}^{3} - G_{ij}(\Theta k) e_i e_j S_0^2
\]

If \( S_0 \) lies in the feasible region we solve the above equation for each layer and each load combination then take the smallest resulting \( S \) as the one that defines the boundary of the feasible region.

The next step in the optimization procedure is to establish a direction vector which will point away from the constraint \( A \) lies on and is parallel to the plane defined by \( \Xi h_k = \text{constant} \). In Figure 1, this direction is shown as \( Z \). Finding \( Z \) first requires calculation of the gradient of the active constraint evaluated at \( A \). Let
\[
C_{k,N} = G_{ij}(\Theta k)(N)(N) + G_{ij}(\Theta k)(N) - 1
\]
where \( k \) and \( N \) correspond to the layer and load combination of the active constraint. A constraint is considered active if
\[
C_{k,N} \geq -E_2
\]
where \( E_2 \) is a small number. Note that more than one constraint may be active. The gradient is then given by
\[
\nabla C_{k,N} = \sum_{L=1}^{L} G_{ij}(\Theta k) \left( \frac{\partial e_i}{\partial h_L} e_j + e_i \left( \frac{\partial e_j}{\partial h_L} \right) + G_{ij}(\Theta k) \frac{\partial e_i}{\partial h_L} \right) h_L
\]
where \( h_L \) is a unit vector. To find the partials of strain we start with the basic equation
\[
\nabla N = |A| \nabla e
\]
0 = \frac{\partial}{\partial h_i} |A| \vec{e} + A \frac{\partial}{\partial h_i} \vec{e}

\frac{\partial \vec{e}}{\partial h_i} = -|A^{-1}| \frac{\partial}{\partial h_i} |A| \vec{e}

and

\frac{\partial}{\partial h_i} |A| = \begin{bmatrix} Q_{11}^{(\theta_i)} & Q_{12}^{(\theta_i)} & Q_{13}^{(\theta_i)} \\ Q_{21}^{(\theta_i)} & Q_{22}^{(\theta_i)} & Q_{23}^{(\theta_i)} \\ Q_{31}^{(\theta_i)} & Q_{32}^{(\theta_i)} & Q_{33}^{(\theta_i)} \end{bmatrix} = [Q_i^{(\theta_i)}]

The gradient vector is normalized to unit length. If more than one constraint is active, the normalized gradients are summed together and the sum is then normalized to one. The negative of the gradient will point away from the constraint, into the feasible region. This vector is now projected onto the plane defined by the unit normal \( \hat{n} \), where

\[ \hat{n} = \frac{1}{L} \sum_{i=1}^{L} \hat{h}_i \]

The projection can be made with a double cross product

\[ \hat{Z} = \hat{n} \times (-\vec{c} \times \hat{n}) \]

With a vector identity, this can be rewritten as

\[ \hat{Z} = (\vec{c} \cdot \hat{n}) \hat{n} - \vec{c} \]

Finally, \( \hat{Z} \) is also normalized to unit length.

Along \( \hat{Z} \), another constraint will eventually be reached (point B in Figure 1). The point is found iteratively by a bisection technique. Since
the bisection method is very time consuming, the constraint line is only found within a relatively large error band. What we are really interested in is a point approximately midway between A and B, which is C in the figure. From point C, the strain ratio technique is used to analytically calculate D. Starting at D, the entire procedure repeats. The program terminates when the distance AB or CD is small (say 1/10 a ply thickness) or the magnitude of $\dot{Z}$ before normalization is very small (implying $\hat{n}$ and $\hat{c}$ are almost parallel).

In some cases, $h_k > 0$ constraint may be reached. When this happens, that orientation is completely dropped from further calculations. Thus, the constraints associated with a zero thickness layer cannot effect the results. Once an orientation reaches zero thickness, it is never reinstated in later iterations.

Figure 1 shows a case where the program reaches the intersection of two constraints. However, simultaneous failure should not be considered a criteria for optimization. Figure 2 shows a case where only one layer approaches failure. The constraint line for the $+45^\circ$ layer is completely in the infeasible region. The line $h^{[45]} + h^{[-45]} = \text{const.}$ has been included to show that point D is the minimum thickness.
References


APPENDIX A

REPRESENTATIVE RESULTS
**Comments**

```
***MATERIAL T300/5200***
TOTAL THICKNESS .006666667 M
AFTER 3 ITERATIONS

N1 = 1000000 N/M
N2 = 0 N/M
N6 = 0 N/M

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<tr>
<th>ANGLE</th>
<th>RATIO</th>
<th>NO. PLIES</th>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-45</td>
<td>0</td>
<td>0</td>
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STRENGTH RATIOS
1=FAILURE; >1=SAFE
PLY LOAD1 LOAD2 LOAD3 LOAD4
0  1.242 1.619 2.398 1.272
45 2.132 1.497 1.263 1.013
-45 1.819 1.474 1.261 2.467
90 1.397 1.294 1.013 1.27

Demonstrates program’s capability to eliminate unnecessary layers.
```

```
***MATERIAL T300/5200***
TOTAL THICKNESS .0076394812 M
AFTER 4 ITERATIONS

N1 = 1000000 N/M
N2 = 1000000 N/M
N6 = 0 N/M

N1 = 2000000 N/M
N2 = 1000000 N/M
N6 = 0 N/M

N1 = 2000000 N/M
N2 = -1000000 N/M
N6 = 0 N/M

N1 = 0 N/M
N2 = 0 N/M
N6 = -1000000 N/M

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<tr>
<td>0</td>
<td>0.3072</td>
<td>18.77</td>
</tr>
<tr>
<td>45</td>
<td>0.2335</td>
<td>14.27</td>
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<tr>
<td>-45</td>
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<td>90</td>
<td>0.2122</td>
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STRENGTH RATIOS
1=ULTIMATE STRAIN; >1=SAFE
PLY LOAD1 LOAD2 LOAD3 LOAD4
0  1.242 1.819 2.398 1.272
45 2.132 1.497 1.263 1.013
-45 1.819 1.474 1.261 2.467
90 1.397 1.294 1.013 1.27
```

