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TRANSIENT HEAT TRANSFER IN COATED SUPERCONDUCTORS

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**ABSTRACT**

Computer simulation of transient heat transfer from coated superconductors to liquid helium have revealed that coating a superconductor with certain new materials, called Laketites, instead of traditional insulation, such as GE7031, substantially improves the ability of the superconductor to withstand transient heat pulses without making the transition to the "normal" i.e. non-superconducting state. In the best cases, the length
20. Abstract Cont'd

of time that constant heating can be withstood is improved twenty fold. This report shows that the most important property of these new materials is their increased thermal conductivity relative to current materials. The highest thermal conductivity materials designated SC-2, SC-3 are the most desirable for further research and development. The increased specific heat of the Laketites is desirable, but not crucial. Furthermore, the thickness of the coating has little effect on the improved ability of the superconductor to withstand heat pulses.

This report recommends a vigorous program of further development of the Laketites. A complete theoretical background of the computer simulation and examples of the programs used are also included in this report.
FORWARD

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I Summary

The objective of this investigation was the computer simulation of transient heat transfer from superconductors coated with the newly developed Lake-tite coatings to liquid helium in order to test the ability of these coatings to improve the heat transfer process at low temperatures and to make recommendations as to which, if any, of the three classes of materials should be selected for further research and development. A secondary objective is the investigation of the changes in heat transfer characteristics due to variation of the thickness of the coatings.

A computer program using the SCEPTRE code was developed to perform the computer simulation. An example of an actual SCEPTRE program is given in Appendix I along with explanatory notes. Since each material has different thermal properties the program must be changed in specified places to reflect these changes in properties. Critical to the operation of the program was a Fortran subroutine which is given in well-documented form in Appendix II. A computer simulation is only accurate to the extent that the equations that are used accurately reflect the physical reality of the process that is occurring.

The heat transfer theory that underlies the equations used in the computer simulation and all the assumptions and approximations used to derive these equations are presented in detail in the theory section of this report. All of the equations, approximations, and assumptions are referenced to existing heat transfer literature to provide a justification for their use.

The computer program thus developed was used to test SC-1C, SC-2B, and SC-3A Laketite coatings with bare copper and GE7031--a typical present-day insulation--used as comparison. The superconductor was represented by a 5 mil thick flat copper plate to imitate the copper outer sheath usually present on superconducting wire. Constant heating with power levels of .05, .1, .2, .5, 1, 2, 3, 4, 5, 10, 20, and 50 W/cm² was used to span the range of heating
power likely to be encountered. Constant heating is produced by the eddy current process due to changing magnetic fields. Coating thicknesses of 1, 2, 4, 10, and 20 thousandths of an inch (mils) were used. The time of heating was varied from .001 seconds to .1 seconds. Despite these particular choices, the program was designed so that it could accept any material whose thermal properties were well characterized; any shape of heating pulse, length of heating, and level of heating, as well as any changes in bath temperatures.

From the voluminous results of the computer simulations, three principle conclusions emerged. 1) The Laketites provide a substantial improvement in the thermal response characteristics over present materials. Thus, a vigorous program of further research and development on these materials should be instituted. 2) The most important parameter for a coating to have is a high thermal conductivity, with specific heat being less important. Thus, the SC-2 and SC-3 materials appear to be the most promising. 3) The thickness of the coating is not a vital parameter and should be governed by mechanical considerations.

Recommendations for further research and development of these coatings and developments of better computer simulations are made.
II Introduction

The Air Force has long been interested in applying the phenomenon of superconductivity for the purpose of lightweight production and storage of energy. Superconductors have the disadvantage that they must be refrigerated below a transition temperature in order to operate. If a section of superconductor heats up above its transition temperature, that section becomes a "normal" high resistance section of wire. The continuing passage of electrical current through the resistive section produces additional ohmic heating which raises more of the superconductor above its transition temperature, leading to a runaway reaction that disables the superconducting device. This problem is particularly severe for devices that are pulsed, since pulsed operation creates rapidly changing magnetic fields that produce large eddy current heating in the copper or bronze matrix that surrounds the superconducting portion of the wire. The shorter the pulse, the worse this problem becomes. As superconducting energy storage devices may well be operated in the pulsed mode, any way to increase the ability to remove or absorb the heat produced during a pulse, thus preventing the superconductor from becoming "normal" is desirable.

In 1980 Lake Shore Cryotronics, a concern that manufactures cryogenic equipment, developed a series of dielectric coatings called Laketites for superconducting wires that have unusually large specific heat and/or thermal conductivity at the low temperatures that are characteristic of superconductivity. Simple calculations indicated that these materials would assist in solving the heating problem by either rapidly carrying away the heat with high thermal conductivity or by absorbing heat by use of high specific heat or by some combination of these properties. A method was needed to choose the best material in this series for further testing and development and to choose the correct thickness of the coating for best results. Experimental testing
of different coatings and thicknesses would require testing numerous samples. Work in the summer of 1980\textsuperscript{2} revealed that there was no simple experimental test with readily available equipment that could screen a large number of samples. As an alternative, this project was established to develop a computer simulation of transient heat transfer using the Air Force-sponsored SCEPTRE computer code. This computer program was then used to evaluate different coating materials and various thicknesses of coatings. Preliminary reports on the early stages of the development of this program have been given at scientific meetings\textsuperscript{3,4} The final results of this project will be presented to the Applied Superconductivity Conference in Knoxville, TN November 30, 1982 and will be published in the IEEE Journal, MAGNETICS in the spring of 1982.

This report describes the final version of the computer program, presents the results of the analysis for the three major types of Laketite coatings, and gives conclusions about their feasibility and recommendations for future work.
III Theory

The basic approach of this project is to use the powerful and versatile SCEPTRE computer code to simulate transient heat transfer processes. The SCEPTRE code was written to solve fast transient electrical problems. To solve heat transfer problems requires translating the thermal process into electrical terms.

Heat transfer in a solid is readily modeled electrically, as a series of resistors and capacitors with the resistors representing the thermal resistance due to a finite thermal conductivity and the capacitor representing the heat capacity of the object. This analogy is mathematically exact, i.e. the equations have exactly the same mathematical form. This exactness represents the basic similarity between the two processes both of which involve transport by particles--electrons for electricity, phonons for heat. Two possible problems can arise from this approach. 1) A basic assumption of this analysis is that the materials are isotropic--the same in all directions. Some materials notably composites are not isotropic. In the absence of contrary evidence, it is assumed that the Laketites and all other materials involved in this project are isotropic. 2) In the electrical case the flow of current is nearly always longitudinal (the long direction) which usually permits the assumption that the cross-sectional area perpendicular to the flow of the current is constant. The flow of heat is often radial (the short direction): If the material is curved, e.g. a wire, the cross sectional area is no longer constant. In this project, the simplest possible geometry--a square thin flat plate was chosen. Such a configuration is the easiest to produce experimentally. Thus the largest amount of experimental heat transfer data comes from thin flat plates. Selecting a flat plate as the basic configuration allows comparison with this existing data, and ease in setting up confirming experiments. However, most actual superconducting projects use
curved wires. The program can be readily modified to solve the problem of a curved wire. Rather than use actual superconducting material as the material that is coated, this simulation assumes that the coating is applied to copper. All superconductors of interest are imbedded in a copper or bronze (copper alloy) matrix. Since copper is more thoroughly thermally characterized than are superconductors or bronze, copper is used to simulate the superconductor.

At the interface between the material and the liquid helium, there is an additional boundary resistance commonly called the kapitza resistance. At first it was thought that this resistance only appeared between materials and superfluid helium. However, experimental evidence has shown that this boundary thermal resistance also is present for non-superfluid helium. Data between materials and normal, i.e. nonsuperfluid, helium is extremely limited, but indicates that the resistance is of the same order of magnitude for both normal and superfluid helium. To account for the kapitza resistance at the liquid helium--solid interface, the algebraic form

$$R_K = C/(T^3(1+1.5\Delta T/T+(\Delta T/T)^2 + .25(\Delta T/T)^3))$$

(1)

was used. This is derived from the basic theory of this effect while allowing the assumption that $\Delta T$ could become large relative to $T$. There is a sound physical theory behind this form. The basic theory assumes that the thermal resistance at the liquid-solid interface arises from scattering due to the difference in phonon frequencies in the material and the helium, i.e. the acoustic mismatch. This theory predicts that the principle temperature dependence of the resistance should be $1/T^3$ which is approximately experimentally correct. It furthermore predicts that temperature dependence is independent of the surface condition of the material but that the actual number depends on surface conditions which is in agreement with observations. While the theory must be modified to achieve detailed agreement with experiment,
it is obviously basically correct. Since nearly the same conditions of acoustic mismatch hold in normal helium as in superfluid, the same basic formula should be correct. In superfluid helium the kapitza resistance dominates heat transfer so it is easily measurable. For normal helium the kapitza resistance is small compared with other effects, thus it is difficult to measure. Assuming the same form of the equation and coefficients for a copper-normal helium boundary as for copper-superfluid helium boundary seems to be the most reasonable procedure. The acoustic properties of the Laketite have never been measured, as they are new materials. Other dielectrics show a $T^{-2}$ dependence for kapitza resistance.\textsuperscript{10} However, the other thermal properties of the Laketites are similar to lead, a metallic substance with a kapitza resistance similar to copper, so it was assumed that the Laketites have the same boundary resistance as copper. Nevertheless, there is the possibility that this assumption may not be correct and may affect the results of this program. If the actual boundary resistance were higher for the Laketites than the program predicts, the inner surface of the coating under consideration would be hotter than predicted if everything else remains the same. This would tend to make both the superconductor and the coating become hotter for the same amount of heat energy input. Conversely if the actual kapitza resistance were lower than the predicted, the superconductor would be cooler than predicted. However, there is some evidence\textsuperscript{11} that hotter surface temperature due to thin insulating coating e.g. ice, result in improved heat transfer; because the insulator promotes the local superheating necessary for nucleate boiling. Simulating this problem by varying the constant in equation 7 by a factor of six changes some numbers, but does not alter the general form of the results or any of the basic conclusions.

