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ELECTRON BEAM-INDUCED ELECTROMAGNETIC WAVES IN A MAGNETOSPHERIC PLASMA

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**Title:** Electron Beam-Induced Electromagnetic Waves in a Magnetospheric Plasma

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**Abstract:**
A theoretical interpretation of waves detected by the SCATHA satellite near geostationary altitude during electron beam operations is presented. The observations consist of emission bands at or between harmonics of the electron gyrofrequency, combining background and beam-induced emissions. Beam-induced emissions often broaden and/or intensify the emission band. A magnetic field component is detected along with the electric field component for most of the emissions. Therefore, these emissions are assumed to be electromagnetic waves.
The linearized Vlasov equation is used to derive the dispersion relation for electromagnetic waves in a warm, homogeneous, uniformly magnetized plasma. The ambient plasma is represented as a Maxwellian distribution and the electron beam as a delta-function in velocity space. The dispersion relation is solved analytically for non-resonant, perpendicularly propagating, electromagnetic waves. Using observational data to quantitatively define the plasma parameters, unstable solutions are found at frequencies consistent with those of the observed emissions.
INTRODUCTION

The question of whether or not electromagnetic cyclotron harmonic waves can be excited in the magnetosphere near geostationary altitude, was raised upon examination of the data from the VLF broadband measurements on the SCATHA satellite during a severe spacecraft charging event (Donatelli et al., 1983). During this event strong emission bands were detected at or near the first two harmonics of the 2 kHz electron gyrofrequency. These emission bands appeared on both the electric and magnetic wave field detectors. While the SCATHA electron gun was in operation at varying current/energy levels attempting to discharge the satellite, there were frequency shifts in the emission band and consistent magnetic field components. This observation prompted the present theoretical study to determine whether electromagnetic electron cyclotron harmonic waves could be generated in a plasma representative of local conditions.

Theoretical and observational studies to date have explained observations analogous to those of the SCATHA data in terms of electrostatic emissions. Using the Harris dispersion relation, Fredricks (1971) showed that non-resonant electrostatic waves could be excited in a plasma in which the electron perpendicular velocity distribution has a narrow region of positive slope, i.e. \( \partial f_0 / \partial \nu_\perp > 0 \). The restriction on the velocity distribution was relaxed by Young et al. (1973) to the requirement that a distribution of cold and warm electrons have a velocity distribution that is non-monotonic in \( \nu_\perp \). They showed that this provides sufficient free energy for driving instabilities. The ratio of cold to hot plasma density, above which instability will not occur, was established by Ashour-Ahalla et al. (1975). The non-
convective and convective nature of the instability is shown to be controlled by the temperature ratio of the cold to hot electron populations (Ashour-Abdalla and Kennel, 1978). Electrostatic upper hybrid waves observed on ISEE-1 have also been explained by this theory (Kurth, et al. 1979). Sentman et al. (1979) show examples of two-component electron distribution functions constructed from ISEE-1 observations of low energy magnetospheric electrons that occur simultaneous with the detection of electrostatic emissions between the electron cyclotron frequency and the upper hybrid resonance.

Electromagnetic emissions within this frequency range, with the exception of the whistler mode, have not been reported prior to the SCATHA observations. Ohnuma et al. (1981) have shown that electromagnetic cyclotron harmonic waves may be generated in a dense plasma; i.e. one in which \( \omega_{pe} \gg \Omega_{ce} \), where \( \omega_{pe} \) and \( \Omega_{ce} \) are the electron plasma and gyro-frequencies, respectively. Although their results were applied to laboratory plasmas, this condition may prevail in the vicinity of SCATHA during beam operations.

Here the problem will be approached in a manner suggested by the work of Tataronis and Crawford (1970). They used the quasi-static approximation in deriving a dispersion relation for a warm magnetoplasma and proceeded to examine under what conditions unstable electrostatic cyclotron harmonic waves (Bernstein modes) exist. They looked at several types of distribution functions and examined ranges of plasma parameters for finding unstable modes. They show that if the analytic solution to the dispersion relation undulates about zero for a given distribution function, non-convective mode coupling instabilities occur. These occur between specific ranges of the two ratios, the plasma frequency/electron
gyrofrequency and the gyroradius/wavelength, for each harmonic pair. For two component distributions, the density and velocity ratios of the two components must also be considered. Similar solutions are sought here for electromagnetic, cyclotron harmonic waves (extraordinary and ordinary modes).