Multiple load capability
Comments

*************************
TOTAL THICKNESS .0055896089 M
AFTER 3 ITERATIONS

N1 = 2000000 N/M
N2 = 1000000 N/M
N6 = 0 N/M

N1 = 1750000 N/M
N2 = 1250000 N/M
N6 = 433012.7 N/M

ANGLE RATIO NO. PLIES
0 0.3546 15.06
90 0.1563 7.08
45 0.3664 13.7
-45 0.1687 8.08

STRENGTH RATIO
1=ULTIMATE STRAIN: >1=SFE
PLY LOAD1 LOAD2
0 1.261 1.123
90 1.001 1.118
45 1.034 1.307
-45 1.2 1

Note that this π/4 laminate and the π/6 laminate on the next page give the same total thickness.

Second load has the same magnitude as the first only with the principle axis rotated 30°.

Bar Graph. Vertical scale is the ply ratio; horizontal is orientation angle.
**Comments**

* SAME LOADS AS PREVIOUS EXAMPLE. 

**MATERIAL T300/5208**
**TOTAL THICKNESS .0055897041 M**
**AFTER 2 ITERATIONS**

- **N1 = 2000000 N/M**
- **N2 = 1000000 N/M**
- **N6 = 0 N/M**
- **N1 = 1750000 N/M**
- **N2 = 1250000 N/M**
- **N6 = 433812.7 N/M**

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<td>90</td>
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<td>4.48</td>
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**STRENGTH RATIOS**
1 = ULTIMATE STRAIN: >1 = SAFE

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<tr>
<th>PLY</th>
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<tr>
<td>-60</td>
<td>1.125</td>
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<td>1.127</td>
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![Bar Chart](chart.png)
**18 layer laminate**

Total Thickness 0.11247529 m

After 10 iterations

N1 = 40000000 N/m
N2 = 10000000 N/m
N6 = 0 N/m

N1 = 12500000 N/m
N2 = 32500000 N/m
N6 = 12990000 N/m

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>RATIO</th>
<th>NO. PLIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>-80</td>
<td>0.009</td>
<td>6.13</td>
</tr>
<tr>
<td>-70</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-60</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-50</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-40</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-30</td>
<td>.0537</td>
<td>48.35</td>
</tr>
<tr>
<td>-20</td>
<td>.099</td>
<td>89.1</td>
</tr>
<tr>
<td>-10</td>
<td>0.1176</td>
<td>105.82</td>
</tr>
<tr>
<td>0</td>
<td>0.1059</td>
<td>95.3</td>
</tr>
<tr>
<td>10</td>
<td>.0742</td>
<td>66.31</td>
</tr>
<tr>
<td>20</td>
<td>.0414</td>
<td>37.22</td>
</tr>
<tr>
<td>30</td>
<td>.0252</td>
<td>22.68</td>
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<tr>
<td>40</td>
<td>.0342</td>
<td>30.8</td>
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<tr>
<td>50</td>
<td>.0639</td>
<td>55.98</td>
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<tr>
<td>60</td>
<td>.0963</td>
<td>86.66</td>
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<tr>
<td>70</td>
<td>0.1138</td>
<td>102.41</td>
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<tr>
<td>80</td>
<td>0.1028</td>
<td>92.52</td>
</tr>
<tr>
<td>90</td>
<td>.0634</td>
<td>57</td>
</tr>
</tbody>
</table>

**Strength Ratios**

1 = Ultimate Strain; >1 = Safe

<table>
<thead>
<tr>
<th>PLY</th>
<th>LOAD1</th>
<th>LOAD2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-80</td>
<td>1.023</td>
<td>1.389</td>
</tr>
</tbody>
</table>

-30 1.449 1.004
-20 1.574 1.015
-10 1.652 1.015
 0 1.649 1.05
 10 1.566 1.11
 20 1.44 1.201
 30 1.311 1.326
 40 1.201 1.481
 50 1.117 1.648
 60 1.055 1.775
 70 1.021 1.801
 80 1.004 1.711
 90 1.005 1.554

0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
Peaks at -10° and 70°.
Comments

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>RATIO</th>
<th>NO. PLIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>2.5215</td>
<td>46.05</td>
</tr>
<tr>
<td>70</td>
<td>0.4786</td>
<td>42.25</td>
</tr>
</tbody>
</table>

Same loads as 18 layer example.

Selecting "peak" orientations results in a laminate with about the same total thickness as the 18 layer case.

<table>
<thead>
<tr>
<th>ANGLE</th>
<th>RATIO</th>
<th>NO. PLIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2911</td>
<td>53.08</td>
</tr>
<tr>
<td>90</td>
<td>0.7009</td>
<td>129.25</td>
</tr>
</tbody>
</table>

With only two orientations, angle sensitivity is important. Here a small change to [0/90] has resulted in a laminate twice as thick as for [-10/70].
APPENDIX B

SUBROUTINE DESCRIPTIONS
500-760 CONSTRAINT TEST
Test each possible failure constraint. If a constraint is violated, set G$ = "FAIL" and return. If no constraints are violated, set G$ = "PASS", make a list of active constraints, and set NC (no constraints).