Experimental work has shown that there is a kapitza resistance even between two solids such as epoxy and metal.\textsuperscript{12} That effect was visible only
at very low temperatures—\( \sim 0.1 \text{K} \), well below the temperature of boiling helium—
\( \sim 4.2 \text{K} \). This solid to solid contact resistance should be negligible in the
higher temperature range, 4 to 10K, of this report unless the acoustic pro-
PERTIES of Laketites differ substantially from that of copper. Thus any such
effect was omitted.

One should note that the technique for applying the SCI compounds that
seems most promising uses a composite glass-Laketite coating. In that mixture
the thermal conductivity appears to be dominated by the kapitza resistance
between the glass and the Laketite coating,\(^6\) indicating the possible complexity
of the heat transfer situation. In the absence of experimental evidence,
the simplest assumption—minimal boundary resistance—0 in the program—between
the superconductor and the coating seems to be the most reasonable approach.
However the program is so structured that any copper-Laketite thermal resis-
tance could be inserted in the computer simulation by adding a resistor
between the copper and the Laketites.

The addition of heat to the wire-coating system is simulated by an ideal-
no resistance-current source distributed throughout the length of the sample.
As explained in the introduction, pulsed operation tends to produce eddy
current heating throughout the length of the superconducting wire. Dielectric
coatings would not have eddy current heating. Neither is the helium subject
to heating by changing magnetic fields, so all of the heat source is in the
superconductor. Eddy current heating for relatively thin matrixes follows
the changing magnetic fields with a time constant of less than a microsecond.
Thus for times longer than a few microseconds—a square wave pulse of duration
equal to the total switching time of the magnet is a good approximation—so
it was used. A square wave pulse assumes linear variation of the magnetic
field during switching which is the usual approximation. However the program
is constructed so that any form of heating pulse can be used.

To complete the equivalent circuit for the solid, three small capacitors
were inserted, one on the inside edge of the outermost surface of the copper, one on the inside edge of the outermost surface of the coating and one at the innermost edge of the liquid helium. These served as near zero mass thermometers, at those three points since the program would automatically compute the voltage across these capacitors--equivalent to temperature--at all times. As a test of their effect the value of these capacitors was increased by a factor of ten, without affecting the numerical results, at the one tenth of one percent level, thus allowing the conclusion that the capacitances of these capacitors had no effect on the results.

The heat transfer to the liquid helium was simulated as two ideal negative (removal of heat) current sources. Only a thin boundary layer of helium would be heated, but the amount of heat transferred into and out of the boundary layer was varied with different conditions. The value of the heat transfer from the solid into the boundary layer was controlled by a lengthy subprogram which calculated the appropriate amount of heat transfer based on the temperature of the helium, the amount of energy in the boundary layer of the helium, the rate of flow of energy into the helium, the rate of flow of heat from the boundary layer to the main helium bath, the elapsed time, and the history of the heat transfer.

Since this is a transient heat transfer problem, it is nearly impossible to add enough energy to heat the main helium bath significantly. Even at a phenomenal 50W/cm$^2$ rate for a 100 cm$^2$ surface, a .001 second pulse only adds .05 J/cm$^2$ to the bath, sufficient to raise one liter of helium only .002K.

The main bath was simulated by a constant ideal voltage source, i.e. it could absorb any amount of heat without changing temperature. This temperature was fixed at 4.0K rather than the normal boiling temperature of 4.21K to approximate operation under slightly reduced pressure as would be likely in an aircraft or spaceship. The topological properties of the SCEPTRE code requires a resistor to shunt a voltage source. The value of
this "dummy" resistor was varied by two orders of magnitude in both directions without detectably affecting the results of the computer program, so its presence seems unimportant. The complete circuit diagram is shown in Figure 1.

The most complex problem in this project is determining the amount of heat transfer from the solid into the boundary layer of the helium. In steady-state processes there are four distinct modes of heat transfer—conduction and/or convection, nucleate boiling, transition boiling, and film boiling. Transient conditions further modify some of these modes.

The conduction-convection region is the most heavily modified by transient conditions. In the steady state low heat flow is dominated by convection. Convective heat transfer is difficult to simulate since it strongly depends on the geometry of the situation; e.g. the orientation of the heater, the size of the channels, the depth of the fluid, etc. However, for short time periods conductive effects dominate. Convection is driven by density differences in a gravitational field. A maximum for the acceleration of a parcel of fluid next to the solid is of the order of \( \frac{0.01g}{g} \) where \( g \) is the acceleration of gravity. Assuming continuing constant acceleration of \( \frac{0.01g}{g} \), the maximum distance that hot fluid could travel in \( 0.001 \) seconds would be \( 4.9 \times 10^{-6} \) cm. This maximum distance of travel also represents the greatest reasonable depth of fluid that can be influenced by convective heat transfer. The maximum distance that is influenced by conductive heat transfer is approximately given by \( \alpha t \) where \( \alpha \) is the thermal diffusivity \( K/\rho c_p \). For helium at 4K, \( \alpha = 3.2 \times 10^{-2} \) cm\(^2\)/sec. So for a time of \( 0.001 \) seconds the conductive heat penetration depth is about \( 5.66 \times 10^{-4} \) cm, much farther than the convective distance. At about \( 0.05 \) second both distances are of the same order of magnitude. On the other hand for very short times the distance that heat penetrates begins to approach the depth of the surface roughness which is usually of the order
FIGURE 1

Electrical circuit model of liquid helium in contact with a heated surface.

Figure 1b. Detail model of Coating and Superconductor.
of $10^{-5}$ cm which invalidates an analysis based on flat plates and flat boundary layers. Thus between some very short time and about $10^{-2}$ seconds, conductive heat transfer is the dominant mode and convection can be ignored.

Before considering the exact mechanism for conductive heat transfer, the thickness of the boundary layer must be determined. The lower limit of this size can be set two ways. Even the highly polished surfaces have irregularities of the order of $10^{-5}$ cm, i.e. 10000Å. Unless the boundary layer is substantially thicker than the irregularities, a model of a flat boundary layer on a flat surface is meaningless. Also, it is desirable to have the same boundary layer serve for nucleate boiling as well as conduction. The minimum diameter for bubbles in boiling in Helium is $10^{-5}$ to $10^{-6}$ cm depending on the theory used. The maximum size of the boundary layer is set by thermal diffusion constraints. Because $\alpha$ is small for helium, the maximum thermal diffusion ranges from about $5 \times 10^{-4}$ cm for one millisecond to about $1 \times 10^{-3}$ cm for 10 milliseconds. The Japanese work showing that grooves of less than 1mm width aid heat transfer can easily be understood as an increase in effective area if the boundary layer is substantially less than one half the groove width. If the boundary layer is thicker than one half the groove width, the boundary layers from opposite sides of the groove touch and there is no increase in effective area. Even a close approach of the boundary layers would inhibit heat transfer in the same manner that narrow channels inhibit boiling heat transfer. The evidence argues for a maximum depth of the boundary layer of $10^{-3}$ cm. Splitting the difference yields a depth of $2 \times 10^{-4}$ cm for the boundary layer. As a test of the sensitivity of the program to this parameter, the depth of the boundary layer was both increased and decreased from this value by a factor of 5 without affecting the results.

Having determined the depth of the boundary layer, the procedure was to construct a temperature profile for the boundary layer. Heat transfer out
of the boundary layer was given by

\[ \frac{Q}{A} - K \frac{dT}{dx} \]  

(2)

with the derivative being evaluated at 90% of the depth of the boundary layer to avoid mathematical singularities at the exact boundary. A temperature profile of the form

\[ \Delta T'(x) = \Delta T_{\text{total}} (1-x/D)^y \]  

(3)

where \( \Delta T'(x) \) is the rise in temperature above the original bath at a distance \( x \) away from the solid surface, \( \Delta T_{\text{total}} \) is the total difference in temperature between the solid and the bath, \( D \) is the thickness of the boundary layer and \( y \) is an exponent to be determined. \( y \) is determined by requiring that the

\[ \int_0^D \rho C_p \Delta T'(x) \, dx = E \]  

(4)

where \( E \) is the total thermal energy present in the boundary layer, \( \rho \) is the density and \( C_p \) the specific heat of the helium in the boundary layer. Since \( E \) is known (the computer is very efficient at keeping track of all heat inputs and outflows from the boundary layer and maintaining a running total of the energy contained in the boundary layer), substituting Equation 3 in Equation 4 determines the exponent \( y \). With \( y \) determined, Equation 3 can be used to determine \( T(x) \). Differentiating \( T(x) \) and using Equation 2 determines the heat flux out of the boundary layer. Clearly heat flux out of the boundary layer is conductive for short periods of time since convective motion can't reach the outer edge of the boundary layer. In steady state both convection and conduction form linear temperature profiles which is the limiting case for this procedure. The details of heat transfer inside the boundary layer are not known. The use of energy consideration to determine heat flow in the absence of detailed knowledge is customary procedure. A polynomial curve is the simplest equation that reduces to the proper steady state limit. As Figure 2 shows, the temperature profiles generated by this
Fig. 2. Temperature Profile in the Thermal Boundary Layer for Conduction Heat Transfer.
method yield maximal heat flow into the boundary layer and very little outflow of heat for short periods of time and approach steady state for longer periods of time. Thus this method of calculating heat flux out of the boundary layer seems physically reasonable. Unfortunately this method can not handle the situation when the surface is cooling, i.e. recovery.

As the temperature of the solid surface increases and the energy contained in the boundary layer increases, eventually nucleate boiling commences. The traditional view of the onset of nucleate boiling is that gas trapped in minute cavities on the surface expands to form bubbles, a process called heterogeneous nucleation. The onset of nucleation requires that temperature TL for the liquid adjacent to the wall be greater than the boiling temperature of the fluid T_{sat}. Most sources use T_{wall}, but that ignores the kapitza resistance. In addition, if the bubble is to expand and grow there must be sufficient energy present for some of the liquid around the bubble to vaporize. Since a single bubble does not affect the heat transfer properties by a measurable amount, exact criteria for the onset of nucleate boiling are hard to develop. Such criteria are necessary for a computer, which has trouble handling a situation where the mode of computation is an indefinite mixture of two situations.

