Attenuation of Seismic Waves at Regional Distances

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This final report deals in part with the use of regional seismic crustal phases, particularly Lg, to discriminate between explosions and earthquakes, to determine m_b values of explosions and earthquakes, and to estimate yields at regional distances. An m_b - log yield calibration relation is developed for NTS events, and then used to estimate the yields of selected explosions at the East Kazakh test sites in the USSR.
The report is further concerned with the attenuation of seismic surface waves at intermediate periods and how the attenuation of those waves is related to that of higher-frequency crustal phases. Significant regional variations of upper crustal Q values are found and higher frequency waves propagate more efficiently than expected on the basis of intermediate-period waves.
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MATTHEW J. KEMPER
Chief, Technical Information Division
The research of O.W. Nuttli on regional waves can be summarized as:

**Attenuation.** Attenuation coefficient values for 1-sec period Lg waves have been determined for paths from NTS to selected stations in the western United States, for paths from the Sahara test site to selected WWSSN stations in Africa, for paths from the East Kazakh test sites to selected WWSSN stations in Scandinavia and southern Asia, and for paths across Iran. Attenuation also was determined for 1-sec period Pn, Pg and Sn waves, and for 3-sec period Lg waves, across Iran. Each of these waves have been used to derive regional magnitude formulas for $m_b$.

**Discrimination.** The discrimination ability of 1-sec period Lg-wave amplitudes is low. For the same $m_b(P)$ value, the earthquake Lg waves are on average only 25% larger (0.1 magnitude unit) than the explosion Lg waves. However, Lg-wave amplitudes from earthquakes vary appreciably with azimuth, whereas Lg-wave amplitudes from explosions show a near-circular radiation pattern. Therefore, the standard deviation of $m_b(Lg)$ should be larger for earthquakes than explosions, which may serve as a regional discriminant.

**Lg Excitation and P-Wave Magnitude Bias.** If the source medium is taken into account, the excitation of 1-sec period Lg waves is the same for explosions of similar yield in various geographic regions. That is, there is no magnitude bias for $m_b(Lg)$ as there is for $m_b(P)$. The Lg data can be used to estimate $m_b(P)$ bias, if $m_b$ (teleseismic P) values are available.
ica as a datum (assume \( m_b(Lg) = m_b(P) \) for that region), the \( m_b(P) \) bias is 0.0 for East Kazakh and 0.4 for NTS, the Sahara and Iran.

**Regional Estimates of Explosion Yield.** The announced yields of explosions at NTS are used to empirically determine a relation between \( m_b(Lg) \) and \( \log_{10} \) yield. For hard-rock sources, the empirical equation is a quadratic log yield, with a slope of near unity at small yields (10 kt) and of about 0.6 at large yields (1000 kt). For alluvium sources at NTS, the curve is almost linear, with a slope of approximately 0.7 for the range of 1 to 100 kt.

The NTS calibration curve for Lg has been used to estimate yields of selected explosions at East Kazakh. From April, 1976 through September, 1980, the largest yield is found to be that of 14 September 1980, with an estimated yield of 195 kt, based on Lg amplitudes.

The research of B.J. Mitchell was primarily concerned with the attenuation of waves at intermediate periods (2-50 s), and the relationship of the attenuation of those waves with that of regional phases at higher frequencies. Methods were determined by which fundamental-mode and higher-mode Rayleigh wave spectra could be utilized to determine \( Q_\beta \) as a function of the depth. The methods are important in regional studies of attenuation because they can be used with a single station and single event. They were used to determine \( Q \) structure in the eastern United States, the Colorado Plateau, and the Basin and Range province in North America, and in a region of Iran and Turkey and in the Barents shelf. The results indicate that \( Q \) varies from region to region in the upper crust.
If the models obtained using intermediate-period waves are used to predict the attenuation of higher frequency regional phases such as Lg, it is found that the models predict much lower Q (greater attenuation) than is observed for those phases. In order to explain that result it is required that the dissipative properties of the crust be frequency-dependent, higher frequencies being characterized by higher Q values. This result is confirmed by both frequency-domain studies and synthetic seismogram analyses of 1 Hz Lg waves.
Publications Resulting from Contract F49620-79-C-0025


Discrimination Potential of 1-sec Period Lg Waves

Otto W. Nuttli

The ratio of $m_b$ (obtained from teleseismic P-wave amplitudes) to $M_S$ (obtained from 20-sec period surface-wave amplitudes) has been proven over the years to be a useful discriminant between underground explosions and earthquakes. Three principal problem areas concerning the effectiveness or usefulness of the discriminant are: 1) For small explosions the amplitude of 20 sec-period surface waves is too small to be detected. This occurs at an $M_S$ value of about 3, corresponding to a contained explosion of approximately 10 kt in hard rock. 2) Some large explosions release tectonic strain and thus have an earthquake-type radiation which is superposed on the radiation of waves from the explosion. 3) Some earthquakes, particularly those in mid-plate environments, are associated with high stress drop. Thus their source spectrum is similar in shape to that of explosions, and their $m_b : M_S$ values are more explosion-like than those of earthquakes. A few Asian earthquakes of this type were described by Nuttli and Kim (1975). The high stress drop of the New Brunswick earthquake of 9 January 1982 had $m_b : M_S$ values which fall in the explosion population in Figs. 10.7b and 10.7c of Dahlman and Israelson (1977). The mainshock, which occurred at 00h14m, had an $m_b = 5.7$ and $M_S = 4.8$, and an aftershock on the same day at 12h53m had $m_b = 5.1$, $M_S = 3.8$ (U.S. Geological Survey, 1982).

The second problem ordinarily is associated with explosions of large yield, so that other means can be used to distinguish them from earthquakes. However, the tectonic release tends to distort yield estimates based on surface-wave magnitude.
The third problem is not so tractable, and may be common for mid-
plate earthquakes. This matter is considered further in the section of
this report entitled "Magnitude Relations and Spectral Scaling."

Because *Lg* is a superposition of higher-mode surface waves, a
number of investigators have considered it to be potentially useful,
when compared with P-wave amplitudes, to discriminate between explosions
and earthquakes for small events for which 20-sec period surface waves
are too small to be detected. The reasoning in support of this deduc-
tion is that explosions, which are essentially compressive radiation
sources, will not generate short-period surface waves as well as earth-
quakes of the same P-wave magnitude. Blandford (1981) concluded that
"the amplitude ratio of the maximum motion before $S_n (P_{\text{max}})$ to the max-
imum after $S_n (Lg)$ is a good discriminant between earthquakes and explo-
sions at regional distances. Experimental data shows that the ratio is
not seriously affected by site or source geology or by event depth.
However, propagation effects can be severe, especially from the USSR to
the South, and can lead to the requirement for regionalization." In
Figure 33 of Blandford (1981) vertical-component *Lg* amplitudes, measured
at NORSAR and equalized according to the P-wave magnitudes of the event,
are plotted versus epicentral distance along with attenuation curves
obtained for the eastern United States. The *Lg* waves whose amplitudes
are plotted were produced by nine Soviet explosions, eight of which were
PNE's in the western part of the country. The remaining event was
located at the East Kazakh test site. These nine data points define an
explosion attenuation curve. *Lg* amplitudes at NORSAR of two Austrian
earthquakes also are plotted in the figure. The one, for an earthquake
of 17 June 1972, falls below the explosion curve, which puts it in the
"explosion" population. The other data point, for the earthquake of 16 April 1972, lies about 0.5 magnitude unit above the curve, in the "earthquake" population. Thus, of the two earthquakes considered, one passes and one fails the discriminant test. Figure 42 of Blandford (1981) shows that the Semipalatinsk earthquake of 20 March 1976, of \( m_b = 5.1 \), had an \( Lg_{\text{max}}/P_{g_{\text{max}}} \) ratio about ten times larger at MSH (Iran) than a 15 January 1976 explosion of \( m_b = 5.2 \).

Pomeroy (Rondout, 1983) reviewed the research on discrimination using regional events. He noted that the Teledyne Geotech group, consisting of Blandford and his co-workers, found the \( P_{\text{max}}/Lg \) amplitude ratio to be a good discriminant, both in Asia and the United States. On the other hand, Pomeroy noted that Bennett and Murphy of S3 found insufficient scatter in the amplitude ratio data for western United States earthquakes that the error bounds of the earthquake data enveloped the explosion data of NTS events, and concluded that the \( P_n/Lg \) amplitude ratio was not a promising discriminant. In review of Mykkeltveit and Husebye's work at NORSAR, Pomeroy noted that earthquakes and explosions in the western and central portions of the USSR showed large scatter in the \( P/Lg \)-type amplitude ratios, and the authors could not find any discriminant of this type that could be applied to their data with any degree of success. Commenting on his own research, Pomeroy observed that western United States earthquakes and NTS explosions as recorded at the Yellowknife array in Canada, do separate, but with considerable overlap, when the ratio of the Lg amplitude to the first significant amplitude in the P wave train is utilized. Pomeroy and Nowak (1978a, b) noted that the \( Lg/P \) amplitude ratio data for the explosion SALMON in the eastern United States fell below similar earthquake data, but that for
similar explosions in the western Soviet Union, the ratios for earth-
quakes and explosions were the same.