In the next section the electromagnetic dispersion relation for a warm, uniformly magnetized, homogeneous plasma is examined. Analytical solutions will be presented for the three perpendicularly propagating modes, the Bernstein, extraordinary, and ordinary modes, using a two component distribution function. The ambient plasma is represented as a Maxwellian; the electron beam is represented as a delta-function in velocity space. Sample calculations show that instabilities may exist for both the Bernstein and the extraordinary modes. The frequencies and wave numbers of the excited modes vary with the beam-to-ambient-plasma density and velocity ratios.

THEORY

The dispersion relation is derived using the linearized Vlasov equation and Maxwell's equations (in cgs units):

\[
\left[\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \frac{((\mathbf{v} \times \mathbf{B}_0)/c) \cdot \mathbf{v}}{m_j}f_{j1} = - (q_j/m_j)[E_1 + (\mathbf{v} \times \mathbf{B}_1)/c] \cdot \mathbf{v}f_{j0}
\right. \\
\left. \mathbf{v} \times \mathbf{E}_1 = -c^{-1} \frac{\partial \mathbf{B}_1}{\partial t} \\
\mathbf{v} \times \mathbf{B}_1 = c^{-1} \frac{\partial \mathbf{E}_1}{\partial t} + (4\pi q_j n_{j1}/c) \int \mathbf{v} df_{j1} \\
\mathbf{v} \cdot \mathbf{E}_1 = 4\pi q_j n_{j1} \int df_{j1} \\
\mathbf{v} \cdot \mathbf{B}_1 = 0
\]

(1)
The equilibrium electron velocity distribution is $f_{j0}(v_\perp, v_\parallel)$ where $\perp$ and $\parallel$ refer to components perpendicular and parallel to the external magnetic field, $R_0 = R_{0z}$. There is assumed to be no external electric field, $F_0 = 0$. The perturbation electric and magnetic fields are $E_1$ and $B_1$, respectively. The perturbed particle distribution and density are $f_{j1}$ and $n_{j1}$, respectively, where particle species is indicated by the subscript $i$. The charge and mass for each particle species are $q_j$ and $m_j$, respectively; $c$ is the velocity of light in a vacuum.

The plasma is assumed to be infinite, spatially homogeneous and uniformly magnetized, since variations in time and space may be neglected. Equations (1) and (2) are solved by introducing a Fourier transform in space, a Laplace transform in time, and integrating along unperturbed particle orbits. This leads to the nine element dielectric tensor, from Krall and Trivelpiece (1973):

$$
\begin{bmatrix}
\eta_{xx} & \eta_{xy} & \eta_{xz} \\
\eta_{yx} & \eta_{yy} & \eta_{yz} \\
\eta_{zx} & \eta_{zy} & \eta_{zz}
\end{bmatrix}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} = 0
$$

where the elements of the dielectric coefficient are $(\text{Im}(\omega) > 0)$:

$$
\eta_{xx} = 1 - \left(\frac{c^2 k_\perp^2}{\omega^2}\right) - \left(2\pi/\omega\right) \lambda_j \lambda_N \omega_j^2 \frac{\sigma_j^2 N^2 \Omega_j^2}{k_\perp^2}
$$

$$
\eta_{xy} = - \left(2\pi/\omega\right) \lambda_j \lambda_N \omega_j^2 \frac{\sigma_j^2 N^2 v_\perp v_\parallel \Omega_j}{k_\perp}
$$

$$
\eta_{xz} = \left(\frac{c^2 k_\perp^2}{\omega^2}\right) - \left(2\pi/\omega\right) \lambda_j \lambda_N \omega_j^2 \frac{\sigma_j^2 N^2 v_\parallel \Omega_j}{k_\perp}
$$