For heterogeneous nucleation a criteria for the beginning of nucleate boiling can be derived from the equation by Han & Griffith for the critical radius of nucleation sites.

$$r_c = \frac{(T_L - T_{sat})}{3(T_L - T_{bath})} \left( \frac{1}{1+\frac{12(T_L - T_{bath})T_{sat} \sigma}{(T_L - T_{sat})^2}} \right)$$  (5)

The critical radii first became real when the term under the square root equals or exceeds zero. For the case where T_{bath} = T_{sat} the beginning of nucleate boiling becomes
\[(T_L - T_{bath}) = \frac{12 T_{bath} \sigma}{\rho_v h_{fg}} \quad (6)\]

This can be related to energy by multiplying by \(C_p\), giving

\[E_{NB} = \frac{12\rho_L C_p T_{bath} \sigma}{\rho_v h_{fg}} \quad (7)\]

For saturated liquid helium at 4K the energy for the beginning of nucleate boiling is \(E_{BNB} = 1.04 \times 10^{-2} \text{ Jm}^{-2}\). To allow for the small range of radii necessary to actually produce boiling, a value of \(2 \times 10^{-2} \text{ Jm}^{-2}\) was used for \(E_{BNB}\) in this program. This value gave results that agreed very well with experimental data. The computer added up the total energy in the boundary layer; when this figure exceeded \(E_{BNB}\) nucleate boiling was assumed to occur. However, experimentally nucleate boiling does not begin with a sharp transition, but rather with a mixture of both nucleate boiling and conductive heat transfer. The smooth joining of the conductive and the nucleate regime was achieved as follows. For energies in the boundary layer of less than four times the minimum energy required for the initiation of nucleate boiling, the heat transfer is calculated as a linear combination of conductive and nucleate boiling heat transfer.

Very recent experimental work\(^{16,17}\) has suggested that heterogeneous nucleation is incorrect for helium because of the absence of trapped gases. If heterogeneous nucleation does not occur, nucleate boiling still exists commencing with a process called homogenous nucleation which occurs when local temperature and energy fluctuations create bubbles in the liquid. The onset of nucleate boiling is very sharp and occurs solely as a function of temperature for a liquid under fixed pressure. The difference in the conditions for onset of nucleate boiling between the two theories is small (barely detectable experimentally). Nevertheless future development of this computer program should
include a changing of the criteria for the onset of nucleate boiling to the homogenous nucleation case since both the physical arguments and the experimental evidence for the existence homogenous nucleation in helium appear compelling.

Once nucleate boiling becomes established it dominates heat transfer because the relation between $q$ and $\Delta T$ becomes much greater than the linear relationship for conductive heat transfer. Nucleate boiling once well established is relatively independent of the material or the condition of the surface of the solid. Correlations such as the one Kutateladze developed based solely on the hydrodynamic properties of the fluid agree fairly well with a wide range of experimental data. Using the available experimental data\textsuperscript{18} and guided by the general temperature variation of the quantities in the hydrodynamic correlations, a form

$$q = B(T_W - T_{sat})^2$$  \hspace{1cm} (8)

was used as the basic formula for heat transfer in the nucleate boiling region.

The constant is evaluated by requiring that a $q$ vs $T$ plot gives the correct results at the peak of the nucleate boiling curve. Because of the dramatic changes in heat transfer at the peak of nucleate boiling, this point is easily determined experimentally. Data for various copper systems similar in configuration to the arrangement in this program cluster around the values used in this program which are $q/A = .6 \text{ W/cm}^2$ and $T_L - T_{sat} = .5K$. The higher values reported in some work are somewhat suspect since no allowance was made for Kapitza resistance.

At the peak heat flux for nucleate boiling, the bubbles merge in an unstable fashion to form a continuous layer of gas. Because the thermal conductivity of the vapor is much lower than that of the liquid--a factor of five for helium--a large resistance to heat flux builds up. With the build-up of this resistance the amount of heat transported across the
boundary layer into the helium bath drops rapidly. If the heat flux remains constant, the temperature of the sample rises rapidly, a phenomenon called burnout. Due to the rapidity of the temperature rise, typically a few microseconds, the exact form of the heat transfer in this region is unknown. From measurements with the surface temperature carefully controlled\textsuperscript{19} it is known that the heat flux for copper would drop to about $q/A = .2W/cm^2$ at a temperature difference $T_L - T_{sat} = 6.8K$. Until recently little study of this region has been made, so that a linear fit between the peak flux and the minimum flux seems to be the best approach for the $q$ vs $\Delta T$ curve. This region, which is known as transition boiling, was modeled as a combination of nucleate and film boiling heat transfer. It was assumed that transition boiling occurred between the peak nucleate boiling flux and the establishment of film boiling. Following the theory of Kalinin\textsuperscript{28} the fraction of nucleate boiling contributing to the heat transfer is calculated from

$$f = (1 - \Delta T^*)$$

(9)

where

$$\Delta T^* = ((EBL(T_L - T_{sat})^3)^{25} - (C_{T1})^{25} / ((C_{T2})^{25} - (C_{T1})^{25})$$

(10)

where $C_{T1}$ and $C_{T2}$ are constants. The data of Steward\textsuperscript{21} for carbon surfaces were used to determine $C_{T1} = 1.$ and $C_{T2} = 15 \times 10^4$ J K$^3$/m$^2$ respectively for the incipience and the completion of transition boiling.

To avoid the problems of the transition region, several attempts have been made to define a criterion for transition from nucleate boiling to film boiling during transient heat transfer.\textsuperscript{22-27} None of the approaches fit the data very well.

The most promising approach seems to focus on the energy contained in the boundary layer. For short, transient heat pulses, less than .01 seconds in duration, the bubbles from nucleate boiling do not have sufficient time to escape.
Since the energy cannot escape from the boundary layer, a simple criterion for the start of film boiling is that the energy in the boundary layer be equal to or exceed the amount of energy required to vaporize all of the liquid in the boundary layer. It produced a formula \( E T^m = \text{constant} \) with \( m \) between 2 and 2.5 depending on the exact assumption of the behavior of the boundary layer. This approach has limited agreement with the available experimental data but not sufficient agreement to use in the computer program. Instead an empirical formula

\[
E_{BL} (T_L - T_{bath})^3 = \text{constant}
\]  

(11)

with the constant being \( 5 \times 10^8 \text{ Jm}^{-2} \) was used, which agrees somewhat better with the experimental data.

Steady state film boiling heat transfer is best modeled by the correlation of Breen & Westwater. This must be modified to account for transient effects as pointed out by Iwasa. To model these transient conditions the following form was used

\[
q_{FB} = (200(T_L - T_{bath})^{1.2}) \cdot (E_{FBSS} + E_{BL})/ 2E_{BL}
\]  

(12)

since energy variations are more easily calculated in the SCEPTRE format than are temperature variations.

Actual techniques of the use of the SCEPTRE code are contained in the instruction manual and the book on the code. An example of an actual SCEPTRE program is given in Appendix I and the critical Fortran subroutine that actually calculates the value of the boiling heat transfer is given in Appendix II.
IV Results

With the computer program operational, the procedure was to run programs using different thicknesses of coatings, different compositions of coatings, different levels of heating and different times. Based on the manufacturer's report on the Laketites, there are three broad classes of these materials, SC-1, SC-2, SC-3 with several different variants in each class. The most promising compositions are SC-1C, SC-2B, and SC-3A so these are the ones that were studied. For comparison purposes the same computer runs were made on bare copper (no coating) and on copper that was coated with GE 7031 varnish. GE 7031 is typical of the usual dielectric materials used to insulate wires at low temperatures. Because of its ease of application, it is the most popular such material and therefore has been the subject of many investigations to accurately determine its low temperature properties. In the case of the SC-i materials, GE 7031 is mixed with the coating material in the application process. Thus the logical choice as the comparison material representing typical existing dielectric coatings at low temperatures is GE 7031. The superconductor is represented by a slab of copper .005" (5mils) = .0125cm thick. This is a typical thickness for the copper or bronze matrix around the superconducting filaments in the normal construction of filamentary superconducting cable. Coating thicknesses of 1, 2, 4, 10, and 20 mils were used (.0025, .005, .01, .025, .05cm). Mechanical constraints in the process of applying coatings to wires limit the coating thickness to this range of values. Superconductors might face a wide variety of heating levels from gentle heating for slow switching and low magnetic fields to severe heating for rapid switching and high magnetic fields. This was simulated by varying the input power level of heating in the copper. The heating levels chosen were 0.05, 0.1, 0.2, 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 10.0, 20.0 and 50.0 W/cm$^2$. The clustering of values in the 1-5 W/cm$^2$ range reflected
crude calculations that this was the actual heating range for some specific Air Force projects that are in the design stage. All values are in W/cm², the usual scientific convention for removing the effect of varying areas in different experimental arrangements. All heating was assumed to be a constant heating rate, with zero turn-on time, i.e. a square wave pulse with a zero turn-off time at the end of the simulation. This is a good approximation or real eddy current heating, but real experiments have a finite turn on time, usually of the order of 2 to 20 microseconds, thus a comparison of these results with experiments where the heating is less than 10⁻⁴ seconds in duration is questionable unless a correction is introduced to account for the finite turn-on time. The length of heating and the total time for the computer simulation of the behavior were identical since this program can not successfully model the problem of recovery--the cooling down after the end of a heating pulse. Three different times were used--0.001 seconds, 0.01 seconds and 0.1 seconds. For time periods substantially shorter than 1 millisecond the actual surface roughness (usually an unknown variable) must be accounted for as the approximation of a flat surface and a flat boundary layer breaks down. For times longer than 100 milliseconds, the basic assumption of a constant thickness boundary layer breaks down because of convective motion. Thus these three times span the time range that is normally meant by the term transient heat transfer.