Research done under Contract F49620-79-C-0025 that is relevant to
the regional discrimination problem was reported by Nuttli (1981). This
study considered 16 explosions and 6 earthquakes in the western and cen-
tral Asia portion of the USSR. Short-period P-wave amplitudes were
measured (as done conventionally for determining \( m_b \)), along with 1-sec
period, vertical-component Lg amplitudes from WWSSN seismograms at 13
stations in Germany, Scandinavia and southern Asia.

The variation of P-wave amplitude with epicentral distance, after
all events were equalized to \( m_b = 5.0 \), showed a scatter of approximately
ten times (one \( m_b \) unit) over the distance range of about 5° to 30°.
Scatter of the explosion amplitudes was comparable to that of the earth-
quake amplitudes. The Veith-Clawson (1972) amplitude attenuation curve
provided a good average fit to the data over the distance range con-
sidered.

The Lg amplitudes individually had to be corrected for the propaga-
tion path from the source to the station, because the coefficient of
anelastic attenuation is different for each path, and because small
errors in the assumed value of this coefficient can lead to orders-of-
magnitude errors in calculated Lg amplitudes at large regional dis-
tances. In this study the attenuation coefficient of 1-sec period Lg
waves was determined from the variation of the wave frequency in the Lg
coda with travel time (Herrmann, 1980), as described in the section of
this report dealing with Lg attenuation in Eurasia.
By use of the Veith-Clawson (1972) P-wave amplitude attenuation curve and of observed Lg attenuation coefficient values for each of the 90 source-to-station paths, theoretical values of log \( (A_{Lg}/A_p) \) were calculated for the 90 path lengths, assuming that all events were earthquakes. These theoretical values were compared with the observed values of log \( (A_{Lg}/A_p) \). The deviation, or difference between the log \( (A_{Lg}/A_p) \) of observed values and the log \( (A_{Lg}/A_p) \) of calculated values, is plotted in Fig. 1. From the figure it can be seen that, in general, the deviation for the explosions is negative and for the earthquakes is near zero or positive. This indicates that, on the whole, the ratio of \( A_{Lg}/A_p \) is larger for earthquakes than for explosions. However, the scatter and intermixing of explosion and earthquake data points in the figure is enough to conclude that the amplitude ratio \( A_{Lg}/A_p \) of 1-sec period waves recorded at regional distances by WWSSN vertical-component seismographs is not an effective discriminant. That is, too many of the events could be misidentified.

In the process of compiling information for this report we have observed, on the basis of a few events only, that Lg amplitudes from earthquakes show greater variation with azimuth than do those from explosions located in the same region and recorded at the same stations. This suggests that Lg has a circular radiation pattern for explosions and a lobed pattern for earthquakes. If this observation proves true, the standard deviation of \( m_b(Lg) \) of explosions will be less than of earthquakes. This idea will be pursued in further studies, to determine if it can serve as an effective discriminant.

On the basis of the present research, however, we conclude that the
Fig. 1. Difference between Observed and Predicted Values of $\log (A_{Lg}/A_p)$ for Earthquakes and Explosions in Eurasia. If Lg waves from earthquakes are not excited as strongly as for earthquakes, the explosion data points should have ordinates of less than zero.
scatter of $A_{Lg}/A_p$ values for explosions and earthquakes in the western and central Asia portions of the USSR is too large to make the ratio useful as a discriminant. This conclusion is in agreement with that of most other investigations, except for Blandford and his colleagues at Teledyne Geotech.
References


Magnitude Relations and Spectral Scaling

Otto W. Nuttli

Magnitudes play an important role in discrimination and yield estimation, both in the application of the \( m_b : M_s \) discriminant and in the use of \( m_b(P) \), \( m_b(Lg) \) and/or \( M_s \) to estimate yields. Therefore, it is of value to understand the factors which affect the estimates of the magnitudes and of the inter-relations among them.

Research performed under Contract F49620-79-C-0025 considered various aspects of these problems, including the relation of \( m_b(P) \) to \( M_s \) for mid-plate and plate-margin earthquakes (Nuttli, 1983a, b), the relation of \( m_b(Lg) \) to \( M_L \) (Herrmann and Nuttli, 1982), and the so-called magnitude bias of \( m_b(P) \) caused by anomalous attenuation of teleseismic P-wave amplitudes in the asthenosphere beneath the source (Nuttli, 1982a, b).

For more than ten years it has been known that the \( m_b \) versus \( M_s \) curve used to separate the explosion and earthquake populations varies regionally. In general, plate-margin earthquakes have larger \( M_s \) values than mid-plate earthquakes, if both have the same \( m_b \) value (even if P-wave magnitude bias is taken into account). The \( m_b : M_s \) relations were determined empirically, and varied somewhat with author and with type of instrument used to obtain the magnitudes.

Nuttli (1983a) used published values of \( m_b \), \( M_s \) and \( M_o \) (seismic moment) to determine spectral scaling relations for plate-margin and mid-plate earthquakes. Following Aki (1967), he assumed that the 1-sec and 20-sec period amplitudes of the far-field spectrum, after correction for attenuation, are proportional to \( m_b \) and \( M_s \), respectively, and that
the spectral amplitude at very long periods is proportional to $M_o$.

Nuttli (1983a) found, by trial and error, sets of spectra which, on average, satisfied the $m_b$, $M_S$, $M_o$ combinations for mid-plate and plate-margin earthquakes. A spectrum of the mid-plate spectra consisted simply of a linear long-period segment of constant amplitude (slope zero) and a linear short-period portion of slope two. The periods at which the two linear segments of the spectrum intersect, called the corner period ($T_c$), satisfied the relation $M_o T_c^{-4} = \text{constant}$. The derived plate-margin spectra were more complex, because for seismic moments greater than about $10^{23}$ dyne-cm they had an intermediate-period linear segment of slope one. For both sets of spectra there is the same one-to-one correspondence between $M_S$ and $M_o$ values. For example, for $M_S = 6$, the spectral amplitude is the same for both plate-margin and mid-plate earthquakes. However, for $m_b$ (as obtained from teleseismic P-wave amplitudes), a value of 0.4 (on average) had to be added to the $m_b$ of plate-margin earthquakes to result in the same spectral amplitude as that corresponding to mid-plate earthquakes. That is,

$$m_b (\text{plate-margin earthquakes}) + 0.4 = m_b (\text{mid-plate earthquakes})$$

This equation suggests that teleseismic P-wave amplitudes of plate-margin earthquakes suffer a 2.5 times ($\log^{-1}0.4$) larger asthenosphere attenuation in the source region than mid-plate earthquakes, on average. This is an example of the so-called magnitude bias of 1-sec period P waves, which must be taken into consideration when $m_b$ (teleseismic P) is used to estimate explosion yield.

Assuming values of average fault displacement and fault rupture area, Nuttli (1983a) showed how the fault rupture length and average stress drop varied with magnitude $m_b$ and $M_S$, and seismic moment $M_o$, for
average mid-plate and plate-margin earthquakes. In general, for a given $M_S$ value, plate-margin earthquakes have lower average stress drop and greater fault rupture length than mid-plate earthquakes. This observation has some relevance to nuclear explosion detection. For mid-plate regions, because the fault rupture lengths of moderate-sized to large earthquakes are relatively small, rather insignificant looking geological structures can produce such earthquakes. Thus for mid-plate environments it is more difficult to identify, from surface geological or satellite studies, potential earthquake source zones. This problem is well known for the eastern United States, and often produces controversy in the siting of nuclear power plants.

Nuttli (1983b) added more data and did more extensive study of mid-plate scaling relations, in a follow-up of the earlier paper (Nuttli, 1983a). In the later paper he concluded that the spectra of mid-plate earthquakes of moment greater than $10^{23}$ dyne-cm required an intermediate-period portion of unit slope. From the two corner periods the fault rupture length and width could be determined as a function of seismic moment. Also, the average fault displacement could be calculated, using equations developed by Savage (1972).