1000-1270 GRADIENT
Find the gradient of the constraint identified by ply P and load N. Normalize the gradient to unit length.

1500-1760 STRAINS
Given a value of S find $|A| = |A_h| + |A_z| \times s$. Invert $|A|$ and calculate laminate strains for each independent loading.

2000-2170 FORM $A_h$ AND $A_z$
\[
A_{h,ij} = \sum_{k=1}^{L} Q_{ij}^k h_k; A_{z,ij} = \sum_{k=1}^{L} Q_{ij}^k z_k
\]
Note that at a distance $S$ along $Z$, $|A| = |A_h| + |A_z| \times s$

2200-2290 FORM Q
Convert C array to 3x3 matrix for ply II.

2400-2498 FORM G
Convert B array to 3x3 matrix for ply II. Also form vector s which contains linear components of failure parameters.

2500-2800 NEW H
Two step procedure. First find approximate distance along vector $Z$ to next constraint. At a point half that distance, use the strain ratio
routine to decrease the total thickness with constant ply ratios until a constraint is found.

3000-3350 NEW DIRECTION
Find a feasible direction vector which is parallel to the iso-thickness plane and leads away from the active constraints.

3500-3650 STRAIN RATIO
Along a vector pointing to origin, find a scalar distance from current location in h space to the nearest constraint.

4100-4400 INITIAL FEASIBLE POINT
Using an assumption of equal thickness plies, find the first point where no constraints are violated. Initialize $A_h$, $A_z$, $h$, strains, and constraint list.

4500-4780 TRANSFORMATIONS
For each ply orientation, transform $Q$ and $G$. Store the results in $B$ and $C$.

5000-5495 INPUT
Prompt user for material, angles and loads.

5500-5658 OUTPUT
Video display of results.

5660-5820 STRENGTH RATIO
Used by OUTPUT and HARDCOPY to print out a list of strain ratios for each loading.
5870-5950 PLY RATIO
Used by OUTPUT and HARDCOPY to print out a list of ply ratios and number of plies.

6500-6670 HARDCOPY
If printer attached, makes a printout of results.

7000-7200 NEW MATERIAL
Prompts user for new material properties.

7500-7740 HISTOGRAM
Generates a bar graph.

8000-8760 INVARIANTS
Given engineering constants and strengths, form invariants.
APPENDIX C

VARIABLE LIST
Arrays
A (3,3) - a matrix for current value of thickness vector
B (18,9) - contains transformed G's (strength parameters in strain space) for each ply in the sequence G_{11}, G_{22}, G_{12}, G_{66}, G_{16}, G_{26}, G_1, G_2, G_3. First subscript is ply number, second is G element.
C (18,6) - contains transformed Q's (modulus components) for each ply in the sequence Q_{11}, Q_{22}, Q_{12}, Q_{66}, Q_{16}, Q_{26}. First subscript is ply number, second is Q element.
D (3,3) - \sum Q_{ij}^k \bar{Z}_k where \bar{Z} is the direction vector
E (4,3) - strains corresponding to each independent loading. First subscript identifies load, second is strain component. (c_1, c_2, c_6)
G (3,3) - strength parameter matrix for a given ply orientation
N (4,3) - loads. First subscript identifies independent loading, second is load element (N_1, N_2, N_6)
P (3,3) - A inverse
Q (3,3) - modulus matrix for a given ply orientation

Vectors
H (18) - thickness for each ply
R (3) - intermediate results
S (3) - linear strength parameter components for a given ply orientation
T (18) - angle of each ply in radians
U (5) - modulus invariants
V (7) - strength invariants (strain space)
W (18) - normalized gradient of a constraint
X (18) - normalized sum of gradients from all active constraints
Y (3) - intermediate results
Z (18) - direction vector
Scalars

E2 - defines minimum move along direction vector before terminating program

E5 - defines an active constraint (if \( G_{ij} c_i c_j + G_i c_i - 1 > -E5 \) then constraint active)

E6 - small factor included in strain ratio routine to guarantee the point found is slightly in the feasible region, despite numerical error

C2 - \( \cos 2\theta \)

C4 - \( \cos 4\theta \)

S2 - \( \sin 2\theta \)

S4 - \( \sin 4\theta \)

S - final scalar distance to be moved

S1 - point in feasible region in bisection routine

S2 - point in infeasible region in bisection routine

SREF - distance to origin used by strain ratio routine

SMAX - distance along direction vector to first \( h = 0 \) constraint

SI1 - units conversion \( \text{lb/in} \rightarrow \text{N/m} \)

SI2 - units conversion \( \text{m} \rightarrow \text{in} \)

NPLY - number of ply orientations

NL - number of independent loadings

NC - number of active constraints

ITER - iteration counter

IMAX - iteration limit

M - identifies material

EX, EY, VX, ES - engineering constants (reused as strength parameters in stress space)

XT, XC, YT, YC, SS - strengths

QXX, QYY, Qxy, Qs - modulus

GXX, GYY, GXY, GSS, GX, GY - strength parameters in strain space
TPLY - thickness of an individual ply
I, J, K, L - loop counters
P, II - ply orientation pointers
N - load pointer
A, B, C, CON, DET, E, NORM, TEST - intermediate calculations
Z - √NPLY* where NPLY* is number of ply orientations for which h_i ≠ 0

Strings
C% (10,2) - list of active constraints, identified by ply and loading
P% (6,19) - material engineering constants
Y% (6,24) - material strengths
M% (6,15) - material names
R% (4,2) - engineering constant labels "EX", "EY", "VX", "G"
S% (5,8) - strength labels "X (Tens.)", "X (Comp.)", etc.
F% - flag to halt program when = "Fail"
G% - flag returned from constraint test routine = "Fail" if a constraint is violated
K% - "Press any key to continue"
U% - units label for load
V% - units label for thickness
E% - when = "E" English units are desired
A%, H% - query responses

Constants entered from keyboard

If the program is loaded from tape, it will be ready to run. If the program is keyed into the computer, certain constants and dimension statements have to be entered before running the program. These have not been defined in the program in order to save memory. When the program is
SAVEd on tape, the constants and dimensions will also be saved. Once the constants have been entered, do not use the RUN command as this will erase all the data.