$$
\eta_{yx} = - \eta_{xy}
$$

$$
\eta_{yy} = 1 - \left[\frac{c^2 (k_\parallel^2 - k_\perp^2)}{\omega^2}\right] - \left(2\pi/\omega\right) \lambda_j \lambda_N \omega_j^2 \frac{\sigma_j^2 N^2 (\Omega_j'^2)}{\lambda_j v_\perp^2}
$$
\[ n_{yz} = (2\pi i / \omega) \frac{1}{\lambda_j n_j} \omega_{pj}^2 II A_j J_N J_N' v_{y1} v_{z1} \]
\[ n_{zx} = (c^2 k_1 k_1 / \omega^2) - (2\pi / \omega) \frac{1}{\lambda_j n_j} \omega_{pj}^2 II x_j J_N^2 v_j \Omega_j / k \]
\[ n_{zy} = (2\pi i / \omega) \frac{1}{\lambda_j n_j} \omega_{pj}^2 II x_j J_N^2 v_j \]
\[ n_{zz} = 1 - (c^2 k_1^2 / \omega^2) - (2\pi / \omega) \frac{1}{\lambda_j n_j} \omega_{pj}^2 II A_j J_N^2 v_j^2 \]

where:

\[ II \equiv \int_0^\infty dv_j \int_0^\infty dv_j \frac{1}{2} v_j / (N \Omega_j + k) - \omega \]
\[ x_j \equiv [1-(k_1 v_j / \omega)](\partial f_0 / \partial v_j^2) + (k_1 v_j / \omega) \]
\[ (\partial f_0 / \partial v_j^2) \]
\[ A_j \equiv (N \Omega_j / \omega)[(\partial f_0 / \partial v_j^2) - (\partial f_0 / \partial v_j^2)] \]
\[ + (\partial f_0 / \partial v_j^2) \]

For simplicity in the above derivation, the wave vector \( k \) is defined as follows:
\[ k = k_1 + k \]
\[ k_1 = k_1 \]

The definition of other terms are:
\[ \omega \] = wave frequency
\[ \omega_{pj} = \text{plasma frequency} = (4\pi n_j^2 q_j^2 / m_j)^{1/2} \]
\[ \Omega_j = \text{particle gyrofrequency} = q_j B_0 / m_j c \]
\[ J_N = N\text{th order Bessel function of the first kind} \]
\[ k_1 v_j / \Omega_j = \text{argument of the Bessel function} \]
\[ J_N' = \text{derivative of } J_N \text{ with respect to the argument.} \]

There is no separation of electrostatic and electromagnetic modes in the dispersion tensor (3). However, by considering parallel propagation \( (k_1 = 0) \) and perpendicular propagation \( (k_1 = 0) \) separately, it is possible to simplify the dispersion tensor and examine some...
characteristic wave modes. Setting \( k_\parallel = 0 \) in (3) singles out the
modes that propagate parallel to the equilibrium magnetic field.
These modes are:

(1) The longitudinal, electrostatic, Landau damped, Langmuir waves
and ion acoustic waves.

(2) The weakly damped transverse electromagnetic Alfven waves at
frequencies below the ion cyclotron frequency; the Whistler waves at
\( \omega = (1/2)\Omega \) and the cyclotron waves which occur at \( \omega = \Omega \). The cyclotron
waves are strongly damped for \( \omega = k_\parallel c \), which is also the band in which
spontaneous emission of radiation occurs.

By taking \( k_\parallel = 0 \), the characteristic waves that propagate perpen-
dicular to the equilibrium magnetic field are singled out. These
modes are:

(1) the almost purely longitudinal, nearly electrostatic
Bernstein modes that correspond to the electrostatic waves discussed at
the beginning of this section.

(2) The transverse, electromagnetic, cyclotron-harmonic waves
which include the extraordinary mode with \( E \perp B_\parallel \) and the ordinary
mode with \( E \parallel B_\parallel \). These modes are normal modes of a high density,
magnetized plasma \( (\omega_p \gg \Omega_e^2) \), (Ohnuma et al., 1981) and are
excited near ion and electron cyclotron harmonics. The SCATHA observa-
tions are within the range of these emissions. It will be determined
if these modes are unstable within the local beam-background plasma.