As explained in the theory section, it was assumed that heating was uniform throughout the copper due to eddy current heating from changing magnetic fields; thus all the heat flow was assumed to be radial. The important case of localized heating, 'hot spot', where both radial and longitudinal heat flow would be present was not considered. The 'hot spot' problem requires a two dimensional solution to the heat equations which is beyond the scope of this project. However, another project which would investigate
this problem would have considerable merit, since the results of this project indicated that these materials are potentially very useful for distributed heating cases and would appear to also provide considerable protection against localized 'hot spots'. If the insulation coating could help provide thermal protection against 'hot spots', the bulk of the superconducting device could be reduced and the reliability of these devices enhanced.

The three classes of Laketites materials are primarily distinguished thermally by their different specific heats and thermal conductivity at low temperatures. The SC-1 materials have very high specific heat, in the 5 to 10K range, and thermal conductivity that is relatively low, only slightly higher than 7031 and other currently existing dielectric materials. The SC-2 materials have relatively high specific heat, though not as large as the SC-1 materials, and very high thermal conductivity, up to one third of that of metallic copper (copper is the universal choice for the highest levels of thermal conductivity at low temperatures). The SC-3 materials have extremely high thermal conductivity, approaching that of metallic copper, but have relatively low specific heat.

The results of this project clearly showed that for the distributed continuous heating case thermal conductivity is the most important parameter. Figure 3 gives an example with a superimposed plot of the change of temperature of the copper, i.e. the superconducting wire, versus time. Each coating is two mils thick, the power level is $1 \text{ W/cm}^2$ and the time is .01 seconds. The wire coated with SC-1C reaches a final rise in temperature of 9.8K, i.e. a final temperature of 13.8K, which exceeds the transition temperature of most superconductors including all of the commonly used superconductors for magnets. In addition, the temperature of the superconductor rises rapidly, reaching a value of 5K above bath temperature in 1.3 milliseconds. Five K
Fig. 3. The temperature of the superconductor (TWIRE) as a function of time for an input power of 1W/cm² for wire coated with different thicknesses of SC-1C coating.
brings the temperature to 9K, which is the absolute 0 field limit for Nb₃Sn wire. In a real magnet such a wire would go normal due to the residual magnetic field. Nb₃Sn is the highest transition temperature superconducting magnet wire, so a temperature rise of 5K is certain trouble. In contrast, wire coated with SC-2B has warmed only .98K, to a temperature of 4.98K, after the full 10 milliseconds of heating--well within the limits of operation of all commonly used superconductors. Running the program for a longer time reveals that SC-2B caused the wire to heat to 5K only after 22 milliseconds, over an order of magnitude difference from SC-1C. Clearly there is a tremendous improvement in performance for SC-2B vs SC-1C. Even at a power of 0.5 W/cm² SC-1C coated wire heats beyond the crucial 5K level in 3.8 milliseconds. Even at power levels as low as 0.1 W/cm² the total temperature rise for 10 milliseconds for SC-1C is an uncomfortably high 2.7K.

The results of SC-1C and 7031 are quite similar to each other, as are the results for the grouping SC-2B, SC-3A and bare copper. SC-1C and 7031 share similar low thermal conductivities while differing sharply in specific heat. SC-2B, SC-3A and bare copper have high thermal conductivity out to the solid-liquid interface but differ substantially in specific heat. Furthermore, the temperature of the boundary layer of helium is remarkably similar (within .05K) for all coated and bare surfaces for all power levels below 2W/cm² and within 10% agreement at higher power levels. The temperature rise of wires coated by SC-1C and GE 7031 seems to be caused by large temperature drops required to transport energy across low thermal conductivity materials.

The same result, that thermal conductivity is the dominant parameter, is evident by comparing results with the same material but differing thicknesses of coating. The total heat capacity depends on the product of the
volumetric specific heat and the volume of the material. The volume of
the coating is directly proportional to the thickness of the coating. As
the thickness of the coating is varied from 1 to 20 mils, the volume and the
total heat capacity change by a factor of twenty. In contrast the effect
of high thermal conductivity is lessened by additional thickness since the
temperature change across a material is directly proportional to the thick-
ness of the material. Thus, a thicker material will 'soak up' more heat
as it becomes thicker, causing the superconductor to have a smaller tempera-
ture rise; but the increased temperature difference required for thermal
conduction due to the increased thickness causes a larger temperature rise
of the superconductor. If specific heat is the dominant factor, the super-
conductor will become much cooler for a given level as the thickness is
increased.

As is shown in the superimposed plot for differing thickness of SC-1C,
this high specific heat material does have a drop in superconductor tempera-
ture and a changing rate of heating with increasing thickness. A further
example, is that for a power level of 1W/cm² for a 1 ms pulse, the maximum
temperature rise of the superconductor falls from 5.0K for a 1 mil coating
of SC-1C to 2.55K for a 20 mil coating. However, this is a small effect
for a 20 times increase in volume, particularly since under the same condition
SC-2B--a medium specific heat coating--causes the temperature rise of the
superconductor to decrease from 0.74K for a 1 mil coating to 0.68K for a
20 mil coating and SC-3A--a lower specific heat material--shows an increase
of only .01K in the temperature rise of the superconductor from 0.74K to 0.75K.
If specific heat were the key variable, a 20 mil coating of an SC-1 material
should give improved performance over a 1 mil coating of SC-2 or SC-3 material.
This does not happen. The high thermal conductivity materials outperform
the ones with low thermal conductivity at all thicknesses. Nevertheless,
Fig. 4. The temperature of the superconductor as a function of time for an input power of 1W/cm² for wire coated with different thicknesses of SC-1C coating.
SC-I materials in thick coatings are definitely superior to 7031 whose performance worsens with greater thickness due to its very low specific heat and thermal conductivity. These results suggest that if SC-1C is the material of choice for further development that the thickest possible coatings be used to best utilize its high specific heat.

The power level of the heating might affect the choice of coating material. Table I presents the maximum temperature rise for the superconductor for 2 mil coating for 1 ms pulse for SC-1C, SC-2B, SC-3A bare copper, and 7031.

TABLE I

Maximum temperature rise of the superconductor versus heater power for a 1 ms pulse and a coating thickness of 2 mil.

<table>
<thead>
<tr>
<th>Power W/cm²</th>
<th>SC-1C K</th>
<th>SC-2B K</th>
<th>SC-3A K</th>
<th>Bare Cu K</th>
<th>7031 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>.05</td>
<td>0.8</td>
<td>.30</td>
<td>.32</td>
<td>.32</td>
<td>.68</td>
</tr>
<tr>
<td>1</td>
<td>1.45</td>
<td>.35</td>
<td>.37</td>
<td>.36</td>
<td>1.10</td>
</tr>
<tr>
<td>.2</td>
<td>2.25</td>
<td>.42</td>
<td>.42</td>
<td>.42</td>
<td>1.85</td>
</tr>
<tr>
<td>.5</td>
<td>3.45</td>
<td>.52</td>
<td>.52</td>
<td>.52</td>
<td>3.8</td>
</tr>
<tr>
<td>1</td>
<td>4.30</td>
<td>.72</td>
<td>.76</td>
<td>.74</td>
<td>6.0</td>
</tr>
<tr>
<td>2</td>
<td>7.1</td>
<td>1.30</td>
<td>1.35</td>
<td>1.35</td>
<td>8.8</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>2.10</td>
<td>2.6</td>
<td>3.0</td>
<td>11.0</td>
</tr>
<tr>
<td>4</td>
<td>11.0</td>
<td>3.6</td>
<td>6.2</td>
<td>9.9</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>12.0</td>
<td>5.0</td>
<td>8.0</td>
<td>11.0</td>
<td>14.0</td>
</tr>
<tr>
<td>10</td>
<td>17.0</td>
<td>9.9</td>
<td>11.5</td>
<td>15.5</td>
<td>18.0</td>
</tr>
<tr>
<td>20</td>
<td>22.5</td>
<td>14.0</td>
<td>15.1</td>
<td>20.5</td>
<td>24.0</td>
</tr>
<tr>
<td>50</td>
<td>31</td>
<td>21.0</td>
<td>22</td>
<td>29</td>
<td>32.0</td>
</tr>
</tbody>
</table>

Clearly, at all power levels the high thermal conductivity materials SC-2B, SC-3A outperform the low thermal conductivity ones, by having a lower temperature rise. The greater specific heat of SC-1C begins to improve its performance relative to 7031 at higher power levels, but otherwise specific heat seems to have little effect on the temperature rise. The results for all
program assumes that the solid-helium interface is being substantially heated and has difficulties when no energy is reaching the interface. Since \( \alpha \) is a key in the treatment of isolated hot spots, it would appear that the large \( \alpha \) materials SC-2B and SC-3A would also give increased protection against localized heating.

Time could also be a factor. Table III shows the maximum temperature rise for 2 mil thick coating at a power level of 0.5 W/cm\(^2\) for different times for each material.

<table>
<thead>
<tr>
<th>Time ( 1 \times 10^{-3} ) sec</th>
<th>SC-1C</th>
<th>SC-2B</th>
<th>SC-3A</th>
<th>7031</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.45</td>
<td>.52</td>
<td>.52</td>
<td>3.8</td>
</tr>
<tr>
<td>10</td>
<td>6.5</td>
<td>.52</td>
<td>.52</td>
<td>4.0</td>
</tr>
<tr>
<td>100</td>
<td>6.5</td>
<td>.52</td>
<td>N.A.</td>
<td>4.1</td>
</tr>
</tbody>
</table>

Note: at 100ms the SCEPTRE program for SC-3A had excessive computation time and failed to reach this power level before exceeding the pre-set computer time limits.