Figure 1 compares the $m_b$ versus $M_S$ relation for mid-plate and plate-margin earthquakes, as determined by Nuttli (1983a, b). The curves, particularly that for plate-margin earthquakes, indicate a more complicated relation than the generally assumed linear one. Also included in the figure is the explosion-earthquake discriminant curve of Sykes and Evernden (1982). From it we can see that the $m_b:M_S$ discriminant is more effective for large events than for moderate-sized ones.
Fig. 1. $m_b$-$M_s$ Relation for Mid-Plate Earthquakes, Plate-Margin Earthquakes and Explosions. Included are the data for two New Brunswick earthquakes.
(m_b of 4.5 to 5.0), particularly for mid-plate earthquakes. Also plotted in the figure are the m_b:M_S values (USGS, 1982) of the 9 January 1982 New Brunswick earthquake and its aftershock. Both practically lie on the discriminant curve of Sykes and Evernden.

Chung and Bernreuter (1981) found for the western United States that

\[ m_b \text{ (teleseismic P)} = 0.99 M_L - 0.39 \]

Also for the western United States, Herrmann and Nuttli (1982) found that

\[ m_b(Lg) + M_L \]

In an earlier publication, Nuttli (1973) defined the m_b(Lg) scale for eastern North American earthquakes in such a way that, for that region,

\[ m_b \text{ (teleseismic P)} = m_b(Lg) \]

From these three relations, and if we assume that m_b (teleseismic P) is reduced by 0.4 units because of abnormal asthenosphere attenuation, we obtain that m_b(Lg) is the same for eastern and western earthquakes of the same 1-sec period spectral amplitude. That is, m_b(Lg) is an estimate of the 1-sec period source spectral amplitude that is independent of region. The conclusion is reasonable, for Lg amplitudes are affected only by transmission through the Earth's crust. The conclusion follows that explosion yield estimates based on m_b(Lg) values should exhibit or have no magnitude bias. It follows that, for a given source rock type, such as hard rock, granite, alluvium, etc., there will be an m_b(Lg) versus yield curve that will be applicable to all continental regions.
Preliminary estimates of the magnitude bias at the Nevada (NTS), East Kazakh (USSR) and Sahara (French) test sites, relative to the eastern United States, were obtained (Nuttli, 1982a, b). For the East Kazakh site, it is approximately 0.0 magnitude units, and for the NTS and Sahara sites it is 0.4 units. That is, if eastern North America is defined to have zero bias, then East Kazakh also has zero bias, whereas NTS and the Sahara sites have a bias of 0.4 units. The surprising result is the large bias of the Sahara test site, similar to that of the western United States rather than to a shield area.
References


If regional seismic waves are to be used to detect, identify and estimate yields of explosions in the USSR, it is essential that their attenuation be accurately known. This is not a simple task, because the attenuation coefficients vary both geographically and with wave frequency. Under Contract F49620-C-79-0025 particular attention was paid to 1-sec period Lg-wave attenuation. Values of this parameter were determined for paths from the East Kazakh test site to the WWSSN seismograph stations in Scandinavia and southern Asia, for paths across Iran and for paths from other explosions and from earthquakes in the USSR to Scandinavian and southern Asian stations. In addition, the attenuation coefficients of 1-sec period Pn, Pg and Sn waves, as well as 3-sec period Lg waves, were determined for paths across Iran.

Several methods were used to determine the values of the attenuation coefficients of the crustal waves. When sufficient data were available, time-domain amplitudes were plotted against epicentral distance and fitted by theoretical curves whose shapes are a function of the attenuation coefficient value. This was the method employed by Nuttli (1980a) for a study of 1-sec and 3-sec period Lg waves and of 1-sec period Pn, Pg and Sn waves across Iran. This method gives average values over broad areas or over long profiles. A second method developed by Herrmann (1980), in an adaptation of Aki's (1969) theory that modeled the Lg coda as scattered waves, was used to determine the average attenuation coefficient for Lg waves of a given period for individual paths from source to receiver. This method has the advantage of
giving the attenuation for a particular path, which is necessary when the attenuation varies rapidly over an area. It is especially useful when there are many seismic events that are located in a small source region, such as a test site, and are recorded at the same group of stations. A second advantage of the Lg-coda method is that the attenuation can be determined from the variation of the wave period in the Lg coda with travel time, and thus is independent of amplitude measurements. There are, however, also some limitations or problems with the coda method for determining attenuation. The principal one is that the value of the attenuation coefficient is frequency dependent, and the coda data often are not available over a sufficient range of periods to determine this frequency dependence precisely. Other methods then have to be resorted to in order to fix the value of $\xi$ in the relation

$$Q(f) + Q_0 f^\xi$$

(Mitchell,)

where $f$ is wave frequency, $\xi$ is taken to be constant over the range of about 0.2 to 10 sec, and $Q$ is the specific quality factor, related to the anelastic attenuation coefficient, $\gamma$, by the equation

$$\gamma(f) + \frac{\pi f}{UQ(f)}$$

where $U$ is the group velocity.

For Iran, Nuttli (1980a) found the value of the anelastic attenuation coefficient of 1-sec period $P_g$, $S_n$ and $L_g$ waves to be 0.0045 km$^{-1}$, approximately the same value that was found for $L_g$ in California by Herrmann (1980). For 3-sec period $L_g$ waves propagating across Iran, the value of $\gamma$ was found to be 0.003 km$^{-1}$. The corresponding approximate relation for $L_g$ waves in Iran is
\[ Q(f) = 200f^{0.6} \]

Making use of the fall-off of amplitude with epicentral distance in Iran, Nuttli (1980a) developed formulas for \( m_b \), valid for distances of 400 to 1500 km, that utilize the amplitude of 1-sec period \( P_g \) or \( L_g \) waves recorded by vertical-component seismographs. The \( m_b \) obtained from these formulas was defined to be the same as that obtained from teleseismic P-wave amplitudes for Iranian earthquakes.

Also it was found that the excitation of 1-sec period \( L_g \) waves for an \( m_b \) (teleseismic P) 5.0 earthquake in Iran is the same as in California, and approximately 0.4 magnitude units greater than for a similar-sized earthquake in eastern North America. An alternative way of stating this is that the teleseismic P-wave amplitudes of seismic events occurring either in Iran or in California suffer an anomalous 0.4 magnitude unit loss of amplitude, relative to eastern North America, in propagating through the asthenosphere beneath the source. Thus, the so-called magnitude bias of Iranian and Californian events, relative to eastern North America, is 0.4 units.

Nuttli (1981) gave values of 1-sec period \( L_g \) attenuation coefficients for paths from 16 explosions in western Russia, the Urals, western Kazakhstan, and western Siberia and from 6 earthquakes in eastern Caucasus to 13 WWSSN stations in southern Asia, Scandinavia and Germany. These values were obtained by application of Herrmann's (1980) coda method.

The coda method also was used by Nuttli (1982) to determine \( Q_0 \) and \( \zeta \) values for paths from the East Kazakh test site to southern Asian and
Scandinavian stations. The values found were

<table>
<thead>
<tr>
<th>Station</th>
<th>Country</th>
<th>$Q_0$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>KBL</td>
<td>Afghanistan</td>
<td>360</td>
<td>0.6</td>
</tr>
<tr>
<td>KEV</td>
<td>Finland</td>
<td>580</td>
<td>0.4</td>
</tr>
<tr>
<td>KON</td>
<td>Norway</td>
<td>700</td>
<td>0.4</td>
</tr>
<tr>
<td>MKI</td>
<td>Iran</td>
<td>380</td>
<td>0.5</td>
</tr>
<tr>
<td>NDI</td>
<td>India</td>
<td>300</td>
<td>0.6</td>
</tr>
<tr>
<td>NIL</td>
<td>Pakistan</td>
<td>380</td>
<td>0.6</td>
</tr>
<tr>
<td>NUR</td>
<td>Finland</td>
<td>580</td>
<td>0.4</td>
</tr>
<tr>
<td>QUE</td>
<td>Pakistan</td>
<td>280</td>
<td>0.6</td>
</tr>
<tr>
<td>SHL</td>
<td>India</td>
<td>340</td>
<td>0.6</td>
</tr>
<tr>
<td>UME</td>
<td>Sweden</td>
<td>620</td>
<td>0.4</td>
</tr>
</tbody>
</table>

In an earlier report (Nuttli, 1980b), $Q_0$ values were estimated for the $Lg$ waves from a number of explosions and earthquakes throughout western and central Eurasia recorded at the same stations listed above. In this early study, the value of $\zeta$ was assumed to be 0.2 for all the source-to-station paths. As a consequence, because this value of $\zeta$ is too small, all the $Q_0$ values given in Nuttli (1980b) were underestimated, more so for the paths to the southern Asian stations than to those in Scandinavia. The more recently obtained $\zeta$ values of 0.4 to 0.6 (see table above) are consistent with $\zeta$ values of 0.5 for $S$ waves in central Asia observed by Rautian and Khlaturin (Aki, 1980), of 0.6 for $S$ waves in the Kurils observed by Fedotov and Boldyrev (Aki, 1980) and of 0.6 and 0.8 for $S$ waves observed by Aki (1980) in Japan.
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EXPLOSION YIELD ESTIMATES

USING REGIONAL Lg WAVES

Otto W. Nuttli

The conventional seismic methods for estimating the yields of underground nuclear explosions make use of body-wave magnitude, $m_b$, and surface-wave magnitude, $M_S$. This is done by setting up a correlation between each of the magnitudes and known explosion yields.