For \( k_\parallel = 0 \), the dispersion tensor (3) reduces to:

\[
\begin{bmatrix}
\Delta_{XX} & \Delta_{XY} & 0 \\
-\Delta_{XY} & \Delta_{YY} & 0 \\
0 & 0 & \Delta_{ZZ}
\end{bmatrix} = 0
\] (4)
At the frequencies of oscillation considered here, ions may be regarded as providing a charge neutral background. The approximation \( \omega_p^2 \ll k^2c^2 \) is also valid since:

\[
\frac{\omega_p^2}{k^2c^2} < \frac{\omega_h^2}{c^2} < \frac{\omega_p^2v_b^2}{\alpha^2c^2} < 0.1
\]

where: \( v_h \) = beam velocity.

Since the \( D_{xy} \) terms are second order in \( \omega_p^2/k^2c^2 \), they may be neglected.

Then (4) may be approximately solved for the three eigenmodes:

1. \( D_{xx} = 0 \): Bernstein mode
2. \( D_{yy} = 0 \): extraordinary mode
3. \( D_{zz} = 0 \): ordinary mode

where the elements of the dispersion matrix reduce to:

\[
D_{XX} = 1 - \left( 4\pi \omega_b^2/k^2 \right) \frac{N^2\Omega^2}{\omega(N\Omega - \omega)} \int dv \int dv \frac{\partial f_0}{\partial v_\perp^2} (5)
\]

\[
D_{YY} = 1 - \left( k^2c^2/\omega^2 \right) - \frac{4\pi \omega_b^2}{2} \left( \omega(N\Omega - \omega) \right)^{-1} \int dv \int dv \frac{\partial f_0}{\partial v_\perp^2} (6)
\]

\[
D_{ZZ} = 1 - \left( k^2c^2/\omega^2 \right) - \frac{4\pi \omega_b^2}{2} \left( \omega(N\Omega - \omega) \right)^{-1} \int dv \int dv \frac{\partial f_0}{\partial v_\perp^2} (7)
\]

Unstable solutions are now sought for each of the eigenmodes using the SCATHA data to evaluate the necessary parameters.

ANALYTICAL SOLUTIONS

The SCATHA observations presented in Donatelli et al. (1983) for 24 April 1974, indicate that beam-injected electrons may create a dense
plasma where electromagnetic cyclotron harmonic instabilities are generated. At the time of interest the magnetospheric plasma was made up of two populations: a low-energy component with a temperature about 300 eV, and a high-energy component about 25 keV (Mullen et al., 1981). The ejection energy of the beam from the electron gun was either 50 eV or 150 eV. After passing through the satellite sheath the beam electrons had energies of 1-3 keV due to acceleration through the vehicle potential. Emission bands were detected consistently at or near the first and second harmonic of the electron gyrofrequency, $\omega$, which was about 2 kHz. From the data, beam-to-background velocity ratios are estimated. A ratio of 2-3 is reasonable for the "artificial" beam (electrons from the beam systems on SCATHA), and a ratio of 9-10 for the natural beam (injections of high energy magnetospheric electrons). The density ratios are assumed to be greater than one, both for the natural beam (Mullen et al., 1981) and the "artificial" beam. Although the current and radius of the artificial beam are known, the effective beam density is not, since the electrostatic forces between beam electrons contribute to rapid spreading as does the external magnetic field (Gendrin, 1973). Furthermore, at these low emission energies the electron beam cannot be highly focussed.

The dimensionless variables to be used in these solutions are defined as follows:

\[ s^2 = 2Kt^2/m^2 = \nu t^2/\Omega^2 \]
\[ x_b = \nu_{lb}/\nu_t \]

where:

\[ \nu_{lb} = \nu_b \sin \theta_b; \]
\[ \theta_b = \text{pitch angle of the electron beam}, 15^0 < \theta < 165^0. \]
Then:

\[ sx_b = kv_b/\Omega = (kv_b/\Omega)\sin \phi_b \]

and:

\[ 0.26 (kv_b/\Omega) < sx_b < kv_b/\Omega \]

where \( sx_b \) is the argument of the Bessel function in equations (5), (6), and (7).