The relative performance of the coating seems to be unaffected by the length of the heating pulse. Obviously for a power level of 0.5W/cm\(^2\) the SC-2B and SC-3A are acceptable, SC-1C and 7031 are not.

The computer program generated far more data—over 10,000 pages of printout and graphs—than can be presented in this report. All of the computer data, both numbers and graphs, has been stored at Saginaw Valley State College and is available to any interested party. The basic SCEPTRE programs for each coating and the Fortran subroutines have been stored on magnetic tape at the Michigan State University Computer center. The basic SCEPTRE language tapes are also stored there. In addition, all of the programming information is stored at Wright-Patterson Air Force Base. These programs can be reactivated on very short notice should new materials be developed or materials other than
other coating thicknesses are essentially the same. Thicker coatings usually lead to lower temperature rises but the high thermal conductivity materials have a much lower maximum temperature rise for all power levels.

Since this is a transient process one might expect that the correct parameter for dealing with it is the thermal diffusivity \( \alpha = \frac{K}{\rho c_p} \), the values of which are given in Table II for a temperature of 5K--roughly the mid-point of the heating process.

**TABLE II**

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha ) (cm(^2)/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SC-1C</td>
<td>.0132</td>
</tr>
<tr>
<td>SC-2B</td>
<td>57.</td>
</tr>
<tr>
<td>SC-3A</td>
<td>974.</td>
</tr>
<tr>
<td>GE7031</td>
<td>.032</td>
</tr>
</tbody>
</table>

The better behavior is associated with the materials with larger thermal diffusivity, as expected. But SC-3A and SC-2B have nearly identical thermal response despite large differences in thermal diffusivity. From the definition of thermal diffusivity it is obvious that the most desirable property is large thermal conductivity and low specific heat. Judging from the computer results this is indeed the correct combination.

Another indication of the importance of the thermal diffusivity is the unusual results for very thick coatings of SC-1C. The surface where the liquid helium meets the coating develops small temperature oscillations, including negative temperature rises--i.e. temperature falling below the bath temperature--for thick (10 & 20 mil) SC-1C coatings at low power levels. The oscillations are small (.001K) and appear to be caused by the inability of the heat pulse to reach the surface due to the low value of \( \alpha \) for SC-1C. This
need to be tested for comparison purposes. Further modifications to the program could readily be made if new experimental data or new theoretical understanding led to changes in heat transfer calculations for liquid helium.
V Conclusions and recommendations

The principal conclusion from this computer simulation is that Laketites show promise for use as dielectric coatings at low temperatures. All of the compounds show promise for improving the thermal response of heated superconductors coated with these materials relative to the performance of superconductors insulated with GE 7031 and other similar epoxies. In particular the SC-3 and SC-2 compounds with their large thermal conductivities should be vigorously developed as they show the greatest promise in keeping superconductors exposed to large transient distributed heating, such as eddy current heating due to changing magnetic fields, as cool as possible. While not tested in this project it would seem likely that the SC-3 and SC-2 materials would also substantially improve the ability of superconductors to withstand localized heating such as that caused by mechanical motion. Superconducting devices made with Laketite coated wires would have greater reliability since they could withstand larger heat pulses for a longer time without heating above the critical transition temperature and 'going normal'. The additional thermal stability might also permit a reduction of mass of the superconducting device by allowing some of the copper and/or bronze usually used for this purpose to be removed. Since the Laketites are less dense than copper, the replacement of copper by a Laketite reduces the mass.

These computer results clearly showed that the most important thermal property for a coating to have is high thermal conductivity. The SC-2 and SC-3 materials have the greatest promise among the Laketite materials. One can also conclude from these results, that thickness is not a very important parameter for thermal properties. Thicker coatings are slightly advantageous because they increase the total specific heat and the total enthalpy. On
the other hand, increasing thickness decreases the utility of high thermal conductivity. The variation of the superconductor temperature with coating thickness is so slow that the thickness should be set by mechanical properties and not by thermal properties.

The most immediate need for research work is for the experimental confirmation of the results of this computer program. Computer simulations should not be regarded as authorities until validated with experimental evidence. Since the computer program in this report assumes that the outer edge of the superconductor is copper, experimental tests with coated copper would be the best method of assessing the validity of the results contained in this report. The parameters for specific heat, thermal conductivity, density etc, of the Laketites were derived from the report of the manufacturers of the Laketites. This data should be verified by experiments run by people not directly connected with the manufacturer of these products to avoid any appearance of inflated specifications.

Because of the promising results of this computer simulation, I recommend a vigorous research and development project for the Laketite materials. Since the thermal characteristics of the Laketites are based on their ferro-electric nature, the thermal properties of these materials should be tested in strong electric fields. The possibility exists that the specific heat and thermal properties might be repressed by electric fields. Because superconductors are used in electrical apparatus, the presence of strong electric fields is always a distinct possibility. These electric tests should include both steady electric fields and pulsed electric fields such as might be present in combat areas due to Electromagnetic Pulses (EMP). Similarly, the manufacturers claims that Laketites are dielectrics and can be used in insulation. This claim should be verified and quantified by experimental tests of the electrical properties of these materials including resistivity, dielectric
constant, A.C. loss parameters, and breakdown resistance. Not only are electrical parameters important, but successful applications of coatings require knowledge of mechanical properties at low temperatures such as thermal expansion, brittleness, strength, bending radius, fracture strength, adhesion to the surface that is coated, etc. This report has assumed that the surface of the coating is mechanically perfect, with no cracks or voids and unaffected by magnetic or electric fields that are producing the heat. The validity of such assumptions need to be tested by actual experimental tests of the materials at low temperatures. In brief all relevant properties of the Laketites should be tested prior to full scale development.

I recommend further development of the computer programs. Two particular problems need to be addressed. One, the process of cooling or 'recovery' after heating needs to be simulated. This will require major modifications of this program, but is particularly important for the evaluation of pulsed systems, where the cooling time is as important as the response to heating. Two, the program needs to be extended to the two dimensional case of localized 'hot spots' since this is a more common source of heating of superconductors than is the case of distributed heating which was covered in this report.
ACKNOWLEDGEMENTS

The author wishes to thank Dr. Charles Oberly at WPAFB for suggesting and guiding this project. A special tribute should be given to Lt. D. Scott Holmes of WPAFB who served as my collaborator on this project. He is listed as co-author on all of the scientific papers resulting from this project in recognition of his performance of a large fraction of the work. In particular, he is largely responsible for the first working versions of both the SCEPTRE program and the Fortran subroutine, FJ. Without his assistance it is doubtful that this project could have been completed in a timely fashion. Valued contributions to the actual operation of the computer programs were given by Mr. Kermit Kranz of the SVSC computer center. The graphics were done by Mr. Wayne Mason an engineering student at SVSC. Without the typing assistance of Carol Shuler and the proofreading of Anne Menard the final copy could not have been produced.
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APPENDIX I
A Typical SCEPTRE Program

MODEL DESCRIPTION
MODEL LIQUID HELIUM (TEMP) (A-G)
A flat surface pool boiling in saturated liquid helium at 4K (.804 ATM).
The program contains peak nucleate boiling and minimum film boiling
information specific to the surface in use. More details and instructions
about the program can be found in the Fortran function FJ listed in
Appendix II of the report. The units are: V(=TEMP IN K), A(=J/S), SEC,
F(=J/K), OHMS(=K/WATT)

ELEMENTS
CL,A-G=1.E-6
JS,A-B=FJ(VCL, VCBL, PA, PTHP, PQP, PTHM, PQM, 1.)
CBL,B-G=1.E-3
JBL,B-C=FJ(VCL, VCBL, PA, PTHP, PQP, PTHM, PQM, 2.)
EO,C-G=4.0
RO,C-G=1.E4

This section sets up the basic model electric circuit for the helium,
note that the Fortran function FJ given in Appendix II is crucial.

DEFINED PARAMETERS
PA=1.
PTHP=.5
PQP=.6
PTHM=6.8
PQM=.2

PTHP and PQP describe peak nucleate boiling, respectively they are
temperature and heat flux, and PTHM and PQM similarly describe minimum
boiling point. These parameters are adjustable.

PTHL=X T(VCL-4.0)
PEBL=X E(CL*PTHL+CBL*(VCBL-4.0))
PTR=X B(PEBL*PTHL**3.)
PLTR=FL(PTR,0.)

INITIAL CONDITIONS
VCL=4.0
VCBL=4.0

CIRCUIT DESCRIPTION
This describes the wire and coating with W indicating wire and C indicating
coating. PTW is the thickness of the wire in centimeters. PTC is the
thickness of the coating in centimeters. Both parameters are adjustable
as is PKAPC which sets the strength of the kapitza resistance.