Dimension Statements

Dim A (3,3)        Dim H (NPLY)
Dim B (NPLY,9)     Dim R (3)
Dim C (NPLY,6)     Dim S (3)
Dim D (3,3)        Dim T (NPLY)
Dim E (NL,3)       Dim U (5)
Dim G (3,3)        Dim V (7)
Dim N (NL,3)       Dim W (NPLY)
Dim P (3,3)        Dim X (NPLY)
Dim Q (3,3)        Dim Y (3)
Dim C$ (10,2)      Dim Z (NPLY)
Dim P$ (6,19)      Dim M$ (6,15)
Dim Y$ (6,24)      Dim R$ (4,2)
Dim S$ (5,8)

Where NPLY is the number of ply orientations allowed and NL is the number of independent loads. NPLY = 18 and NL = 4 will use all available memory in the ZX81.

CONSTANTS

M9(1)="T300/5208"
P9(1)="181.,10.3,0.28,717"
Y9(1)="1500,1500,40.0,246.,68.0"
M9(2)="BORON15505"
P9(2)="204.,18.5,0.23,5.59"
Y9(2)="1260,2500,61.0,202.,67.0"
M9(3)="AS/3501"
P9(3)="138.,8.96,0.30,7.10"
Y9(3)="1447,1447,51.7,206.,93.0"
M$(4) = "Scotchply/1002"
P$(4) = "38.6, 8.27, 0.26, 4.14"
Y$(4) = "1062, 610, 31.0, 118, .72.0"
M$(5) = "Kevlar 49/Epoxy"
P$(5) = "76.0, 5.50, 0.34, 2.30"
Y$(5) = "1400, 235, 12.0, 53.0, 34.0"

Let S11 = 175.1567
Let S12 = 39.37008
Let R%(1) = "EX"
Let R%(2) = "EY"
Let R%(3) = "VX"
Let R%(4) = "S"
Let S%(1) = "X(TENS.)"
Let S%(2) = "X(COMP.)"
Let S%(3) = "Y(TENS.)"
Let S%(4) = "Y(COMP.)"
Let S%(5) = "SHEAR"
Let K$ = "PRESS ANY KEY TO CONTINUE"
Let E2 = 1E-5
Let E5 = 1E-1
Let E6 = 1E-6
Let TPLY = 1.25E-4
Let IMAX = 10
APPENDIX D

NOTES ON SINCLAIR BASIC
The version of BASIC used on the ZX81 should be easily translatable to other machines. There are some nonstandard features however, which may require explanation.

SLOW, FAST - The ZX81 uses these commands to control whether video display is continuous or goes blank during computations. They can be ignored for other machines.

A$(a to b) - TO is the string slicing command and replaces the standard LEFT$, MID$ and RIGHT$. Note that string slicing is used to define material properties. This is used since the ZX81 lacks READ, DATA and RESTORE.

LPRINT - Sends string to printer.

COPY - Sends entire video display to printer

PAUSE 40000 - An indefinite pause, broken by pressing any key

Displays in the program are designed for a screen that has 21 lines with 32 characters.
MAIN

110 LET ITER=1
130 GOSUB 5000
132 PRINT "CORRECTIONS ? (Y/N)"
134 INPUT A$
136 IF A$="Y" THEN GOTO 130
140 IF H=7 THEN GOSUB 7000
150 GOSUB 8000
160 GOSUB 4500
140 GOSUB 3000
160 IF F$="FAIL" THEN GOTO 5500
190 GOSUB 2500
300 IF F$="FAIL" THEN GOTO 5500
302 LET ITER=ITER+1
306 IF ITER>IMAX THEN GOTO 5500
320 GOTO 250

505 LET G$="PASS"
510 LET NC=0
520 FOR P=1 TO NPLY
525 IF H(P)=0 THEN GOTO 750
526 LET II=P
527 GOSUB 2400
530 FOR N=1 TO NL
535 LET CON=-1
540 FOR K=1 TO 3
545 FOR J=1 TO 3
550 LET CON=CON+G(K,J)*E(N,J)*E(N,K)
555 NEXT J
560 NEXT N
570 IF CON=0 THEN GOTO 750
575 NEXT K
580 IF CON<0 THEN LET G$="FAIL"
590 IF CON>-E5 THEN GOTO 700
600 LET NC=NC+1
610 LET C$(NC,:)=CHR$ P
620 LET C$(NC,2)=CHR$ N
700 IF G$="FAIL" THEN RETURN
740 NEXT N
750 NEXT P
760 RETURN

CONSTRANT TEST

525 - If ply thickness zero, ignore constraints associated with it
526-527 - Set up G matrix for ply being tested
560-610 - Solve con = G_ij * e_j