Both methods suffer from certain complicating factors. For example, $m_b$ values are affected by the type of rock in the source region. In addition, a so-called magnitude bias is caused by anelastic attenuation of 1-Hz P-wave amplitudes in the asthenosphere. The value of this bias term varies from one geographic region to another. Also, focusing and defocusing of short-period P waves by lateral crustal variations at a test site can produce anomalous $m_b$ values. Empirical correlation between $m_b$ and yield should be corrected for these factors, which may not be well known for different test sites.

Because the waveguide for 20-sec period surface waves is rather uniform over much of the Earth, the problems that are encountered for P waves are not so serious for surface waves. But there are two other types of problems. The first is that 20-sec period waves do not have sufficient amplitude to be used for estimating yields of less than 10 kt. When large amplitude surface waves of period about 10 sec are used to overcome this limitation, it is found that they exhibit large lateral variations. The second, and more serious problem, is that nuclear explosions can release tectonic strain and produce an earthquake-type radiation pattern superposed on the isotropic radiation field of an
explosion. Depending upon the distribution of seismograph stations, the net result can be either to increase or decrease the estimate of the explosion yield.

As a result of these problems, \( m_b \) and \( M_s \) values can sometimes produce significantly different values of the estimated yield of a particular explosion.

A third type of wave, the higher-mode surface wave \( L_g \), shows promise of providing an independent estimate of yield. It has certain advantages and disadvantages with respect to methods using \( P \) and surface waves. An obvious disadvantage is that it only can be applied for continental paths from source to receiver, because 1-sec period \( L_g \) waves do not propagate across oceanic paths. A second disadvantage is that it is limited to regional distances, which in general restricts its application to distances of less than 1000 km for neotectonic areas and about 5000 km for ancient shield areas.

The advantages of using \( L_g \) amplitudes for yield estimates are that the method can be applied to small events (less than 1 kt), that there appears to be no magnitude bias, such as observed for \( P \) waves, and that apparently the method is not affected by the earthquake-type radiation produced by the release of tectonic strain, so that a single calibration curve can be used for estimating yield for all geographic regions. However, this latter point needs to be tested by further study and documented before it can be accepted.

For the past ten years \( L_g \) amplitudes have been used routinely in eastern North America for estimating body-wave magnitude, called \( m_b(L_g) \).
In general, 1-sec Lg amplitudes for eastern earthquakes give \( m_b \) values equal to \( m_b \) from 1-sec period P-wave amplitudes, as intended. They scale in the same way, because both are a measure of the 1-sec period spectral amplitude. The Lg method has proven useful because it can be applied to events of \( m_b \) as small as 2.5, when WWSSN seismograms are used.

Fig. 1 illustrates how Lg amplitudes are used to determine magnitude. The maximum sustained amplitude at a period near 1 sec and a group velocity near 3.5 km/sec is measured on the vertical-component seismograms, and converted to ground motion. The amplitude then is extrapolated back to 10-km epicentral distance according to the formula (adapted from Ewing et al., 1957)

\[
A(10 \text{ km}) = A(\Delta) \left( \frac{\sqrt{10}}{\sin(10/111)/\sin(10/111)} \right)^{1/2} \exp\left[\gamma(\Delta - 10)\right]
\]

where \( \Delta \) is epicentral distance in kilometers. The quantity \( \gamma \) is called the coefficient of anelastic attenuation. It is given by

\[
\gamma(f) = \frac{\omega f}{UQ(f)}
\]

where \( f \) is wave frequency, \( U \) is the group velocity of the wave whose amplitude was read, and \( Q \) is an apparent specific quality factor. \( Q \) is assumed to be frequency dependent according to the relation proposed by Mitchell (1980)

\[
Q(f) = Q_0 f^\xi
\]

where \( Q_0 \) is the value of \( Q \) at 1-sec period.

For earthquakes in eastern North America, an \( m_b \) (tel. P) = 5.0 corresponds to an \( A(10) \) value of the vertical component of Lg of 110 microns. Thus, on the condition that \( m_b \) (Lg) and \( m_b \) (tel. P) scale in
\[ A(10) = A(\Delta) \left( \frac{\Delta}{10} \right)^{1/3} \left[ \frac{\sin(\Delta/1000)}{\sin(10/1000)} \right]^{1/2} \exp \left[ \gamma(\Delta - 10) \right] \]

\[ \gamma(t) = \pi t \int f U(f) Q(t) \quad \text{and} \quad Q(t) = Q_0 t^\xi \]

For \( m_b = 5.0 \), \( A(10)_2 = 110 \) microns
\( m_b(Lg) = 5.0 + \log \left[ \frac{A(10)}{110} \right] \)

Fig. 1. Methodology for Using Lg Amplitudes and Frequencies to Determine \( m_b(Lg) \)
the same way,

\[ m_b(Lg) = 5.0 + \log[A(10)/110] \]

where \(A(10)\) is the extrapolated value of the 1-sec period, vertical-component Lg ground motion (in microns) at a distance of 10 km.

In the early work \(\gamma\) (the coefficient of anelastic attenuation) was obtained by measuring the decrease of amplitude with epicentral distance. This method was used to obtain \(\gamma\) values for 1-sec period Lg waves in eastern North America and in Iran, and of shorter period waves in the Mississippi Valley and in California. Later we observed that the coda theory of Aki and Chouet (1975), as adapted by Herrmann (1980), gave the same values of \(\gamma\) as the fall-off of amplitude with distance. The coda method is ideally suited for nuclear explosion studies, for it gives the average \(Q_o\) and \(\zeta\) values for the path from source to receiver.

Fig. 2 shows an example of the fit of some theoretical curves to the data for paths from NTS to the stations DUG (\(\Delta = 450\) km), BKS (\(\Delta = 550\) km), and TUC (\(\Delta = 700\) km).

Measurements of coda waves for sources in the various parts of the NTS (e.g. Yucca Flats in the east central and Pahute Mesa in the northwest portions of the Test Site) suggest that the \(Q_o\) and \(\zeta\) values for the paths to the stations DUG, BKS and TUC do not vary with source location. Thus, lateral variations in crustal structure over the NTS do not appear to affect 1-sec Lg amplitudes, in contrast to what is observed for 1-sec P-wave amplitudes.

Springer and Kinnaman (1971, 1975) published yields for a number of explosions at NTS, along with information about location, origin time,
Fig. 2. $Q_0$ and $f$ Values for Paths from Yucca Flats, NTS to BKS, DUG and TUC
source rock type and depth of the shot. By examination of the Lg waves from these explosions we found that DUG, EKS and TUC were the best WWSSN stations available for this study. However, they were not ideal. For example, TUC operates at a magnification of 200,000 at 1-sec period, which results in the Lg waves going off the edges of the seismogram for explosions of about 50 kt and greater. DUG, which is closer to NTS than TUC, normally operates at a 1-sec magnification of 200,000 or 400,000. Therefore, the motion "whited out" or went offscale for events of yield as low as 20 kt, except when the gain was reduced before some of the larger explosions.

Fig. 3 shows the $m_b$ (Lg) versus log yield data and curve for explosions of announced yield in hard rock at NTS. The curve is a quadratic least-squares fit to the data, given by the equation

$$m_b = 3.901 + 1.185 \log Y - 0.102 (\log Y)^2$$

where $Y$ is the announced yield in kilotons. Considering log $Y$ to be error-free, the standard deviation of $m_b$ is 0.077. The slope of the curve at small yields (10 kt) is 0.98, very close to unity as is to be expected from the spectral scaling relations for explosions given by Mueller and Murphy (1971). At 1000 kt the slope has a value of 0.57, which also is in agreement with the spectra scaling relations.