The ambient density is between 0.5 and 1.0 \( \text{cm}^{-3} \). If the density is set at 1.0 \( \text{cm}^{-3} \), the value of the following non-dimensionalized parameters are:

\[
\begin{align*}
\left( \omega_b/\Omega \right)^2 &= 20 \\
\left( \omega_b/kc \right)^2 &= 0.1
\end{align*}
\]

The distribution function is approximated as follows:

\[
f_0 = n_p (m/2\pi K T)^{3/2} \exp[-(m/2KT)(v_t^2 + v_{\perp}^2)] + \\
(n_b/2\pi v_{\perp h}) \delta(v_t - v_{th}) \delta(v_{\perp} - v_{\perp h})
\]

where:

\[ T = \text{temperature (°K)} \]
\[ K = \text{Boltzmann constant} \]
\[ n_p = \text{low energy ambient electron density} \]
\[ n_b = \text{density of electron beam} \]

This distribution function is a Maxwellian combined with a ring distribution in velocity space and including motion parallel to the magnetic field. The Maxwellian represents the ambient plasma, and the delta-function represents the beam, with the ring distribution describing the portion of the monoenergetic beam electrons moving in the plane perpendicular to the magnetic field, uniformly distributed in gyrophase angle. Tataronis and Crawford (1970) conducted a numerical
study of the propagation characteristics of perpendicularly propagating electrostatic waves in a combined ring-Maxwellian distribution. They state that instability occurs if the analytic function for the dispersion relation is undulatory about zero. Their results show that node coupling is a feature of the ring distribution leading to strong non-convective instability. Combining the ring and Maxwellian distributions leads to non-convective instability with higher growth rate and lower instability threshold than for the ring distribution alone.

The electromagnetic dispersion relation may be solved for each of the three eigenmodes of perpendicularly propagating waves. Solutions to the dispersion relation may be found as functions of \( x_b \) and \( n_b/n_p \), using preceding definitions.

A. The Bernstein Mode

The dispersion relation for the Bernstein mode, obtained by substituting equation (8) into equation (5) and integrating, is the following:

\[
\eta_{XX} = 1 - \left(4\omega_b^2/\omega^2\right) \sum_{N=1}^\infty N^2 \Omega^2 A_{NX}/(\omega^2 - N^2 \Omega^2) = \eta
\]  

(9)

where:

\[
A_{NX} = s^{-2}\exp(-s^2/2)I_N(s^2/2) + (n_b/n_p) s x_b J_N J_N'
\]

\( I_N \) is the modified Bessel function of the first kind with argument \( s^2/2 \). The prime denotes the derivative of the Bessel function with respect to the argument. \( \sum \) is now the summation from \( N=1 \) to \( \infty \). Since \( A_{NX} \) is undulatory about zero, unstable solutions are anticipated (Tataronis and Crawford, 1970). In Appendix A it is shown that in using a three-term approximation to equation (9), unstable solutions are found for:

\[
n_b/n_p = 2 \text{ and } x_b = 2
\]

such that:
\[ \omega/\Omega = 1.7 + 0.9i \]  \hspace{1cm} (10) 

and:

\[ n_b/n_p = 4.5 \text{ and } x_b = 10 \]

such that:

\[ \omega/\Omega = 1.8 + 1.1i \]  \hspace{1cm} (11)

The Bernstein Mode is nearly a pure electrostatic mode. The solutions are equivalent to those obtained by Tataronis and Crawford (1970) using the electrostatic approximation to the dispersion relation. They found unstable solutions in this frequency range with the growth rate, \( \gamma \), a finite fraction of the real part of the frequency, \( \omega_r \). In equation (10), with \( \omega_r = 1.7 \Omega, \gamma = 0.5\omega_r \); for equation (11), \( \gamma = 0.6\omega_r \). In both equations (9) and (10) \( \omega_r \) is in the range of emissions detected by the SCATHA broadband receiver (Donatelli et al., 1983). However, this mode is not expected to have the observed magnetic field component; the presence of a magnetic component requires the existence of electromagnetic extraordinary and/or ordinary modes.