640-670 - If con is close to zero, identify constraint as active. C$ form a list of constraints in terms of ply and load
1010 LET NORM = 0
1012 LET II = P
1014 GOSUB 2400
1020 FOR L = 1 TO NPLY
1025 IF H(L) = 0 THEN GOTO 1200
1026 LET II = 0
1027 GOSUB 2200
1030 FOR J = 1 TO 3
1035 LET R(J) = 0
1040 FOR K = 1 TO 3
1045 LET R(J) = R(J) - 0(J,K) * E(N,K)
1050 NEXT K
1055 NEXT J
1100 FOR J = 1 TO 3
1105 FOR K = 1 TO 3
1110 LET Y(J) = Y(J) + R(J,K) * R(K)
1115 NEXT K
1120 NEXT J
1130 NEXT L
1140 FOR L = 1 TO NPLY
1150 LET U(L) = W(L) / NORM
1160 NEXT L
1170 RETURN

1510 FOR I = 1 TO 3
1520 FOR J = 1 TO 3
1530 LET E(I,J) = A(I,J) + D(I,J) * S
1540 NEXT J
1550 NEXT I
1560 LET DET = E(1,1) * E(2,2) * E(3,3) - E(1,2) * E(1,3) * E(2,3) - E(1,1) * E(2,3) * E(3,2) + E(1,3) * E(2,1) * E(3,2) - E(1,2) * E(1,3) * E(2,2)
1570 LET P(1,1) = (E(2,2) * E(3,3) - E(2,3)) / DET
1580 LET P(1,2) = (E(1,2) * E(3,3) - E(1,3)) / DET
1590 LET P(1,3) = (E(1,3) * E(2,2) - E(1,2)) / DET
1600 LET P(2,1) = (E(2,1) * E(3,3) - E(2,3)) / DET
1610 LET P(2,2) = (E(1,1) * E(3,3) - E(1,3)) / DET
1620 LET P(2,3) = (E(1,3) * E(2,1) - E(1,1)) / DET
1630 LET P(3,1) = (E(2,1) * E(1,3) - E(2,3)) / DET
1640 LET P(3,2) = (E(1,2) * E(1,3) - E(1,1)) / DET
1650 LET P(3,3) = (E(1,1) * E(2,2) - E(1,2)) / DET
1660 FOR I = 1 TO N
1670 FOR J = 1 TO 3
1680 LET E(I,J) = 0
1690 FOR K = 1 TO 3
1700 LET E(I,J) = E(I,J) + P(J,K) * N(I,K)
1710 NEXT K
1720 NEXT J
1730 RETURN

1012-1014 - Form G matrix for designated ply
1026-1027 - For each ply, form Q matrix
1030-1070 - \( \hat{R} = -\frac{2}{\Delta h} |A| \hat{e} \)
1080-1130 - \( \hat{Y} = |A^{-1}| \hat{R} \)
1150-1200 - \( \hat{v}(CON) = [G_{ij} (c_i \frac{\partial e_j}{\partial h_k}) \hat{h}_k + \frac{\partial e_i}{\partial h_k} \hat{e}_j] \hat{h}_k \)
1210-1260 - Normalize \( \hat{v}(CON) \)
1510-1550 - |E| is temporarily the A matrix at a point along the Z vector, a distance S from the current position in h space
1570-1660 - Invert A
1680-1750 - Solve \( \hat{e} = |A^{-1}| \hat{N} \) for each independent loading

FIND STRAINS

GRADIENT

32
FORM A_h AND A_z

2120 - A_h is the A matrix at the current position in h space

2130 - A_{ij} = \sum_{k=1}^{NPLY(k)} Q_{ij} Z_k

FORM Q

2210-2290 - Convert C array into 3x3 Q matrix for a ply designated by II

FORM G

2410-2490 - Convert B array into 3x3 G matrix for a ply designated by II

2492-2496 - Place linear terms of G in vector S

33
NEW POSITION

2501 LET SMAX=1E10
2502 FOR I=1 TO NPLY
2503 IF Z(I)<0 THEN LET S=-H(I)/Z(I)
2504 IF S>0 AND S<SMAX THEN LET SMAX=S
2505 NEXT I
2506 LET FS="""
2507 IF SMAX>10 THEN LET FS="FAIL"
2510 LET S2=SMAX
2512 IF FS="FAIL" THEN RETURN
2514 LET S1=0
2520 IF NC=0 THEN GOTO 2680
2530 LET S=S2
2540 GOSUB 1500
2550 GOSUB 500
2555 IF FS="FAIL" THEN LET S2=S
2557 IF FS="PASS" THEN LET S1=S
2558 IF S1=SMAX THEN GOTO 2625
2560 IF (S2-S1)<E2 AND S1=0 THEN
2562 IF FS="FAIL" THEN GOTO 2760
2564 IF S1=(S2-S1)<4 THEN GOTO 2760
2566 LET SREF=SREF+H(I)*H(I)
2568 LET SREF=SREF+H(I)*H(I)
2569 NEXT I
2570 FOR I=1 TO NPLY
2580 LET K(I)=H(I)+Z(I)*S
2590 IF H(I)<0 THEN LET H(I)=0
2600 LET SREF=SREF+H(I)*H(I)
2625 LET SREF=SREF+H(I)*H(I)
2660 LET SREF=SREF+H(I)*H(I)
2670 NEXT I
2680 FOR I=1 TO NPLY
2690 LET H(I)-H(I)*Z(I)*S
2700 LET H(I)-H(I)*Z(I)*S
2710 LET SREF=SQR SREF
2720 GOSUB 350
2725 LET SREF=SQR SREF
2730 IF SREF-S<2 THEN LET FS="FAIL"
2735 FOR I=1 TO NPLY
2740 LET H(I)-H(I)*S/SREF
2750 NEXT I
2760 LET S=0
2770 GOSUB 2000
2780 GOSUB 1500
2790 GOSUB 500
2800 RETURN