If SHOAL and PILE DRIVER, which were shot in granite, are not included in the quadratic least-squares fit, the resulting equation is

$$m_b = 3.901 + 1.155 \log Y - 0.091 (\log Y)^2$$

with a standard deviation of $m_b$ of 0.066. The resulting curve is so similar to that of Fig. 3 that it is not replotted. The slope at a
Fig. 3. $m_b(Lg)$ Versus Explosion Yield for Sources in Hard Rock at NTS. The curve is a least-squares quadratic fit to the data.
yield of 10 kt is 0.97, and at 1000 kt is 0.61.

Fig. 4 gives the \( m_b(L_g) \) versus log yield data and curve for explosions in alluvium at NTS. Although a quadratic equation was used to fit the data, the curve almost is linear. Its equation is

\[
m_b = 4.041 + 0.726 \log Y - 0.009 (\log Y)^2
\]

with a standard deviation of 0.048 for \( m_b \), assuming \( \log Y \) is error-free. For \( Y = 10 \) kt, \( m_b(L_g) \) for a source in alluvium is approximately 0.2 units less than for a source in hard rock. At \( Y = 1000 \) kt, the difference in \( m_b \) values increases to about 0.4 units.

Table 1 shows the differences between the \( m_b \) values obtained from teleseismic P-wave amplitudes and regional Lg-wave amplitudes. On average, \( m_b(L_g) \) at NTS is 0.31 units larger than \( m_b(P) \). From earthquake magnitude studies, Chung and Bernreuter (1981) and Herrmann and Nuttli (1982) concluded that \( m_b(P) \) is reduced by about 0.4 units because of anelastic attenuation of P waves in the asthenosphere beneath the western United States. Therefore, for earthquakes, \( m_b(L_g) \) should be 0.4 units larger than \( m_b(P) \). The difference between 0.4 and 0.31 may be considered a measure of the discrimination potential of 1-sec period Lg waves, i.e., how much the amplitude of explosion-generated, 1-sec period Lg waves differs from that of earthquake-generated Lg waves.
Fig. 4. $m_b(L_g)$ Versus Explosion Yield for Sources in Alluvium at NTS. The curve is a least-squares quadratic fit to the data.
### TABLE 1

**NTS HARD ROCK SITES**

<table>
<thead>
<tr>
<th>Date</th>
<th>Name</th>
<th>Rock Type</th>
<th>m_b(Lg)</th>
<th>Station</th>
<th>m_p(P-ISC)</th>
<th>Announced Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/05/62</td>
<td>Mississippi</td>
<td>tuff-above</td>
<td>5.78</td>
<td>BKS</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>09/13/63</td>
<td>Bilby</td>
<td>tuff-below</td>
<td>6.06</td>
<td>BKS</td>
<td>235</td>
<td></td>
</tr>
<tr>
<td>10/26/63</td>
<td>Shoal</td>
<td>granite</td>
<td>5.19</td>
<td>BKS,TUC</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>11/05/64</td>
<td>Handcar</td>
<td>dolomite</td>
<td>5.04</td>
<td>BKS,DUG,TUC</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>12/16/64</td>
<td>Mudpack</td>
<td>tuff-above</td>
<td>4.32</td>
<td>BKS,TUC</td>
<td>2.7</td>
<td></td>
</tr>
<tr>
<td>04/14/65</td>
<td>Palanquin</td>
<td>rhyolite</td>
<td>4.67</td>
<td>BKS,DUG,TUC</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>02/24/66</td>
<td>Rex</td>
<td>tuff-below</td>
<td>5.28</td>
<td>BKS,TUC</td>
<td>19</td>
<td></td>
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<tr>
<td>04/14/66</td>
<td>Duryea</td>
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<td>BKS,DUG</td>
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<td>05/27/66</td>
<td>Discus Thrower</td>
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<td>06/02/66</td>
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<td>06/30/66</td>
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<td>12/20/66</td>
<td>Greely</td>
<td>tuff-below</td>
<td>6.49</td>
<td>BKS,DUG</td>
<td>825</td>
<td></td>
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<td>05/20/67</td>
<td>Commodore</td>
<td>tuff-below</td>
<td>6.00</td>
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<td>250</td>
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<td>05/23/67</td>
<td>Scotch</td>
<td>tuff-below</td>
<td>5.96</td>
<td>BKS</td>
<td>150</td>
<td></td>
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<tr>
<td>05/26/67</td>
<td>Knickerbocker</td>
<td>rhyolite</td>
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<td>01/26/68</td>
<td>Cabriolet</td>
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<td>03/12/68</td>
<td>Buggy I</td>
<td>basalt</td>
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<td>BKS,DUG,TUC</td>
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<td></td>
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<td>04/26/68</td>
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<td>rhyolite</td>
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<td>BKS</td>
<td>1300</td>
<td></td>
</tr>
<tr>
<td>12/08/68</td>
<td>Schooner</td>
<td>tuff-above</td>
<td>5.32</td>
<td>BKS,DUG,TUC</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>12/19/68</td>
<td>Benham</td>
<td>tuff-below</td>
<td>6.61</td>
<td>BKS,DUG</td>
<td>1100</td>
<td></td>
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<tr>
<td>10/29/68</td>
<td>Calabash</td>
<td>tuff</td>
<td>5.88</td>
<td>BKS,DUG</td>
<td>110</td>
<td></td>
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<tr>
<td>03/26/70</td>
<td>Handley</td>
<td>tuff-below</td>
<td>6.58</td>
<td>BKS,DUG</td>
<td>&gt;1000</td>
<td></td>
</tr>
<tr>
<td>05/26/70</td>
<td>Flask</td>
<td>tuff-above</td>
<td>5.86</td>
<td>BKS,DUG</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>10/17/70</td>
<td>Carpetbag</td>
<td>tuff-below</td>
<td>6.14</td>
<td>BKS,DUG</td>
<td>220</td>
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<tr>
<td>07/08/71</td>
<td>Miniata</td>
<td>tuff-above</td>
<td>5.94</td>
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<td>04/26/87</td>
<td>Starwort</td>
<td>tuff-below</td>
<td>5.73</td>
<td>BKS,DUG,TUC</td>
<td>85</td>
<td></td>
</tr>
</tbody>
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**NTS ALLUVIUM SITES**

<table>
<thead>
<tr>
<th>Date</th>
<th>Name</th>
<th>Rock Type</th>
<th>m_b(Lg)</th>
<th>Station</th>
<th>m_p(P-ISC)</th>
<th>Announced Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/09/64</td>
<td>Pan</td>
<td>alluvium</td>
<td>5.23</td>
<td>BKS,DUG,TUC</td>
<td>4.8</td>
<td>38</td>
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<td>12/16/64</td>
<td>Parrot</td>
<td>alluvium</td>
<td>4.06</td>
<td>DUG,TUC</td>
<td>1.2</td>
<td></td>
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<tr>
<td>02/16/65</td>
<td>Merlin</td>
<td>alluvium</td>
<td>4.72</td>
<td>BKS,DUG,TUC</td>
<td>10</td>
<td></td>
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<tr>
<td>06/11/65</td>
<td>Pet: l</td>
<td>alluvium</td>
<td>4.19</td>
<td>DUG,TUC</td>
<td>1.2</td>
<td></td>
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<td>05/05/66</td>
<td>Cyclamen</td>
<td>alluvium</td>
<td>4.86</td>
<td>BKS,DUG,TUC</td>
<td>13</td>
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<td>09/21/67</td>
<td>Marvel</td>
<td>alluvium</td>
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<td>05/14/68</td>
<td>Pommord</td>
<td>alluvium</td>
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<td>DUG,TUC</td>
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<td>09/26/72</td>
<td>Delphinium</td>
<td>alluvium</td>
<td>4.87</td>
<td>BKS,TUC</td>
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\[
[m_b(Lg) - m_b(Lg)]_{average} = 0.31 \pm 0.10 \text{ (n = 23)}
\]
Regional Lg waves from East Kazakh (Degelen Mountain and Shagan River) explosions are observed at a number of WWSSN stations in Scandinavia and southern Asia. They can be used to determine $m_b(Lg)$. Assuming that the relation between $m_b(Lg)$ and yield is the same for hard-rock explosions at NTS and East Kazakhstan, the yields of East Kazakhstan explosions can be estimated.

Table 2 shows the $Q$ values, the upper and lower limits of $Q$, obtained by the coda data, and the change in $Q$, required to cause a 0.9 unit change in $m_b(Lg)$, or a 25% change in estimated yield for the small yields (<100 kt) and a 50% change in estimated yield for the large yields (>400 kt).

