ABSTRACT

A number of results were obtained pertaining to signal detection and block truncation coding for image compression. These results led to improved performance over previous approaches, with special attention given to methods which required less statistical knowledge and which were easier to implement. In particular, use of robustness techniques was employed to allow the exploitation of whatever knowledge was available, while retaining insensitivity to the remaining inexactness in knowledge.
A number of results were obtained pertaining to signal detection and block truncation coding for image compression. These results led to improved performance over previous approaches, with special attention given to methods which required less statistical knowledge and which were easier to implement. In particular, use of robustness techniques was employed to allow the exploitation of whatever knowledge was available, while retaining insensitivity to the remaining inexactness in knowledge.
PERSONNEL

PRINCIPLE INVESTIGATOR: Don R. Halverson

GRADUATE RESEARCH ASSISTANT: Julius P. Bleker

STUDENT TECHNICIAN: Michael W. Thompson


SUMMARY OF RESEARCH

The research supported by Grant AFOSR-82-0033 was primarily concerned with signal detection, however some effort was also directed toward the area of block truncation coding for image compression. Results obtained pertaining to the latter will be summarized first.

Block truncation coding (BTC) is a technique for image compression which is easy to implement and often possesses good performance characteristics, relative to other approaches, even in the presence of many channel errors. Furthermore, the BTC algorithm does not depend on knowledge of the specific underlying statistical distributions, but rather on sample moments. The basic BTC technique employs a two level quantizer whose output levels are obtained by matching sample moments. The quantizer's threshold is either taken a priori to equal the sample mean (and thus the output levels are chosen so that the first two sample moments are preserved), or the threshold is chosen so as to preserve a third sample moment. The latter approach, while somewhat more complicated to implement, often yields improved performance since three moments are preserved instead of two.

The developers of the technique presented formulas specifying the quantizer threshold and output levels so that the first three sample moments were supposedly preserved. Unfortunately, however, the formula specifying the quantizer threshold contained an error, and one result we therefore obtained was a correction of this quantity. Moreover, the developers of the technique noted that rounding a certain quantity to an integer was at times necessary in practice, however, there existed ambiguity in precisely at what point in the algorithm the rounding should occur. We therefore investigated the perturbation of the relevant
quantity, and found that it was fortunately preferable from the standpoint of all three moments for the rounding to be done consistently in one way. We furthermore investigated the effects of rounding the quantity on the performance and noted that for very many cases it should not be serious. These results were delineated in #3 of the publication list.

Noting that the BTC approach preserves sample moments (two moments with one method and three with the other), we also investigated whether or not improved performance could be obtained by employing moments other than the first two or three. We found that the original BTC scheme could be generalized to yield a family of moment preserving quantizers using higher sample moments. In particular, we found closed form expressions for the quantizer output levels which preserved the n-th and 2n-th moments when a threshold equal to the $L_n$ norm of the samples was employed; we also found closed form expressions specifying the quantizer output levels and threshold which preserved the n-th, 2n-th, and 3n-th moments. We then applied these results to various example images and found that improvement in performance was obtainable by the use of higher moments, both from the standpoint of mean absolute and mean squared error. Finally we noted that there was a subclass of this family of moment preserving quantizers for which practical difficulties in implementation exist; we then showed that frequently this subclass could be avoided to still obtain good performance. These results were delineated in #1 of the publication list.

A number of results pertaining to the detection of signals in non-Gaussian noise were obtained. For example, a classical detector of
time varying deterministic signals in dependent noise is the matched
filter, which maximizes the output signal to noise power ratio; it is
also Neyman-Pearson optimal in the particular case when the noise is
Gaussian. In many situations it is reasonable to expect that the
signal will be known at discrete instants, thus admitting the design of
the discrete time matched filter. However, it is often a very different
matter to assume that the signal will be known exactly as a closed form
analytical expression over, for example, an interval of time, which would
be necessary for the design of a matched filter in continuous time. While
the signal may thus be incompletely known, it is reasonable to expect
that in many cases it could be modeled as bandlimited. If we further-
more assume the signal is known at a fixed number of instants, we might
hope that a continuous time filter could be designed which would be
insensitive to the remaining inexactness in our knowledge of the signal.
Such a filter might also have the potential for improved performance over
the discrete time filter. Using a saddlepoint criterion, we obtained
a result which specified the form of such a filter. We then showed
that the saddlepoint criterion did indeed impart robustness into the
filter, in fact the output signal to noise power ratio was shown to be
invariant over all appropriate continuous time signals considered.
Moreover, we also showed that the performance (as measured by output
signal to noise power ratio) of this robust continuous time filter upper
bounded that of the discrete time filter. Finally, we showed by way of
example that strict improvement in performance over the discrete time
filter was possible. Employment of continuous time would thus seem to
be useful even in some cases which appear appropriate to discrete time.
These results were delineated in #5 of the publication list.