2501-2505 - Find distance along Z to first h_i=0 constraint
2540-2610 - Bisection method to find distance to next constraint
2585 - If no constraints are violated at S=SMAX then stop search and use that point
2620 - Take a point halfway in between constraints
2625-2670 - Update h vector at that point and calculate distance to origin
2690-2700 - Update strains at the mid-point
2720 - Use strain ratio routine to find nearest constraint along a line from the midpoint to the origin
2730-2750 - Update h vector to new point near constraint
2770 - Update A matrix
2780 - Update strains
2790 - List of active constraints
NEW DIRECTION

3005 LET NORM=1
3006 LET Z=0
3020 FOR I=1 TO NPLY
3030 LET X(I)=0
3040 LET Z=Z+SGN H(I)
3050 NEXT I
3060 IF NC=0 THEN GOTO 3225
3070 FOR I=1 TO NC
3071 LET F=CODE C(I,1)
3072 LET N=CODE C(I,2)
3080 GOSUB 1000
3100 FOR J=1 TO NPLY
3101 LET X(J)=X(J)-W(J)
3110 NEXT J
3140 FOR I=1 TO NC
3141 LET Z+=Z*G
3150 NEXT I
3170 NEXT I
3180 FOR J=1 TO NPLY
3181 LET X(J)=X(J)/NORM
3220 NEXT J
3225 GOTO 3225
3226 LET X(I)=X(I)/NORM
3240 FOR I=1 TO NC
3241 LET Z(I)=X(I)-TEST*Z*G
3260 NEXT I
3290 LET NORM=SQR NORM
3300 FOR I=1 TO NPLY
3301 LET Z(I)=Z(I)/NORM
3320 GOSUB 2000
3350 RETURN

STRAIN RATIO

3510 FOR P=1 TO NPLY
3520 IF H(P)=0 THEN GOTO 3640
3522 LET II=P
3524 GOSUB 2400
3530 FOR N=1 TO NL
3540 LET B=0
3545 LET C=0
3550 FOR I=1 TO 3
3555 FOR J=1 TO 3
3560 LET C=C-REF*1RF*1RF*E(N,I)*E(N,J)
3565 NEXT J
3570 NEXT I
3580 NEXT N
3590 LET B=B-REF*1RF*E(N,I)*E(N,J)
3600 NEXT P
3610 LET SVRL=(-SOR(D*5-4*C*(2-(1-E5S)))*1.6)
3620 IF SVRL>S THEN LET S=SM,
3630 NEXT N
3640 RETURN

3522-3524 - Form G matrix
3522-3560 - For each constraint solve
\[
g_{ij} e_i e_j (SREF)^2 \]
\[
\frac{S}{S - 1} = E6
\]
for S

3620 - Take smallest value of S
(closet constraint)
INITIAL FEASIBLE PT.

4110 LET Z = 1/5 OR NPLY
4120 FOR I = 1 TO NPLY
4130 LET Z(I) = Z
4140 NEXT I
4150 GOSUB 2000
4160 LET S = 0
4170 LET SREF = 1
4180 GOSUB 1500
4190 GOSUB 3500
4330 FOR I = 1 TO NPLY
4340 LET H(I) = H(I) * S
4350 NEXT I
4360 LET S = 0
4375 GOSUB 2000
4390 GOSUB 500
4400 RETURN

TRANSFORMATIONS

4510 FOR I = 1 TO NPLY
4520 LET C2 = COS(2 * T(I))
4530 LET C4 = COS(4 * T(I))
4540 LET S2 = SIN(2 * T(I))
4550 LET S4 = SIN(4 * T(I))
4560 LET B(I, 1) = U(1) + C2 * V(2) + C4 * V(3)
4570 LET B(I, 2) = U(1) - C2 * V(2) + C4 * V(3)
4580 LET B(I, 3) = U(4) - C4 * V(3)
4590 LET B(I, 4) = U(5) - C4 * V(3)
4600 LET B(I, 5) = S2 / 2 * V(2) + S4 * V(3)
4610 LET B(I, 6) = S2 / 2 * V(2) - S4 * V(3)
4620 LET C(I, 1) = U(1) + C2 * U(2) + C4 * U(3)
4630 LET C(I, 2) = U(1) - C2 * U(2) + C4 * U(3)
4640 LET C(I, 3) = U(4) - C4 * U(3)
4650 LET C(I, 4) = U(5) - C4 * U(3)
4660 LET C(I, 5) = S2 / 2 * U(2) + S4 * U(3)
4670 LET C(I, 6) = S2 / 2 * U(2) - S4 * U(3)
4680 NEXT I
4690 RETURN

4660-4670 - Transform failure parameters in following order
B(I,1)=G11  B(I,5)=G16
B(I,2)=G22  B(I,6)=G26
B(I,3)=G12  B(I,7)=G1
B(I,4)=G66  B(I,8)=G2
        B(I,9)=G3

4680-4730 - Transform modulus in following order
C(I,1)=Q11  C(I,4)=Q66
C(I,2)=Q22  C(I,5)=Q16
C(I,3)=Q12  C(I,6)=Q26