Table 3 gives preliminary estimates of $m_b(Lg)$ and yield for selected East Kazakh events. Of the 42 explosions listed in Table 3, the average $m_b(Lg) - m_b(tel. P) = -0.14$. For NTS, the average difference was +0.31 units. The difference between -0.14 and +0.31, or 0.45 units, could be caused by a different relation between $m_b(Lg)$ and yield for NTS and East Kazakh explosions (different excitations of Lg) or by an anomalous absorption of P amplitudes at NTS (relative to East Kazakhstan). The latter explanation appears more likely, as it would approximately agree with Chung and Bernreuter's (1981) and Herrmann and Nuttli's (1982) estimate of anomalous P-wave attenuation in the asthenosphere beneath the western United States.
<table>
<thead>
<tr>
<th>Station</th>
<th>Average Distance (km)</th>
<th>$Q_0$</th>
<th>$\xi$</th>
<th>$\Delta Q_0^*$</th>
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<tr>
<td>KBL</td>
<td>1900</td>
<td>360 (340-380)</td>
<td>0.6</td>
<td>17</td>
</tr>
<tr>
<td>KEV</td>
<td>3500</td>
<td>580 (550-620)</td>
<td>0.4</td>
<td>28</td>
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<td>KOW</td>
<td>4300</td>
<td>700 (650-760)</td>
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<td>MHF</td>
<td>2100</td>
<td>380 (350-410)</td>
<td>0.5</td>
<td>18</td>
</tr>
<tr>
<td>NDI</td>
<td>2400</td>
<td>300 (270-320)</td>
<td>0.6</td>
<td>13</td>
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<tr>
<td>NIL</td>
<td>1900</td>
<td>380 (360-410)</td>
<td>0.6</td>
<td>21</td>
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<tr>
<td>NUR</td>
<td>3500</td>
<td>580 (520-650)</td>
<td>0.4</td>
<td>31</td>
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<tr>
<td>QUE</td>
<td>2200</td>
<td>280 (260-310)</td>
<td>0.6</td>
<td>10</td>
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<tr>
<td>SHL</td>
<td>2900</td>
<td>340 (320-350)</td>
<td>0.6</td>
<td>11</td>
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<tr>
<td>UME</td>
<td>3700</td>
<td>620 (610-640)</td>
<td>0.4</td>
<td>30</td>
</tr>
</tbody>
</table>

$\Delta Q_0 = \text{change in } Q_0 \text{ to cause } 0.10 \text{ unit change in } m_b(Lg).$
### TABLE 3

\(m_b(Lg)\) AND PRELIMINARY ESTIMATED YIELDS FOR
SELECTED EAST KAZAKH EXPLOSIONS

<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th>(m_b(Lg))</th>
<th>Number of Stations</th>
<th>(m_b(P))-ISC</th>
<th>Estimated Yield (kt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-15-65</td>
<td>Shagan</td>
<td>6.02</td>
<td>2</td>
<td>5.8</td>
<td>166</td>
</tr>
<tr>
<td>11-02-72</td>
<td>S</td>
<td>6.04</td>
<td>1</td>
<td>6.1</td>
<td>178</td>
</tr>
<tr>
<td>12-10-72</td>
<td>S</td>
<td>6.06</td>
<td>2</td>
<td>6.0</td>
<td>186</td>
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<tr>
<td>04-19-73</td>
<td>Degelen</td>
<td>5.18</td>
<td>2</td>
<td>5.4</td>
<td>17</td>
</tr>
<tr>
<td>07-10-73</td>
<td>D</td>
<td>5.23</td>
<td>2</td>
<td>5.2</td>
<td>19</td>
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<td>6.22</td>
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<td>5.77</td>
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<td>01-30-74</td>
<td>D</td>
<td>5.27</td>
<td>2</td>
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<td>4.94</td>
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\[ \text{avg. } m_b(Lg) - m_b(P) = 0.15 \pm 0.19 \ (N = 41) \]
REFERENCES


Multi-mode Surface-wave Propagation

and $Q_p$ in the Continental Crust

by

Brian J. Mitchell

Recent studies have indicated that $Q$ structure varies significantly from region to region throughout continents as well as oceans. Mitchell (1975) showed that $Q_p$ values in the upper crust over a broad region of western North America were about half as large as those values in the upper crust of eastern North America.

In order to understand the regional variations of $Q_p$ in detail it is necessary to study the attenuation of seismic waves over regions which are as small as possible. A single source-single station method has been developed in order to study the anelastic properties of the crust along relatively short paths (Cheng and Mitchell, 1981). By utilizing short paths, effects of lateral variation of $Q$ along the path, as well as the effects of refraction, reflection, and multi-pathing of surface waves can be minimized. We have now applied the method to three regions in North America, the eastern United States, the Colorado Plateau, and the Basin-and-Range Province, as well as to a region of Iran and Turkey and the Barents shelf.
Method

Because of the overlap in the group arrival times of the several higher modes which comprise the phase Lg in continental regions, it is usually not possible, using single station recordings, to separate those modes for individual study. To avoid that problem, Cheng and Mitchell (1981) introduced the concept of multi-mode spectra. Using that concept, theoretical spectra, consisting of the contributions of several higher modes, can be compared to observed spectra corresponding to the composite of higher modes which form the phase Lg on long-period seismograms. An example appears in Figure 1.

There are four assumptions which are inherent in the multi-mode method: (1) that the shape of the source spectrum of the seismic event can be estimated within certain bounds, (2) that velocity structure along the path of propagation is approximately known, (3) that the mechanism which causes seismic waves to attenuate affects both the fundamental mode and higher modes in the same way and that the attenuation can be described by $\exp[-U_RTQR]$ where $U_R$ is the group velocity, and $Q_R$ is the Q value for a given Rayleigh mode at period $T$, and (4) that $Q_R$ is independent of frequency.

Assumption (1) requires, in the case of earthquakes, that the focal depth and fault-plane solution are known, and since we usually assume the source to be a step-function in time, that the observed spectra do not depart greatly from those predicted for such a source. For explosion sources, we should take account of a small maximum in the source spectrum at short periods as discussed later. The absolute level of the source spectrum need not be known, since we can attempt to fit the
Figure 1. Example of a comparison between observed and theoretical multimode spectra for a synthetic seismogram corresponding to a path across the Barents shelf. Circles denote "observed" higher-mode spectra and plus signs denote "observed" fundamental-mode spectra.
fundamental- and higher-mode spectral data by adjusting seismic moment, as well as the value of $Q$ in the upper crust.

Assumption (2) usually poses no difficulties since we can use the group velocities observed for the analyzed seismograms to determine a crustal velocity model. In our inversion, we neglect the dependence of surface wave velocities on $Q$ structure in the earth and invert for $Q$ structure alone, assuming our velocity model is reasonably well determined.

The validity of assumption (3) is the most difficult of our assumptions to assess. It is possible, for instance, that the energy in one mode might be reflected or converted to other types of waves more easily than that of another mode. For this reason, we attempt to use relatively short paths across regions which are devoid of obvious major lateral changes. In addition, we attempt to use several paths in different locations and directions and to find a model which explains all of the data as well as possible.

Assumption (4) may not be correct since $Q_P$ appears to increase with decreasing period, at least at periods shorter than 2 or 3 seconds (Mitchell, 1980). As shown in that study, however, it is not likely, using surface wave data and presently available methods, that we could detect a frequency dependence of $Q_P$ at periods between about 3 and 50 seconds. In the absence of data at shorter periods, therefore, we must restrict our study to the determination of frequency-independent models.

For the regions discussed below, we make yet another assumption, that the value of $Q_B$ in the lower crust is much higher than that in the
upper crust. This assumption is based both upon previous determinations of $Q_\beta$ in the central United States where the best short-period surface wave attenuation data have been obtained, and upon the idea that fluid-filled cracks and pore-space in the crust, which are likely contributors to crustal attenuation close at mid-crustal depths. Although we have assumed that $Q_\beta$ is 1000 in the lower crust, we shall see that our observed spectra are not very sensitive to lower crustal $Q_\beta$ values.

The multi-mode method considers the amplitude spectrum of the combined higher modes, as well as the spectrum of the fundamental mode. If $Q_\beta$ in the lower crust is much higher than $Q_\beta$ in the upper crust, then the fundamental-mode spectrum is more strongly affected than the higher-mode spectrum by changes of $Q_\beta$ in the upper crust (Cheng and Mitchell, 1981). There is, of course, some effect of the shapes of both spectra by changes in $Q_\beta$. Given the scatter in the data, however, it is not possible to assign significance to small changes in spectral shape. We therefore look at two things: the ratio of the fundamental-mode spectrum to that of the higher modes and the gross shape of the spectra. In situations, such as those discussed below, where $Q_\beta$ values in the lower crust are higher than those in the upper crust, the higher-mode spectrum serves mainly as a reference to which the fundamental-mode spectrum is compared. Of course, large differences between the shapes of the observed and theoretical spectra would lead us to try other $Q$ models. In later paragraphs we will see that both the ratios and shapes of the spectra are primarily sensitive to the $Q_\beta$ values in the upper crust.