R. The Extraordinary Mode

The dispersion relation for the extraordinary mode is obtained by substituting equation (8) into equation (6) and integrating to obtain:

\[ n_{yy} = 1 - (k^2c^2/\omega^2) - (2\omega_p^2\Delta_0/\omega^2) - 4\omega_b^2 \left\{ A_{NY}/(\omega^2 - N^2\Omega^2) \right\} \]  \hspace{1cm} (12) 

where:

\[ A_{NY} = (N^2/s^2) \exp(-s^2/2)I_N(s^2/2) + (n_b/n_p)[(J_N')^2 + (s\lambda^2/2)J_N'J_N''] \]

The function \( A_{NY} \) is undulatory about zero, therefore, meeting the instability condition of Tataronis and Crawford (1970). Using a four-term approximation to the dispersion relation, instabilities are found
for (see Appendix B):
\[ n_b/n_p = 10 \text{ and } x_b = 9 \]
such that:
\[ \omega/\Omega = 1.5 + 0.2i \]
and:
\[ n_b/n_p = 10 \text{ and } x_b = 3 \]
such that:
\[ \omega/\Omega = 1.8 + 0.6i \]
Here it is shown that unstable solutions exist within the desired frequency range. The larger beam-to-background density ratios required to support the extraordinary mode are consistent with the work of Ohmura et al. (1981).

C. The Ordinary Mode

The dispersion relation for the ordinary mode is obtained by substituting equation (8) into equation (7) and integrating to obtain:

\[ n_{zz} = 1 - (k^2 c^2/\omega^2) - (A_{oz} \omega_p^2/\omega^2) - 2\omega_p^2 \int A_{NZ}/(\omega^2 - N^2\omega^2) \]

where:

\[ A_{NZ} = \exp(-s^2/2)I_n(s^2/2) + (n_p/n_p)J_n^2 \]

Since \( A_{NZ} \) is always positive, no unstable solutions are anticipated.

CONCLUSIONS

The analytical solutions presented here show that electromagnetic non-resonant instabilities may be excited in a plasma represented as a Maxwellian background with a monoenergetic beam of electrons. The electromagnetic dispersion relation was solved for \( k_z = 0 \), using the
approximation \( \omega_r^2 \ll k^2c^2 \). These simplifications permit the elements of the dispersion relation to be separated and solved as three distinct eigenmodes of perpendicularly propagating waves:

1. Bernstein Mode; \( \mathbf{E} \perp \mathbf{B_0}, \mathbf{k} \parallel \mathbf{E} \)

2. Extraordinary Mode; \( \mathbf{E} \parallel \mathbf{B_0}, \mathbf{k} \perp \mathbf{E} \)

3. Ordinary Mode; \( \mathbf{E} \parallel \mathbf{B_0}, \mathbf{k} \parallel \mathbf{E} \)

In the first two cases it is shown that cyclotron harmonic modes may couple between the first two harmonics of the electron gyrofrequency, exciting non-convective instabilities with growth rates, \( \gamma \), that are a finite fraction of \( \omega_r \). In the third case no unstable solutions exist. Sample solutions presented for the Bernstein and extraordinary modes are shown to depend on the ratios \( n_b/n_p \) and \( x_h = v_{\parallel b}/v_T \). These ratios must be greater than one.

The ratio, \( n_b/n_p \), for the nearly electrostatic Bernstein mode can be compared quantitatively to the \( \alpha \) of Tataronis and Crawford (1970) and the ratio \( N_c/N_H \) of Ashour-Abdalla et al. (1975) by considering the Maxwellian portion of the electron distribution as the "cold" component and the delta-function as the "hot" or "ring" component. The results for \( n_b/n_p = 4.5 \) and \( x_h = 10 \) agrees with the results for \( \alpha = 0.2 \) and \( N_c/N_H = 0.2 \). These are values associated with non-convective electrostatic instabilities at frequencies between the first two harmonics of the electron gyrofrequency.