Note, transformations for all orientations calculated and stored
CLS
5007 PRINT "ENTER PLY ORIENTATION": PRINT 
5008 FOR I=1 TO N
5009 PRINT "I","M$(I)
5010 NEXT I
5011 PRINT "NEW"
5012 SLOU
5013 PRINT AT 15,5,"SELECT MATERIAL:
5014 INPUT M
5015 SLOU
5016 PRINT "HOW MANY PLY ANGLES?"
5017 INPUT NPLY
5018 CLS
5019 PRINT "ENTER PLY ORIENTATION": PRINT 
5020 FOR I=1 TO NPLY
5021 PRINT "I","T$(I)
5022 INPUT T$(I)
5023 LET T$(I)=T$(I)/PI/180
5024 NEXT I
5025 PRINT "ENTER NUMBER OF INDEPENDENT LOADING CONDITIONS"
5026 INPUT NL
5027 LET U$="IN"
5028 LET U$="M"
5029 PRINT "ENGLISH OR SI UNITS? (E$)"
5030 INPUT E$
5031 IF E$="E" THEN LET U$="LBS"
5032 IF E$="E" THEN LET U$="IN"
5033 FOR I=1 TO NL
5034 CLS
5035 PRINT "LOADING CONDITION": PRINT 
5036 FOR J=1 TO 3
5037 LET L=J
5038 IF J=3 THEN LET L=6
5039 PRINT "N","L","="
5040 INPUT N$(I,J)
5041 PRINT AT J,4;N$(I,J);U$
5042 IF E$="E" THEN LET N$(I,J)=N$(I,J)*511
5043 NEXT J
5044 NEXT I
5045 CLS
5046 FAST
5047 RETURN

5090-5160 - List available materials
5120 - SLOU = number of layers
5210 - NPLY = number of layers
5230-5290 - Enter orientations
5250 - computer requires angles in radians, so convert degrees to radians
5330-5428 - Establish proper units labels
5450-5480 - Enter loads. If loads are in lbs/in, convert to N/m
5505 LET TEST=0
5506 LET E=1
5510 FOR I=1 TO NPLY
5515 LET TEST=TEST+T(I)
5530 NEXT I
5540 PRINT "TOTAL LAMINATE THICK"
5550 PRINT TAB 10,TEST*E;V$
5555 PRINT "AFTER ";ITER;" ITERATIONS"
5560 PRINT AT 21,0,K$
5570 CLS
5580 GOSUB 5870
5590 PRINT AT 21,0;K$
5600 CLS
5615 GOSUB 5660
5620 PRINT AT 21,0;K$
5630 CLS
5640 INPUT H$
5645 IF H$="Y" THEN GOSUB 6500
5650 PRINT "HISTOGRAM (Y/N)?"
5660 INPUT "$$
5665 PRINT "ULTIMATE STRAIN: ")
5666 PRINT "STRENGTH RATIO"; N
5667 PRINT "SAFETY"
5670 PRINT "FLY"
5671 FOR I=1 TO NL
5672 PRINT AT 2,I*7-1;"LOAD"; I
5673 NEXT I
5680 FOR P=1 TO NPLY
5685 IF H(P)=0 THEN GOTO 5810
5690 LET E=0
5700 GOSUB 2400
5710 FOR N=1 TO NL
5720 FOR K=1 TO 3
5730 LET A=R+G(J,K)*E(N,J)*E(N,K)
5740 NEXT K
5750 LET B=6+S(J)*E(N,J)
5760 NEXT J
5770 LET A=(-B+5QR (B=6+4*A)) / (2
5780 LET A=INT (A+5/1E3);1E3)
5790 PRINT AT P+2,N*7-1;A
5860 PRINT "HRRDCOPY (Y/N)?";
5870 INPUT "$$
5880 IF "$S=X" THEN GOSUB 7500
5890 GOTO 1
5900 PRINT "STRENGTH RATIOS"
5910 PRINT "ULTIMATE STRAIN: ")
5915 PRINT "STRENGTH RATIO"
5920 PRINT "ULTIMATE STRAIN: ")
5930 PRINT "STRENGTH RATIO"
5940 PRINT "ULTIMATE STRAIN: ")
5950 PRINT "STRENGTH RATIO"
5960 PRINT "ULTIMATE STRAIN: ")
5970 PRINT "STRENGTH RATIO"
5980 PRINT "ULTIMATE STRAIN: ")
5990 PRINT "STRENGTH RATIO"
6000 PRINT "ULTIMATE STRAIN: ")
6010 PRINT "STRENGTH RATIO"
6020 PRINT "ULTIMATE STRAIN: ")
6030 PRINT "STRENGTH RATIO"
6040 PRINT "ULTIMATE STRAIN: ")
6050 PRINT "STRENGTH RATIO"
6060 PRINT "ULTIMATE STRAIN: ")
6070 PRINT "STRENGTH RATIO"
6080 PRINT "ULTIMATE STRAIN: ")
6090 PRINT "STRENGTH RATIO"
6100 PRINT "ULTIMATE STRAIN: ")
6110 PRINT "STRENGTH RATIO"
6120 PRINT "ULTIMATE STRAIN: ")
6130 PRINT "STRENGTH RATIO"
6140 PRINT "ULTIMATE STRAIN: ")
6150 PRINT "STRENGTH RATIO"
6160 PRINT "ULTIMATE STRAIN: ")
6170 PRINT "STRENGTH RATIO"
6180 PRINT "ULTIMATE STRAIN: ")
6190 PRINT "STRENGTH RATIO"
6200 PRINT "ULTIMATE STRAIN: ")
6210 PRINT "STRENGTH RATIO"
6220 PRINT "ULTIMATE STRAIN: ")
6230 PRINT "STRENGTH RATIO"
6240 PRINT "ULTIMATE STRAIN: ")
6250 PRINT "STRENGTH RATIO"
6260 PRINT "ULTIMATE STRAIN: ")
6270 PRINT "STRENGTH RATIO"
6280 PRINT "ULTIMATE STRAIN: ")
6290 PRINT "STRENGTH RATIO"
6300 PRINT "ULTIMATE STRAIN: ")
6310 PRINT "STRENGTH RATIO"
6320 PRINT "ULTIMATE STRAIN: ")
6330 PRINT "STRENGTH RATIO"
6340 PRINT "ULTIMATE STRAIN: ")
6350 PRINT "STRENGTH RATIO"
6360 PRINT "ULTIMATE STRAIN: ")
6370 PRINT "STRENGTH RATIO"
6380 PRINT "ULTIMATE STRAIN: ")
6390 PRINT "STRENGTH RATIO"
6400 PRINT "ULTIMATE STRAIN: ")
6410 PRINT "STRENGTH RATIO"
6420 PRINT "ULTIMATE STRAIN: ")
6430 PRINT "STRENGTH RATIO"
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6650 PRINT "STRENGTH RATIO"
6660 PRINT "ULTIMATE STRAIN: ")
6670 PRINT "STRENGTH RATIO"
6680 PRINT "ULTIMATE STRAIN: ")
6690 PRINT "STRENGTH RATIO"
6700 PRINT "ULTIMATE STRAIN: ")
6710 PRINT "STRENGTH RATIO"
6720 PRINT "ULTIMATE STRAIN: ")
6730 PRINT "STRENGTH RATIO"
6740 PRINT "ULTIMATE STRAIN: ")
6750 PRINT "STRENGTH RATIO"
6760 PRINT "ULTIMATE STRAIN: ")
6770 PRINT "STRENGTH RATIO"
6780 PRINT "ULTIMATE STRAIN: ")
6790 PRINT "STRENGTH RATIO"
6800 PRINT "ULTIMATE STRAIN: ")
6810 PRINT "STRENGTH RATIO"
6820 RETURN