ELEMENTS
J1,1-2=Q J(T JTOT(TIME),2.)
CW1,2-1=Q CW(PTW,8.9,VCW1,PA,2.)
RW1,2-3=Q R(PTW,T KCU(VCW1),PA,1..2.)
J2,1-3=Q J(T JTOT(TIME),2.)
CW2,3-1=Q CW(PTW,8.9,VCW2,PA,2.)
RW2,3-21=Q R(PTW,T KCU(VCW2),PA,1..2.)
CC1,21-1=Q C(PTC,1..T CPC3A(VCCI),PA,3.)
RC1,21-22=Q R(PTC,T KC3A(VCCI),PA,1..3.)
APPENDIX (part 2)

CC2,22-1=Q C(PTC,1.,T CPC3A(VVC2),PA,3.)
RC2,22-23=Q R(PTC,T KC3A(VVC2),PA,1.,3.)
CC3,23-1=Q C(PTC,1.,T CPC3A(VCC3),PA,3.)
RC3,23-88=Q R(PTC,T KC3A(VCC3),PA,1.,3.)
RK,88-99=Q RK(PKAPC,PTS,PDT)
H,99-1=MODEL LIQUID HELIUM
DEFINED PARAMETERS
PA=1.
PTW=.0125
PTC=.005
PKAPC=15.
PTS=X1(1.*VCC3)
PTHW=Q TH(VCW1)
PTH=Q TH(PTS)
PDT=X2(Abs(PTS-VCLH))
PSEC=FL(TIME,0.)
PLTHS=FL(PTS,4.0)
PLEBL=FL(PEBLH,0.)
OUTPUTS
IRK(QS),PLOT(PTHS(THS))
JSH(QINBL),PLOT(PLESEC(LOGSEC))
JBLH(QOUTBL),PLOT(PLESEC(LOGSEC))
PTHW(TWIRE)
PTH(S(THS)
PTHLH(THL)
PTHS(THS),PLOT(PSEC(LOGSEC))
PEBLH(EINBL)
JSH(QINBL),PLOT(PTHLH(THL))
PLEBL(LOGEBL),PLOT(PSEC(LOGSEC))
PLTHS(LOGTHS),PLOT(PSEC(LOGSEC))
PLTRH(LOGTR),PLOT(PSEC(LOGSEC))
PSEC(LOGSEC)
INITIAL CONDITIONS
VCW1=4.0
VCW2=4.0
VCC1=4.0
VCC2=4.0
VCC3=4.0
FUNCTIONS
EQUATION J(I,N)=(I/N)
EQUATION R(D,K,A,GFCTR,NE)=(D*GFCTR/(K*A*NE))
EQUATION C(D,RHO,CP,A,NE)=(D*RHO*CP*A/NE)
EQUATION CW(D,RHO,T,A,NE)=(D*RHO*A*
((1.08E-5*T)+30.6*(T/344.5)**3.))/NE)
EQUATION TH(TEMP)=(TEMP-4.0)
EQUATION RK(C,T,DT)=(C/(T**3.*(1.+1.5*(DT/T)
+(DT/T)**2.+0.25*(DT/T)**3.)))

JTOT sets the heating level and the shape of the heating pulse. It
is piecewise linear between discrete points. The tables of Cp and K
are likewise piecewise linear between discrete points.

TABLE JTOT
0,0.0,0.05,0.001,0.5
APPENDIX I (part 3)

TABLE KCU
4,2,5,5,3,8,5,10,6,12,7,15,8,7,18,10,20,10,8,25,11,8,
30,12,35,11,2,40,10,50,7,7,60,6,3,70,5,5,80,4,9,100,4,5

TABLE CPC3A
4,.0018,4.5,.0027,5,.0039,6,.0076,7,.0137,8,.023,9,.036,
10,.055,12,.107,15,.216,20,.448,30,1.35,40,3.2

TABLE KC3A
4,1.2,5,1.4,6,1.3,7,1,8,8,9,6,10,5,15,2,
20,.125,.1

RUN CONTROLS
STOP TIME=.1
INTEGRATION ROUTINE=IMPLICIT
TERMIRATE IF(PTHS.GT.50.)

The reruns allow the program to be performed again with a single
variable changed.

RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,1,.001,.1
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,2,.001,.2
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,5,.001,.5
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,1,.001,.1
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,2,.001,2
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,3,.001,3
RERUN DESCRIPTION
FUNCTIONS
TABLE JTOT
0,0,0,4,.001,4
END
APPENDIX II

Fortran Program for Function FJ

FUNCTION FJ(TL,TBL,AREA,THLPNBS,QPNBSS
1,THLMFBS,QMFBSS,JNUM)

C

A DEPENDENT CURRENT SOURCE MODEL FOR HEAT TRANSFER
BETWEEN A SURFACE AND A LIQUID HELIUM BOUNDARY LAYER
TAKES INTO ACCOUNT THE SURFACE TEMP, CONDITION OF
THE BOUNDARY LAYER, VARIOUS MODES OF HEAT TRANSFER,
TIME HISTORY, AND THE KAPITZA RESISTANCE.

DEFINITION OF SOME TERMS (THE VARIABLE NAMES AS
THEY APPEAR IN THE SCEPTRE PROGRAM ARE ENCLOSED
IN <> BRACKETS):

JNUM = A NUMBER DESIGNATING THE DEPENDENT CURRENT
SOURCE USING THE SUBPROGRAM - MAKES A
DIFFERENCE IN THE HEAT FLUX <JS FOR JNUM=1.
AND JBL FOR JNUM=2.>

TL = TEMPERATURE OF THE LIQUID ADJACENT TO THE
SURFACE. <VCL>

TBL = EFFECTIVE AVERAGE TEMPERATURE OF THE BOUNDARY
LAYER. EQUALS THE VOLTAGE ON CAPACITOR CBL
(USE TO CALCULATE ENERGY IN BOUNDARY LAYER). <VCBL>

CS = A SMALL CAPACITANCE IN THE SURFACE DIRECTLY
ADJACENT TO THE LIQUID. USED TO GIVE THE
TEMPERATURE DIRECTLY AT THE SURFACE.

CL = A SMALL CAPACITANCE IN THE LIQUID DIRECTLY
ADJACENT TO THE SURFACE. USED TO GIVE THE
TEMPERATURE OF THE LIQUID AT THIS POINT. <VCL>

CBL = HEAT CAPACITY OF THE BOUNDARY LAYER. DEFINED
SUCH THAT EBLPUA=EPNBSS WHEN=THPNBSS IN
STEADY STATE. <CBL>

AREA = SURFACE AREA OF THE OBJECT (OFTEN TAKEN AS
UNIT AREA). <PA>

DLTAMAX = THICKNESS OF THE FULLY DEVELOPED THERMAL
BOUNDARY LAYER FOR CONDUCTION AND CONVECTION
HEAT TRANSFER.

THPNBSS = STEADY STATE THETA-SURFACE AT THE PNB POINT,
I.E. THE WALL TEMPERATURE MINUS THE BATH TEMPERATURE AT PEAK
NUCLEATE BOILING HEAT FLUX. <PTHP>

QPNBSS = STEADY STATE HEAT FLUX AT THE PNB POINT. <PQP>

THLMFBS = STEADY STATE THETA-SURFACE AT THE MINIMUM FILM BOILING
POINT. <PTHM>
APPENDIX II (part 2)

C QMFBSS = STEADY STATE HEAT FLUX AT THE MINIMUM FILM BOILING POINT. <PQM>
C EBNB = THE VALUE OF DELTA * THL AT WHICH NUCLEATE BOILING BEGINS - DIMENSIONS OF CM*K - THIS NUMBER IS FROM NBS DATA BUT IS CLOSE TO THE VALUE PREDICTED BY THEORY (3E-5). <PBNB>
C TR1 = THE VALUE OF (EBLPUA*THL**3) AT WHICH TRANSITION BOILING BEGINS - THE THL TERM SEEMS TO COME FROM THE TEMPERATURE DEPENDENCES OF THE DENSITY, HEAT CAPACITY, AND SURFACE TENSION IN THE HELIUM. THE NUMBER FOR TR1 WAS DETERMINED FROM NBS DATA.
C TR2 = THE VALUE OF (EBLPUA*THL**3) AT WHICH FILM BOILING BEGINS - CAME FROM LOOKING AT THE TEMPERATURE DEPENDENCE OF THE MFB POINT AND TRYING TO FIND AN UNDERLYING PRINCIPLE WITHIN THE GAS FILM STABILITY ARGUMENTS FOR THE MINIMUM FILM BOILING POINT. THE NUMBER FOR TR2 WAS GUESSED AT, USING NBS DATA AS A BASE.
C EPNBSS = THE ENERGY IN THE BOUNDARY LAYER (CAPACITOR CBL) AT WHICH PNB OCCURS IN STEADY STATE.
C EMFBSS = THE ENERGY IN THE BOUNDARY LAYER AT WHICH MFB OCCURS IN STEADY STATE HEAT TRANSFER.

TO CHANGE THIS PROGRAM

CHANGE THE DATA POINTS FROM THE QDOT/A VS. THETA SURFACE STEADY STATE HEAT TRANSFER GRAPH FOR HEAT TRANSFER FROM A PARTICULAR SURFACE TO LIQUID HELIUM TO MAKE THIS DEPENDENT CURRENT SOURCE MODEL REFLECT THE PROPERTIES OF THE SURFACE.

>NOTE: THL REPRESENTS THE LIQUID TEMPERATURE AT THE SURFACE, WHICH IS DIFFERENT FROM THE SURFACE TEMPERATURE DUE TO THE KAPITZA TEMPERATURE DROP AT THE SURFACE. DO NOT JUST PLUG IN THE NUMBERS OFF AN ORDINARY Q VS. THS PLOT.

PNB: Q<PQP>, THL<PTHP>

MFB: Q<PQM>, THL<PTHM>

THE HEAT TRANSFER EQUATIONS USED HERE SHOULD ALSO BE SCRUTINIZED FOR APPLICABILITY AND THE DATA CHECKED FOR ACCURACY.

THE KAPITZA RESISTANCE OF THE SURFACE IN CONTACT
APPENDIX II (part 3)

WITH LIQUID HELIUM I MUST BE CHANGED IF THE SURFACE
CHANGES. THE KAPITZA RESISTANCE IS ASSUMED TO BE
OF THE FOLLOWING FORM:

\[ RK = \frac{PKAPC}{(T^3) \times (1 - 1.5(DT/T) + (DT/T)^2 - 0.25(DT/T)^3)} \]

WHERE TI IS THE SURFACE TEMPERATURE AND DT IS THE KAPITZA
TEMPERATURE DROP BETWEEN TS AND TL. THE SURFACE IS THUS
REPRESENTED BY ONLY THE CONSTANT <PKAPC> IN THE MAIN
PROGRAM. PKAPC = 91.52 FOR GRAPHITE
AND PKAPC = 12. FOR COPPER SURFACES.

>NOTE THAT A T**(-3) RELATIONSHIP IS APPROACHED FOR
SMALL VALUES OF DT. THIS EXPRESSION USES THE FULL
EXPANSION AND NOT JUST THE FIRST TERM.