We also investigated a situation where inexact knowledge of the statistics of the noise was present for the discrete time detection of time varying deterministic signals in i.i.d. non-Gaussian noise. In earlier work employing the canonical form of the locally optimal detector, the authors showed how the design of the asymptotically robust detector for the Huber-Tukey mixture class of noises could be obtained. As might have been expected, the results led to censoring the factors of the test statistic to impart robustness, which led to the employment of a detector nonlinearity which limited observations of large magnitude. However, this work did not take into account the common situation where more is known about a noise density near the origin than on the tails. In fact, the noise often arises in practice from the sum of a number of "nearly" independent sources, and thus frequently the noise density resembles the Gaussian near the origin but may differ markedly on the tails. Such knowledge had the potential to be exploited for improved performance, where the robustness is imparted to account for our lack of knowledge of the tails of the noise density. We investigated this situation and obtained results which specified an asymptotically robust detector for the case where the noise density was known on an interval about the origin. We then showed by way of example that the approach led to improved performance over the previous one; in fact, the improvement was quite dramatic in certain cases. These results were delineated in #2 of the publication list.

The area of memoryless discrete time detection of signals in φ-mixing noise has been studied by us in the past for both the case when
the signal was constant and when it was also a random process. The
detector structure employed consisted of a memoryless nonlinearity
followed by an accumulator and threshold comparator, which was Neyman-
Pearson optimal in the independent noise case. Using the criterion of
asymptotic relative efficiency, these earlier results accounted for
the dependency by specifying the detector nonlinearity which optimized
performance. This nonlinearity appeared as the solution of an integral
equation which unfortunately involved second order statistics of the
random processes present. Since such statistical knowledge may often
be incomplete, we investigated alternative approaches which required
less statistical knowledge and which were easy to implement.

For example, we considered the detection of a constant signal
in weakly dependent $\phi$-mixing noise using two approaches. For the
first, the locally optimal detector was employed, resulting in the need
to know only the univariate noise density. Since this approach ignores
the dependency, it is of course important to inquire into its perfor-
mance. We therefore obtained a bound on the degradation in performance
which was expressed in terms of the $\phi$-representation of the noise.
This bound showed quantitatively how close the performance of the
locally optimal detector was to optimal for various $\phi$-representations.
The second approach considered a more general form, consisting of the
locally optimal nonlinearity plus a linear correction term. Such a
structure has been shown to yield improved performance under a moving
average weak dependence model. We then showed that in the much more
general $\phi$-mixing case, such improved performance is unfortunately
not possible uniformly over the class of noise processes possessing a
given $\phi$-representation. In fact, for any appropriate $\phi$-representation,
we showed that there must exist such a process for which the locally
optimal detector outperforms the one with the linear correction term. Because of this result, we might in practice simply wish to employ the
locally optimal detector in the absence of the required statistical
knowledge. These results were delineated in #4 of the publication list.

Finally, we considered an alternative approach which was applied
to memoryless discrete time detection of random signals in noise, where
in this case both the signal and noise were random processes (not
necessarily independent of each other) whose maximal correlation
coefficient sequences were summable. Such processes are strong mixing,
moreover this class includes as a proper subset the class of $\phi$-mixing
processes. In some earlier work we showed that the asymptotically
optimal nonlinearity again satisfied an integral equation, which in this
case involved second order statistics of both the signal and the noise.
Noting that finding an exact solution was often limited in practice by
the numerical techniques employed as well as the expected inexact
statistical knowledge, it was concluded that the actual nonlinearity
obtained was only approximately optimal. We therefore wondered whether
other approximations might offer certain advantages, and therefore
pursued an investigation into the general problem of approximating a
detector nonlinearity. This led to a result which established sufficiency
conditions for the performance of a sequence of approximations to converge
to the performance of a given detector. We then applied these conditions
to the case where the optimal detector nonlinearity was approximated by
a quantizer or by a polynomial. Our results specified the form of the
optimal M-level quantizer, moreover it was shown that the performance of
the resultant detector converged to optimal as the number of quantization levels approached infinity. Since one may be interested in quantizing to implement the detector digitally, the use of such quantizers might be attractive. We also specified the form of the optimal polynomial of a given degree, and found that it could be obtained simply by solving a set of linear equations which involved joint moments rather than densities. We then showed that for a large class of noises the resultant detector's performance converged to optimal as the degree of the polynomial approached infinity. Because of these convergence results, we therefore concluded that the employment of a quantizer or polynomial detector allowed performance arbitrarily close to optimal. These results were delineated in #6 of the publication list.
DIRECTIONS OF CURRENT RESEARCH

Current research is directed toward the areas of signal detection and block truncation coding for image compression. Most of this work has as a goal the admission of non-Gaussian processes with dependency. In addition, we are heavily interested in approaches which require only moderate amounts of statistical knowledge, thus yielding greater utility in practice. This is resulting in the attractiveness of robust or nonparametric schemes. In particular, approaches featuring robustness appear desirable because they frequently have the potential to exploit whatever available statistical knowledge might be present.