For the extraordinary mode a larger density ratio, \( n_b/n_p = 10 \), is required to excite instabilities, consistent with the results of Ohnuma et al. (1981). These instabilities may be excited for velocity ratios of 3 and 9, representing ratios of the "artificial" and "natural" electron beam densities, respectively, to the ambient density. These
electromagnetic instabilities are non-convective, non-resonant, with large growth rates. They may be excited by electron beams, given sufficient beam-to-ambient density and velocity ratios. The full range of parameters over which they may occur will be explored through numerical calculations. For further understanding of the relationship of these waves to effects observed in the SCATHA data and in the magnetosphere, the full electromagnetic dispersion relation, including the $k$ terms, must be solved for conditions pertaining to space vehicles in space plasmas.

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APPENDIX A

Solution for Bernstein Modes:

The dispersion equation for the Bernstein modes is:

\[ \eta_{xx} = 1 - \frac{(4\omega_p^2/\Omega^2)}{N^2\bar{n}^2A_{Nx}/(\omega^2 - N^2\Omega^2)} \] (A-1)

where:

\[ A_{Nx} = s^2\exp(-s^2/2)I_N(s^2/2) + \left(\frac{n_b}{n_p s x_b}\right)J_N(s x_b)J_N'(s x_b) \]

This may be approximated:

\[ \eta_{xx} = 1 - \frac{(4\omega_p^2/\Omega^2)\left[(A_1\Omega^2/(\omega^2 - \Omega^2)) + (4A_2\Omega^2/(\omega^2 - 4\Omega^2))\right]}{N^2\bar{n}^2A_{Nx}/(\omega^2 - N^2\Omega^2)} \] (A-2)

Setting \( \eta_{xx} = 0 \) leads to the fourth order equation:

\[ \frac{\omega^4/\Omega^4}{N^2\bar{n}^2} - [5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2)] \frac{\omega^2/\Omega^2}{N^2\bar{n}^2} + 4[1 + (4\omega_p^2/\Omega^2)(A_1 + A_2)] = 0 \] (A-3)
This may be solved as a quadratic, then transformed to polar coordinates, to obtain:

\[ \omega/\Omega = (Kr)^{1/2}[\cos(n\varphi + \pi m) + i\sin(n\varphi + \pi m)], \quad n = 0, 1 \quad (A-4) \]

where:

\[ x = [5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2)]/2 \]
\[ r^2 = x^2 + y^2 \]
\[ \theta = \tan^{-1}(y/x) \]
\[ x = 1 \]
\[ y = [[16(1 + (4\omega_p^2/\Omega^2)(A_1 + A_2))/(5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2))^2] - 1]^{1/2} \]

For instability, the following condition must be satisfied:

\[ 0 < [5 + (4\omega_p^2/\Omega^2)(A_1 + 4A_2)]^2 < 16[1 + (4\omega_p^2/\Omega^2)(A_1 + A_2)] \quad (A-5) \]

\( A_1 \) and \( A_2 \) are evaluated as functions of \( n_b/n_p \) using tabulated values of Bessel functions from Ahranowitz and Stequn (1970) for the approximate range of the argument, as defined by the constraints on \( s_x_b \). Substituting these and the value of \( \omega_p/\Omega \) into the inequality (A-5), determines the constraints on \( n_b/n_p \).

For \( s_x_b = 3.6; \ x_b = 20; \ n_b/n_p = 2: \)
\[ A_1 = 0.0464; \ A_2 = -0.0132. \]

Substituting in equation (A-4) and solving for \( \theta \) and \( Kr \):

\[ \omega/\Omega = 1.95(0.89 + 0.45i) = 1.73 + 0.886i \quad (A-6) \]

A second solution is found for \( s_x_b = 3.2, x_b = 10. \) Then with \( n_b/n_p = 4.5; \ A_1 = 0.078, \ A_2 = -0.0223; \) and

\[ \omega/\Omega = 1.83 + 1.14i \]

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APPENDIX B

Solution for Extraordinary Modes:

The dispersion equation for the extraordinary mode is:

\[ n_{yy}^2 = 1 - \left( \frac{k^2 c^2}{\omega^2} \right) - \left( 2 \frac{\omega_p^2 A_0}{\omega^2} \right) - 4 \frac{\omega_p^2}{\omega^2} \chi_H/(\omega^2 - \frac{c^2}{\omega^2}) \]  \hspace{1cm} (B-1)

where:

\[ \chi_H = \left( N^2 / s^2 \right) \exp(-s^2/2) I_H(s^2/2) + \left( 2 n_b / n_p \right) \left[ (J_H'(s x_b))^2 + s x_b J_H'(s x_b) J_H''(s x_b) \right] \]

This may be approximated:

\[ n_{yy}^2 = 1 - \left( \frac{k^2 c^2}{\omega^2} \right) - \left( 2 \frac{\omega_p^2 A_0}{\omega^2} \right) - 4 \frac{\omega_p^2}{\omega^2} \left[ (A_1/(\omega^2 - \frac{c^2}{\omega^2})) + (A_2/(\omega^2 - 4\omega^2)) \right] \]  \hspace{1cm} (B-2)

Setting \( n_{yy} = 0 \) leads to the sixth order equation:

\[ (\omega^4 - 5(\omega / k c)^2 + 4\omega^4)[\omega^2/(k c)^2] - 1 - (2 \frac{\omega_p^2 A_0}{k^2 c^2}) \]

\[ - (4 \frac{\omega_p^2}{k^2 c^2})[\omega^4(A_1 + A_2) - (\omega / k)^4(4A_1 + A_2)] = 0 \]  \hspace{1cm} (B-3)

This may be reduced to a fourth order equation by using the approximation \( \omega^2 / k^2 c^2 \ll 1 \):

\[ 1 + (2 \frac{\omega_p^2 A_0}{k^2 c^2}) + (4 \frac{\omega_p^2}{k^2 c^2})(A_1 + A_2) \]  \hspace{1cm} \( (\omega / k)^2 - \)

\[ 5(1 + (2 \frac{\omega_p^2 A_0}{k^2 c^2}) + (4 \frac{\omega_p^2}{k^2 c^2})(A_1 + A_2)) \]  \hspace{1cm} \( (\omega / k)^2 + \)

\[ 4(1 + (2 \frac{\omega_p^2 A_0}{k^2 c^2}) \]  \hspace{1cm} \( = 0 \)  \hspace{1cm} (B-4)

This may be solved as a quadratic, then transformed to polar coordinates, analogous to the solution for the Bernstein mode of Appendix A, to obtain:

\[ \omega / \Omega = (kr)^{1/2}[\cos(0/2 + m\pi) \pm isin(0/2 + m\pi)], \quad m = 0, 1 \]  \hspace{1cm} (B-5)

where:
For instability, the following condition that must be satisfied:

\[ n < \left[ \left( 1 + 2 \omega_p^2 \frac{A_0}{k^2 c^2} \right) + (4 \omega_p^2 / k^2 c^2) (A_1 + A_2) \right]^2 \]

\[ < 16 \left( 1 + 2 \omega_p^2 \frac{A_0}{k^2 c^2} \right) \left[ 1 + 2 \omega_p^2 \frac{A_0}{k^2 c^2} + (4 \omega_p^2 / k^2 c^2) (A_1 + A_2) \right] \]  

(A - 6)

\( A_N \) and \( A_{N+1} \) are evaluated as functions of \( n_h/n_p \) using the tabulated values of Bessel functions from Abramowitz and Stegun (1970) for the appropriate range of the argument as defined by the constraints on \( sx_b \). Substitute these and the value of \( \omega_p^2 / k^2 c^2 \) into the inequality (A-6) to obtain the constraints on \( n_h/n_p \).

For \( sx_b = 4.0; \ x_b = 9.0; \ n_h/n_p = 10 \):

\( A_0 = 1.05, \ A_1 = -0.714, \ A_2 = 2.77 \). Substituting and solving for \( \theta \) and \( KR \):

\( \omega/\Omega = 1.55(0.99 + 0.14i) = 1.53 + 0.22i \)  

(A - 7)

A second solution is found for \( sx_b = 4.4; \ x_b = 3. \) Then with \( n_h/n_p = 10 \): \( A_0 = 3.06, \ A_1 = -2.4, \ A_2 = 2.79 \), and

\( \omega/\Omega = 1.8 + 0.6i \)