REFERENCE: NBS TECHNICAL NOTE 385 BY N.S. SNYDER

THE HEAT FLUX INPUT TO THE SURFACE IS CHANGED BY
ALTERING THE TABLE JTOT IN THE MAIN PROGRAM. TIME
AND WATT/CM**2 COORDINATES ALTERNATE IN THIS TABLE,
TIME BEING THE INDEPENDENT VARIABLE.

IF THE CIRCUIT DESCRIPTION IS CHANGED, BE SURE TO
RENAME ALL REFERENCES TO THE SURFACE CAPACITANCE
WHICH IS USED TO PROVIDE THE SURFACE TEMPERATURE.
LOOK IN: <PTS> AND IN THE LUMPED THERMAL
RESISTANCES NEAR THE SURFACE ELEMENT.
ALSO CHECK THE INITIAL CONDITIONS SECTION.

A CHANGE IN THE LHE BATH TEMPERATURE WOULD PROBABLY REQUIRE
CHANGES IN <PTHP>, <PQP>, <PTHM>, <PQM>, BNB, BTB,
BFB, RHOL, CPL, KL, AND HFG. BE VERY CAREFUL ABOUT
DOING THIS.

"SSHT" STANDS FOR STEADY STATE HEAT TRANSFER
"PNBSS" STANDS FOR PEAK NUCLEATE BOILING POINT IN SSHT.
"MFBSS" STANDS FOR MINIMUM FILM BOILING POINT IN SSHT.
"Q" AS A PREFIX STANDS FOR A HEAT FLUX RELATED TERM;
WHEN FOLLOWED BY PNB OR MFB THE NUMBER IS PART OF THE
COORDINATE FOR EITHER THE PNB POINT OR THE MFB POINT
ON A QDOT/A VS. THETA SURFACE SSHT GRAPH.
"TH" AS A PREFIX STANDS FOR A THETA VALUE, THAT IS, THE
DIFFERENCE BETWEEN THE SURFACE TEMPERATURE AND THE
LIQUID HELIUM BATH TEMPERATURE.
"L" STANDS FOR THE LIQUID.
APPENDIX II (part 4)

"BL" stands for boundary layer and usually refers to the thermal boundary layer in the liquid helium. "BNB" stands for beginning of nucleate boiling, as extracted from equation 1.13 of Hsu & Graham's book on two-phase fluid flow and boiling heat transfer. I extracted \( THL \times \Delta T \) as being the important parameter and called this BNB. A direct theoretical calculation of BNB for this system gives a value of \( 1.45 \times 10^{-6} \) cm\(\cdot\)K. Assuming a linear temperature profile and constant properties, the energy in the BL per unit area would be:

\[
E/A = \rho_0c_p \times (1.45 \times 10^{-6})/2 = 4.5 \times 10^{-7} \text{ J/cm}^2
\]

Other liquid such as water require higher values than the critical value before boiling is observed so I'm not surprised that a value of 2.E-6 seems to give the best results (4.4 times larger than the theoretical critical value).

Preliminary data introduction, modification, and checking of the input temperature.

```fortran
REAL KL, JNUM
DATA RHOL/.129/, CPL/4.19/, KL/1.95E-4/
1, DLTAMAX/2.E-4/, CBL/1.E-3/, CL/1.E-7/ 2, EBNB/2.E-6/, TRI/1.E-4/ 3, HFG/21.47/, RHOV/.01356/
C TBATH=4.0
C FJ=0.
C PATHTR2=0.
PATHNB=0.
PATHTB=0.
PATHFB=0.
C EXPQNB=2.
EXPFQB=1.2
C THL=TL-TBATH
THE=TBL-TBATH
C IF(THL.LE.0.)GOTO 900
IF((THE.LE.0.).AND.(JNUM.GT.1.5))GOTO 900
IF(THE.LT.0.)THE=0.
C TRPNB=5.*TRI
EPNBSS=TRPNB*(THLPNBS**3.)
C EPNBSS IS 4.E-3 FOR THLPNBS=.5 AND TRI=1.E-4
```
APPENDIX II (part 5)

TH = THLMFBS
PATHTR2 = 2
GOTO 530

5 PATHTR2 = 0.
EMFBSS = EFBSS
TRMFB = EMFBSS * (THLMFBS**3.)
TR2 = TRMFB * 5

C EMFBSS IS 9.57E-3 FOR THLMFBS = 6.8 AND TR2 = 15.04

HEAT TRANSFER MODE DETERMINATION

1 - CALCULATE THE ENERGY IN THE BL PER UNIT SURFACE
AREA AND DETERMINE IF FILM OR TRANSITION BOILING
EXISTS AT THE SURFACE.

EBLPUA = (THE*CBL + THL*CL)/AREA
(EBLPUA = ENERGY IN THE BOUNDARY LAYER
PER UNIT SURFACE AREA)
IF(EBLPUA .LE. 0.) GOTO 900

WITH NO ENERGY IN THE BL BUT A POSITIVE THS
HAVE THE CASE OF SIMPLE CONDUCTION LIMITED
ONLY BY THE KAPITZA RESISTANCE - NO CONDUCTION
DEEP INTO THE BL IN THIS CASE.

FIRST, CHECK TO SEE IF FILM OR TRANSITION
BOILING IS OCCURRING. THE PRODUCT OF THE
ENERGY IN THE BOUNDARY LAYER TIMES THE PEAK
TEMPERATURE OF THE LIQUID SEEMS TO BE A
CONSTANT FOR THE TWO BOUNDARIES OF THE
TRANSITION BOILING REGION.
IF(JNUM.GT.1.5) GOTO 50
EXTL = EBLPUA * (THL**3.)

IF (EXTL .GE. TR2) GOTO 500
IF (EXTL .GT. TR1) GOTO 400

CHECK FOR NUCLEATE BOILING. EBNB IS AN ENERGY
PER UNIT SURFACE AREA (J/CM**2) WHICH
CORRESPONDS TO DELTA * THL * RHOL * CPL
THE ACTUAL VALUE USED FOR EBNB IS ABOUT 2
TIMES GREATER THAN THE THEORETICAL VALUE AND
WAS TAKEN FROM NBS DATA USING CARBON SURFACES.
IF(EBLPUA .LE. EBNB) GOTO 100
GOTO 300

50 IF(EBLPUA .LE. EBNB) GOTO 100
IF(EBLPUA .LE. EPNBSS) GOTO 350
IF(EBLPUA .LT. EMFBSS) GOTO 450
GOTO 500

MODE I: CONDUCTION HEAT TRANSFER

CALCULATION OF THE BOUNDARY LAYER TEMPERATURE PROFILE AND THE CONDUCTION HEAT TRANSFER.

1-TO FIND THE HEAT FLUX OUT OF THE SURFACE, ASSUME A TEMPERATURE PROFILE OF THE FORM:

\[ T_h = T_hL \left( \frac{1}{1+L} \right) \]

AND THEN USE A RELATION EQUATING THE ENERGY IN THE BL TO \( \rho C_p T \) TIMES THE INTEGRAL OF \( T(h) \) OVER \( x=0 \) TO \( \Delta T \) TO SOLVE FOR \( \Delta T \).

2-IN THE EVENT THAT TOO MUCH ENERGY IS IN THE BL FOR THIS SORT OF ANALYSIS, GET SOME SORT OF ANSWER BY GOING TO MODE II: CONVECTION HEAT TRANSFER

3-IF SOME CONDUCTION OR CONVECTION IS OCCURRING, FIND THE SLOPE AT \( 0.9 \Delta T \) AND USE THIS VALUE TO CALCULATE A CONDUCTION HEAT TRANSFER USING:

\[ Q_{cond} = -k \frac{dT}{dx} |_{x=0.9 \Delta T} \]

WHERE \( \frac{dT}{dx} \) IS THE SLOPE OF THE TEMPERATURE PROFILE IN THE BL AT \( x=0.9 \Delta T \).

100 \[ T_{ave} = \frac{E_B L P U A}{\Delta T \rho C_p} \]

\[ T_{ratio} = \frac{T_hL}{T_{ave}} \]

IF \( T_{ratio} \leq 1.1 \) GOTO 200

\[ \Delta T = T_{ratio} - 1 \]

IF \( J_{num} > 1.5 \) GOTO 150

X=0

GOTO 180

150 IF \( \Delta T > 4 \) GOTO 900

\[ T_{ave} = \frac{E_B L P U A}{\Delta T \rho C_p} \]

\[ T_{ratio} = \frac{T_hL}{T_{ave}} \]

IF \( T_{ratio} \leq 1.1 \) GOTO 200

\[ \Delta T = T_{ratio} - 1 \]

IF \( J_{num} > 1.5 \) GOTO 150

X=0

GOTO 180

180 \[ \frac{dT}{dx} = -\frac{\Delta T}{\Delta T \Delta x} \frac{dT}{dx} \]

\[ Q_{cond} = -k \frac{dT}{dx} \]

IF \( PATHNB \geq 1 \) GOTO 330

FJ(QCOND*AREA)

GOTO 900

MODE II: CONDUCTION HEAT TRANSFER WITH CONVECTION

-ASSUME CONVECTION SETS A LIMIT OF \( \Delta T \) ON DELTA AND ASSUME A LINEAR TEMPERATURE PROFILE BETWEEN \( T_hL \) AND 0. AT \( \Delta T \). CALCULATE THE HEAT TRANSFER AS FOR ORDINARY CONDUCTION.
APPENDIX II (part 7)