We are limited in our interpretation by two factors, (1) the relatively poor resolution obtainable using seismic wave amplitudes, even if
we have high-quality data, and (2) the scatter in our data due to departures from ideal sources and ideal propagation conditions. We therefore attempt to find only very simple Q models of the crust. These models are much cruder than velocity models which can be obtained using modern inversion methods. Our purpose is, however, to look only for major variations of $Q_p$ in the upper crust of various regions of the world. Fortunately, the regional variation of $Q_p$ at those depths can be quite large and the differences from region to region can often be easily detected.

The procedure for employing the multi-mode method begins by obtaining spectral amplitudes of the fundamental- and combined higher-mode Rayleigh waves using the multiple-filter method (Dziewonski et al., 1969). The fundamental mode is usually delineated without difficulty. Local amplitude maxima are taken to correspond to higher modes if they are within ±0.1 km/sec of the theoretical group velocity for the appropriate velocity model. These observed spectral amplitudes are then compared to theoretical spectra computed for various $Q_p$ models utilizing the formulation of Levshin and Yanson (1971) to compute spectra and the equations of Anderson et al. (1965) to include Q effects.

As shown in the following section, the multi-mode method is primarily sensitive to $Q_p$ in the upper crust. Therefore that is the only crustal parameter which is searched for, and it is not difficult to obtain a satisfactory model by trial and error. Our initial procedure, in this study, is to find the model which provides the best trial-and-error fits to all of the spectra. Our only adjustable parameters are therefore $Q_p$ in the upper crust and the seismic moment for each of the
events used in the study. The seismic moment is only a scale factor which sets the level of the spectra, whereas the Q model affects both the shape of the spectra and the ratio between higher- and fundamental-mode spectral values.

The possible sources of error inherent in the multi-mode method may be separated into two groups: those which are associated with uncertainties in the source spectrum and those which may occur because of propagation effects between the source and receiver. Errors due to source effects include uncertainties in focal depth, strike, dip, and slip of the fault on which an earthquake occurs, as well as the effects of finiteness of the fault and deviations from the assumed source time function. For an ideal explosion, the factors pertaining to the geometry of the fault plane (strike, dip, slip, finiteness) do not, of course, enter in. If, however, an explosion causes motion on a fault, then the possible effect of those factors on the spectra cannot be ignored. Errors due to propagation effects include those due to an incorrect velocity model and lateral variations of elastic and anelastic properties along the path of propagation.

Tsai and Aki (1970) showed that the effects of uncertainties in the angles of strike, dip and slip upon the shape of amplitude spectra are relatively small, if those angles are known to within 15°. The effect of uncertainties in focal depth is larger and variations in focal depth can produce significant changes in spectral shape. In the present study, the focal depths are considered to be known quite well, but if they are not, it is still possible using the locations of spectral minima in some cases, to estimate the depth.
The effect of source finiteness should be considered when using earthquake sources and also when using explosion sources which produce significant fault movement. Ben-Menahem (1961) showed that the amplitude of the finiteness factor can be written as \( \frac{\sin X}{X} \), where \( X = \frac{\omega L}{2(1/v - \cos \theta/c)} \). \( L \) and \( v \) are the length and rupture velocity of the fault, respectively, \( \omega \) is the phase velocity, \( \omega \) is angular frequency, and \( \theta \) is the angle measured from the propagation direction. Assuming a rupture velocity of 3.0 km/sec and a fault length of 1 km, values representative of an intraplate earthquake of magnitude 4.5 (Nuttli, 1981), we find the finiteness effect to be insignificant at the periods used in this study. If the fault is assumed to be 3 km long, then the finiteness factor at the shortest period used in this study (3.5 sec) is about 0.64 for the fundamental mode and 0.73 for the first higher mode. Considering the uncertainty in our amplitude data, those factors are of little consequence to our results. For explosions, there will be no effect of source finiteness unless the explosion causes significant fault movement. Although that possibility may occur, fault movement cannot be very large in the cases we have considered since little Love wave particle motion was found on the horizontal component seismograms.

We assume the source time function for earthquakes to be a step. That assumption is consistent with observations from numerous studies of small shallow-focus earthquakes. Departures from a step-function source, for the present purpose would produce differences in spectral shape but would not affect the ratio of higher- to fundamental-mode amplitude spectra. The source spectrum for explosions differs from that of earthquakes by including a small maximum near the short-period end of the spectrum. The effect of that maximum on observed spectra is
discussed in later paragraphs.

The effects of changes in the velocity model upon spectral amplitudes has been studied previously (e.g. Mitchell, 1975). These effects have been found to be insignificant compared to those produced by changes in the Q model as long as the velocity model is reasonably close to the true structure. In the present study the velocity models explain the surface wave dispersion data very well, so uncertainties in the velocity model are not expected to adversely affect our results. The effects of lateral changes in elastic and anelastic properties are more difficult to assess. Lateral changes may produce lateral refraction, multi-pathing and scattering which can distort the recorded waveforms and spectra. The multiple-filter method at least partially mitigates adverse effects of those factors by separating late-arriving phases.
The method was first applied to three regions of North America: the eastern United States, the Colorado Plateau, and the Basin and Range Province, as well as to Iran and the Barents shelf. $Q_\text{p}$ values in the upper crust were found to be about 250 for the eastern United States, 150 for the Colorado Plateau, and less than 80 for the Basin and Range, using a trial-and-error process. An automatic inversion procedure produced slightly higher values (Figure 2). The value for the eastern United States is consistent with that found independently by other methods (Herrmann and Mitchell, 1975).

Upper crustal $Q_\text{p}$ values for the region of Iran and Turkey were found to be similar to those of the Basin and Range. The data in that region required smaller values in the lower crust however. The model which provides the best fit to amplitude spectra throughout most of the Iranian Plateau and Turkey consists of a 13 km thick upper crust with a $Q_\text{p}$ value of 85 overlying a lower crust with a $Q_\text{p}$ value of 300 (Figure 3). Data for the region around Meshed, Iran, however, are better explained by a model in which the upper crust is 18 km thick and has an average $Q_\text{p}$ value of 75. The complexity of several of the spectral shapes in this area indicates substantial lateral complexity of the crustal structure there.

The multi-mode method was also used to study the attenuation properties of the crust throughout the Barents shelf. The sensitivity of the spectra to various source factors and to propagation factors which might characterize a shelf region were thoroughly studied. For plane-layered models, the method was found to be very sensitive to $Q_\text{p}$ in the
Figure 3. Two-layer crustal models obtained for Iran and Turkey. The solid lines indicate a model appropriate for most of the region and the dashed lines indicate a model appropriate for the region near Meshed, Iran.
upper crust and to $Q_B$ of sedimentary layers if those layers are thick and highly attenuating. It is far less sensitive to $Q_B$ in the lower crust and to $Q$. The spectra, for shallow strike-slip earthquakes, were very insensitive to changes in strike, slip, and dip angles of the fault throughout the entire period range of interest, but are very sensitive to variations in focal depth except in the long-period range of the fundamental mode and the short-period range of the higher modes. The fundamental-mode, but not the higher mode, spectra for one path across the Barents shelf were, apparently, strongly distorted at short periods by lateral heterogeneities along the travel path. Simple two-layer, trial-and-error inversions yield a model of the Barents shelf with an upper crustal $Q_B$ value of about 80 (Figure 4). Three-layer inversions with a sedimentary layer, however, yield an upper crustal $Q_B$ value of about 250, if the sediments 1 km thick are characterized by $Q_B$ values of 40 (Figure 5).
Figure 4. Two-layer crustal model of the Barents shelf. The model is overlain by a water layer 0.3 km thick.
Figure 5. Three-layer model (overlain by water) of the Barents shelf.
References


Frequency Dependence of $Q_p$

in the Continental Crust

Most studies which obtain models of shear wave $Q_p(Q_p)$ as a function of depth assume that that quantity is independent of frequency. That assumption is computationally convenient and is based largely upon laboratory $Q$ measurements made at low pressures and temperatures. During the period of this contract, however, evidence has been found that $Q_p$ in the upper crust varies with frequency, at least at frequencies near 1 Hz. This result has implications for observations of regional phases and yield determinations from them since it means lower attenuation than that which would be predicted by frequency-independent models obtained from the analysis of lower-frequency surface waves.