5510-5535 - Sum for total thickness and establish units conversion

5615 - Call ply ratio printout

5626 - Call strength ratio printout

5700-5780 - Solve for R in

\[
\begin{bmatrix}
G_{11} & N & N \\
N & G_{22} & N \\
N & N & G_{33}
\end{bmatrix} R^2 + \begin{bmatrix}
G_{11} \\
G_{22} \\
G_{33}
\end{bmatrix} R - 1 = 0
\]

for each loading
PLY RATIO

For each orientation, find ply ratio

\[ \text{ratio} = \left( \frac{h_i}{\sum h_i} \right) \text{ and number of plies} \]

\[ \text{plies} = \frac{h_i}{\text{(ply thickness)}} \]

HARDCOPY

5560-5610 - Print loads with given units

5630 - Call ply ratio
5650 - Call strength ratio

NEW MATERIAL

7030 - R$ contains prompts \( E_x, E_y, V_x, G \)
7060 - String concatenation to store properties in p$ array
7110 - S$ contains prompts X(TENS.), X(COMP.), Y(TENS.), Y(COMP.), SHEAR
7150 - String concatenation to store strengths in Y$ array
HISTOGRAM

7500 LET Z=0
7505 FOR I=1 TO NPLY
7510 IF H(I)/TEST>Z THEN LET Z=H(I)/TEST
7520 NEXT I
7525 FOR I=1 TO NPLY
7530 LET Z(I)=INT (H(I)*4.2/TEST
DELTA+.5)
7540 NEXT I
7550 FOR J=3 TO Z(I)+2
7560 FOR K=-1 TO 1
7570 LET DELTR=INT (Z/9*.1110+1)/1
7580 LET Z(J)=INT (H(I)*4.2/TEST
DELTA+.5)
7590 NEXT K
7600 NEXT J
7610 NEXT I
7625 FOR I=1 TO 7
7626 IF J=16 THEN LET J=J+1
7630 PRINT AT 21,J;"-120X*30
7640 NEXT I
7650 LET Z=11*DELTA
7660 FOR I=0 TO 18 STEP 2
7670 LET Z=Z-DELTA
7680 LET A=INT ((Z+.005)*1E2)/1E2
7690 LET Z=Z+DELTA
7700 PRINT AT 1,0;A
7710 NEXT I
7720 IF H$="Y" THEN PAUSE 40000
7730 IF H$="Y" THEN COPY
7740 RETURN
INFORMATS

3160 LET EX=VAL P$(H,1 TO 4)*1E9
3170 LET EY=VAL P$(H,6 TO 9)*1E9
3180 LET UX=VAL P$(H,11 TO 14)
3190 LET ES=VAL P$(H,16 TO 19)*1E9
3200 LET XT=VAL Y$(H,1 TO 4)*1E6
3210 LET XC=VAL Y$(H,6 TO 9)*1E6
3220 LET YT=VAL Y$(H,11 TO 14)*1E6
3230 LET YC=VAL Y$(H,16 TO 19)*1E6

8160-8240 - Extract material properties from string arrays VAL converts string to floating point number

8540-8580 - modulus invariants

8590-8630 - Definitions of quadratic strength parameters. Note reuse of variables EX, ET, etc.
8630-8680 - Convert from stress space parameter to strain space

3690-3750 - Failure parameter invariants
Figure 1. Optimization Trajectory for [0/90] Laminate.

\[
\begin{align*}
N_1 &= 2 \times 10^6 \text{ N/m} \\
N_2 &= 1 \times 10^6 \text{ N/m} \\
N_6 &= 0
\end{align*}
\]
Figure 2. Optimization Trajectory for $[\pm 45]$ Laminate.
Figure 3. Flow Chart