C
200 OCV=THL*KL*AREA/DLTAMAX
IF(PATHNB.GT.1.)GOTO 330
FJ=OCV*AREA
GOTO 900
C
C MODE III: NUCLEATE BOILING HEAT TRANSFER
C
C PEAK NUCLEATE BOILING DATA FOR A SPECIFIC
C MATERIAL (REPRESENTED BY QPNBSS AND THPNBSS) ARE
C TAKEN INTO ACCOUNT WHEN CALCULATING THE HEAT
C TRANSFER (WHICH GOES ROUGHLY AS THE SQUARE OF
C THE TEMPERATURE DIFFERENCE THS).
C TRANSIENT BOILING HEAT TRANSFER MAY BE HIGHER
C OR LOWER THAN STEADY STATE VALUES, DEPENDING ON
C PREVIOUS HISTORY. THIS IS TAKEN INTO ACCOUNT
C BY MULTIPLIERS (AN ENERGY RATIO).
C
FTR1 IS THE RATIO OF THLTR1 AT TRANSITION
INCIPIENCE TO THLPNBS AT PEAK NUCLEATE
BOILING (WHICH IS NOT PURE NB).
300 FTR1=.9
THLTR1=FTR1*THLPNBS
CQNB=QPNBSS/(THLTR1**EXPQNB)
C
QNBSS=CQNB*(THL**EXPQNB)
ENBSS=EPNBSS*(THL/THLPNBS)
C
AN ASSUMPTION IS MADE THAT THE HEAT
TRANSFER IN NUCLEATE BOILING IS DETERMINED
BY THE TEMPERATURE OF THE LIQUID AT
THE SURFACE AND IS AFFECTED BY THE AMOUNT
OF ENERGY ACTUALLY IN THE BOUNDARY LAYER.
C
IF(THL.GE.THLTR1)GOTO 310
EXPERNB=((1.-((THL/THLTR1))))
QNB=QNBSS*(EBLPUA/ENBSS)**EXPERNB
GOTO 320
310 QNB=QNBSS
C
IF THE DELTA * THL PRODUCT IS NOT MUCH
GREATER (4X) THAN THE THRESHOLD VALUE FOR
NUCLEATE BOILING, THEN CALCULATE HEAT TRANSFER
FOR A COMBINATION OF CONDUCTION AND NUCLEATE
BOILING. THE IDEA IS TO EASE INTO NB AS MORE
NUCLEATION SITES BECOME ACTIVE.
320 TAU=EBLPUA/ENB
APPENDIX II (part 8)

GOTO 100
330 PATHNB=0.
    QNB=QNB+(QCOND-QNB)*(4.-TAU)/3.
    GOTO 380
C
C NUCLEATE BOILING HEAT TRANSFER OUT OF THE BOUNDARY LAYER.
C THE ENERGY IN THE BL IS USED TO DETERMINE THE
C HEAT TRANSFER OUT OF THE BL BY CALCULATING AN
C EQUIVALENT STEADY STATE TEMPERATURE DIFFERENCE (THLSS).
C THIS EFFECTIVE TEMPERATURE IS THEN USED IN THE STANDARD
C STEADY STATE NUCLEATE BOILING CORRELATION DEVELOPED ABOVE.
C
350 QNB=QPNBSS*(( EBLPUA/EPNBSS)**EXPQNB)
C
380 IF(PATHTB.GT.1.)GOTO 410
    FJ=QNB*AREA
    GOTO 900
C
C
C MODE IV: TRANSITION BOILING
C
TRANSITION BOILING IS ASSUMED TO BE A
COMBINATION OF NUCLEATE AND FILM BOILING
WITH THE PROPORTION OF EITHER HEAT
TRANSFER MODE BEING DETERMINED BY EXTL
AS RELATED TO TR1 AND TR2.
EQUATION (6) FROM KALININ, BERLIN,
KOSTYUK, AND NOSOVA IN: ADVANCES IN
CRYOGENIC ENGINEERING, VOL.21 (1976),
P. 273-277 WAS USED WITH A SMALL ADDITIONAL
TERM TO MATCH THE BOUNDARY CONDITIONS OF
FRACTNB=1 AT PNB AND FRACTNB=0 AT MFB.
THE 1/4 POWER OF EXTL AND THE TRANSITION
CONDITIONS ARE USED TO PRODUCE A RELATION
WHICH IS LINEAR IN TEMPERATURE. A
DIMENSIONLESS VARIABLE DTSTAR IS USED IN
THE EQUATION FOR FRACTNB WHICH SHOULD BE
SIMILAR TO THE DIMENSIONLESS TEMPERATURE (5) USED
IN THE ABOVE REFERENCE.
C
400 PATHTB=2.
    GOTO 300
410 GOTO 500
420 PATHTB=0.
C
    DTSTAR=((EXTL**.25)-(TR1**.25))/
    1((TR2**.25)-(TR1**.25))
    FRACTNB=(1.-DTSTAR)**10
    FRACTFB=(1.-FRACTNB)
APPENDIX II (part 9)

QTB=QNB*FRACTNB+QFB*FRACTFB
FJ=QTB*AREA
GOTO 900

TRANSITION BOILING - OUT OF THE BOUNDARY LAYER

OTHSS (THE STEADY STATE HEAT FLUX IN TRANSITION BOILING) IS GIVEN BY LINEAR INTERPOLATION BETWEEN THE PEAK NUCLEATE BOILING AND MINIMUM FILM BOILING POINTS.

\[
450 \quad \text{EXPQTSS}=\frac{\text{ALOG(QMFBS}SS)-\text{ALOG(QPNBSS))}}{1(\text{ALOG(EMGBSS)}-\text{ALOG(EPNBSS))}}
\]

\[
\text{CQTSS}=\frac{\text{QPNBSS/EPNBSS}**\text{EXPQTSS}}{\text{QTBS}=\text{CQTSS*EBLPUA**EXPQTSS}}
\]

\[
\text{FJ}=\text{QTBS*AREA}
\]

GOTO 900

MODE V: FILM BOILING

\[
500 \quad \text{TH}=\text{THL}
\]

QFBSS (THE HEAT FLUX FOR STEADY STATE FILM BOILING AT A GIVEN THS) IS AN EXPONENTIAL FUNCTION OF THS. THE EQUATION IS AN EMPIRICAL FIT TO DATA FOR THE NBS GRAPHITE SURFACES WHICH DID NOT EXHIBIT A MINIMUM FILM BOILING HEAT FLUX BELOW THAT OF PNB. FOR RADII OF CURVATURE>.5 CM.

\[
\text{NPASS}=0
\]

\[
530 \quad \text{QFBSS}=(.02*\text{TH}**1.2)
\]

\[
\text{NPASS}=\text{NPASS}+1
\]

CALCULATE THE STEADY STATE VAPOR LAYER THICKNESS (DELTASS) BY USING AN EQUATION IN YUKIO IWASA'S CRYOGENICS PAPER (CRYOGENICS, MAY 1978, P .273 Eq.16).

UNITS HAVE BEEN CHANGED TO GIVE DELTA IN CM. THS WAS REPLACED BY THL AND A TERM STATED AS (THS-.5) WAS REDUCED TO THL. HFG IS THE LATENT HEAT OF VAPORIZATION PER UNIT VOLUME OF SAT. VAPOR (J/CM**3).

\[
\text{HFGVOL}=\text{HFG}^*\text{RHOV}
\]

\[
\text{DELTASS}=\left\{\frac{5.7^*\text{TH}}{\text{HFGVOL}}\right\}^*\left\{\frac{5.7^*\text{TH}}{\text{HFGVOL}}\right\}
\]

THE ENERGY IN THE FILM LAYER IS CALCULATED BY USING THE FOLLOWING RELATION FOR THE
VOLUMETRIC ENTHALPY (J/CM**3):

\[ \text{HVOL} = 0.609 \times (T^{-0.25}) \]

which has been integrated from the surface out to \( \Delta \), assuming a linear temperature profile between \( T_L \) and \( T_{BATH} \).

\[ \text{EFILM} = (0.812 \times \Delta_{SS}/T_H) \times (((T_{BATH}+T_{BATH})^{0.75}) - (T_{BATH}^{0.75})) \]

\[ \text{ENBSSFB} \]

is calculated assuming that the same energy is added as for regular nucleate boiling with a heat flux equal to \( Q_{FBSS} \).

\[ \text{ENBSSFB} = \text{EPNBSS} \times ((Q_{FBSS}/Q_{PNBSS})^{(1./\text{EXPQNB})}) \]

\[ \text{EFBSS} = \text{EFILM} + \text{ENBSSFB} \]

If \( \text{PATHTR2} \leq 1. \) GOTO 5

\[ \text{ERATIO} = \text{EBLPUA}/\text{EFBSS} \]

If \( \text{JNUM} > 1.5 \) GOTO 550

Assume a linear temperature profile so that the temperature slope at the surface is directly proportional to the ratio of \( \text{EFBSS}/\text{EBLPUA} \) since this is given. This also assumes that any temperature dependence of the fluid properties will drop out in the ratio.

\[ \text{QFB} = \text{QFBSS} \times (\text{EFBSS}/\text{EBLPUA}) \]

If \( \text{PATHTB} \leq 1. \) GOTO 420

\[ \text{FJ} = \text{QFB} \times \text{AREA} \]

GOTO 900

FILM BOILING - OUT OF THE BOUNDARY LAYER

Calculate an equivalent \( T_{LSS} \). Assume that film boiling heat transfer out of the boundary layer is determined by \( \text{EBLPUA} \). This is to meet the condition that \( Q_{INBL} = Q_{OUTBL} \) when \( EBLPUA = \text{EFBSS} \).

550 IF((ERATIO.GT..9999) .AND. (ERATIO.LT.1.0001)) GOTO 580

\[ \text{TH}_1 = \text{TH} \]

\[ \text{TH} = \text{TH}_1 \times (\text{EBLPUS}/\text{EFBSS}) \]

If \( \text{NPASS} \leq 10 \) GOTO 530

\[ \text{TH} = \text{TH}_1 \times ((\text{EBLPUA}/\text{EFBSS})+1.)^{1/2}. \]

If \( \text{NPASS} \leq 50 \) GOTO 530

\[ \text{TH} = \text{TH}_1 \times ((\text{EBLPUA}/\text{EFBSS})+2.)^{1/3}. \]

If \( \text{NPASS} \leq 100 \) GOTO 530
APPENDIX II (part 11)

C 580 FJ=QFBSS*AREA
   GOTO 900
C
C 900 RETURN
C
C
C END
END
DATE
FILMED
7-83
DTIC