The frequency dependence of $Q_p$ in the crust of the eastern United States has been studied using both frequency-domain and time-domain methods. Only the results of the frequency-domain method will be discussed in this section. The results of the time-domain method are discussed in the following section on the synthesis of short-period Lg waves. It is shown there that approximations made in the frequency-domain method (mainly the use of only a few modes) lead to correct estimates of 1 Hz Lg attenuation coefficient values, but to underestimates of absolute amplitudes.
Frequency-Domain Method

The Rayleigh wave attenuation data of Herrmann and Mitchell (1975) were inverted to obtain $Q_p^{-1}$ models of the crust assuming various types of frequency dependence. The attenuation coefficient values predicted for higher modes for those various models were then compared with attenuation coefficients observed for 1 Hz $L_g$ waves reported by various workers in the eastern and central United States.

In computations of higher mode attenuation, it was assumed that observed attenuation coefficient values at periods of 3 seconds and larger consist of only the first higher mode, that the value at 2 seconds is the average of the first and second higher mode values, and that at a period of 1 second, the observed value is the average of the attenuation coefficient values for the first four higher modes. Since differences in the attenuation coefficient values among the various higher modes are not great for the eastern United States, the results of this study do not depend critically upon the number of modes employed at any period.

The ability to separate the effects of frequency-dependence of $Q_p$ from that of depth-dependence is possible because higher modes sample a different depth interval in the crust than does the fundamental mode for a given period. The higher modes at 1 second period sample the same depth interval as that sampled by the fundamental mode at a period of 6 seconds (about 20 km). Likewise, the first higher mode at a period of 6 seconds samples a depth interval similar to that sampled by the fundamental mode at a period of 25 seconds (about 60 km).
The initial step in the procedure employed is to invert the fundamental mode data to obtain models of $Q_B^{-1}$ as a function of depth. If it is assumed that compressional wave $Q$ values, $Q_p$, are twice as large as $Q_B$, then the Rayleigh waves attenuation coefficient values, $\gamma_R$, are related to $Q_B^{-1}$ and the elastic properties by

$$
(1)
$$

where $c$ and $\beta$ are compressional and shear wave velocities, $C_R$ is Rayleigh wave phase plus velocity, $T$ is period, and $i$ is a layer index (adapted from Anderson et al., 1965). The subscripts $\omega$, $c$, $\beta$, and $i$ refer to frequency, compressional velocity, shear velocity, or density being held constant. The assumption that $Q_p = 2Q_B$ has little affect on our results since surface wave attenuation is so slightly affected by $Q_p$. The above equation is usually employed assuming that $Q_p$ is independent of frequency. In the present study, however, it will be assumed that $Q_p$ depends upon both depth and frequency, so that in each layer,

$$
(2)
$$

where $q$ can be constant through the entire frequency range of interest or can vary with frequency. Note that if $q = 0$, then $Q_p$ is independent of frequency in the commonly assumed way.

The fundamental-mode data are inverted assuming various values for $q$. The fits of the theoretical curves to the data of Figure 1 for $q = 0.0$ and $0.5$ indicate that the fundamental mode data can be explained equally well by a wide range of modes corresponding to different values. This situation occurs because variations in frequency dependence represented by different values of $q$ in (1) can be offset by variations in values of $C$ at different depths in the model.
The models obtained can now be used to compute attenuation coefficient values for the higher modes with a forward calculation using equation (1). The resulting values can then be compared with the higher mode data of Figure 1.
Figure 1. Fit of theoretical higher-mode attenuation coefficients to observed values for various values of $\xi$, assuming that $\xi$ is constant over the entire period range 1-40 seconds.
Results

The results of computations for five values of $\eta$, between 0.0 and 1.0, appear in Figure 1. It is immediately clear that a frequency-independent model ($\eta = 0.0$) does not even come close to explaining the 1-second $L_g$ data. Although $\eta$ values between 0.3 and 0.5 provide better fits, none of these curves for which $\eta$ is constant over the entire frequency range provide very good fits to the entire data set between 1 and 10 seconds.

It is interesting to note that all curves for $\eta$ between 0.0 and 0.5 are consistent with the data at periods of 4 seconds and more. This consistency supports the frequency-independent internal friction models of Mitchell (1973) and Herrmann and Mitchell (1975) as being representative of the crust in eastern North America. It can also be seen that small negative values of $\eta$ would also explain the data at periods greater than 4 seconds.

Further calculations were performed in which $\eta$ was allowed to vary with period. Although it is possible to achieve excellent fits to the data in some of these cases, it was considered that because of the large uncertainties in the data, that it is sufficient to consider only cases where $\eta$ is constant in this report.

Models for which $\eta$ is constant over the entire period range appear in Figure 2. Although it does not adequately explain the higher-mode data, a frequency-independent model ($\eta = 0.0$) is presented, along with its standard deviations. A second model appears for the case where assumes the constant value of 0.3 at all periods. Other models are pos-
Figure 2. (left) Frequency-independent model of $Q^{-1}$ in the eastern United States. (right) Frequency-dependent model in which $Q$ varies with frequency as $\alpha = 0.3$. 

\( \alpha = 0.0 \quad \alpha = 0.3 \)
sible, and perhaps even preferable, in which \( \psi \) assumes other constant values or varies with frequency. In the following section on the synthesis of short-period \( L_g \), it appears that \( y = 0.5 \) explains synthetic data reasonably well.

The computations of the present study indicate that \( Q_{ph}^{-1} \) in the crust in eastern North America is frequency dependent. Several possible models can, however, explain fundamental- and higher-mode Rayleigh wave attenuation coefficient data at periods of 1 second and greater. The models differ in the nature of their dependence upon frequency. One reason for the non-uniqueness of the results is the occurrence of a gap in the higher-mode data at periods between 1 and 4 seconds. If reliable data can be obtained for that range, it would be possible to exclude some of the models we have considered. Later work (Mitchell, 1981) does indeed suggest that we can eliminate models which produce peaks in attenuation at periods between 1 and 4 seconds.
References


Synthesis of Short-period Lg
in the eastern United States
by
Brian J. Mitchell

Recent work (e.g. Bache et al., 1981; Wang, 1981) has allowed us, for the first time, to compute short-period seismograms utilizing modal summation. This allows us to compare computed Lg waveforms with those actually observed on short-period records.

Early computations were performed to see how many modes are required to obtain useful attenuation coefficient values. It was found for sources more than a few km deep that as few as 5 modes gave useful values for an eastern United States model. For very shallow source depths (less than 1 km) and a thick low-velocity sediment layer, more modes are needed, but 10 modes seem sufficient in all cases. Absolute amplitude values, however, require more modes to be realistic. It appears that at least 15 modes are needed and, in some cases at least, as many as 30 or more are desirable to obtain realistic results.

Later work showed that it was necessary to include a surface layer with relatively low velocities if we are to compute realistic short-period seismograms of the Lg phase. Figures 1 and 2 compare a model having low-velocity surface sediments with a model lacking such a layer (see Table 1). The latter model produces seismograms having spikelike arrivals which are not observed on real seismograms.
Figure 1. Vertical component synthetic seismograms at 4 distances for the Simplified EUS model. $Q_a = 2000$ and $Q_B = 1000$ at all depths.
Figure 2. Vertical component synthetic seismograms at 4 distances for the velocity model of McEvilly (1964). $Q_α = 2000$ and $Q_β = 1000$ at all depths.
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<td>2.90</td>
</tr>
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<td>3.98</td>
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<td>180.00</td>
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Table 1
Eastern and Central United States Velocity Models

Simplified EUS Model

McEvilly (1964)
Comparison with Observed Seismograms

Seismograms recorded at several stations for a small central United States earthquake of 14 August 1965 were digitized for comparison with synthetic records. Synthetics were computed for three models: one in which $Q_p$ is independent of frequency, one in which $Q_p$ varies with frequency as $u^{0.5}$, and one in which $Q_p$ varies with frequency as $u^{0.5}$ but which includes a layer of low-$Q$ sediments. Examples are shown in Figures 3 and 4 for Rayleigh waves and Love waves, respectively. It can be seen that most, if not all, of the observed coda can be explained by synthetic seismograms in plane-layered models. It is possible to explain the gross features of the coda, such as duration, absolute amplitude, and perhaps envelope shape in some cases, but not the details of the wave form.

At short distances, all frequency-dependent models tested produced a more prominent and more regular fundamental-mode Rayleigh wave train than that which was observed. At larger distances the time interval on observed records corresponding to the fundamental mode is comprised of more irregular wave forms than the synthetics. This time interval may contain the remnant of the fundamental mode after attenuation by intrinsic $Q$ and scattering as well as converted modes - fundamental to higher modes - and vice versa.

Figures 5 and 6 present comparison of observed and theoretical amplitudes from Rayleigh and Love waves, respectively, for two $Q$ models. A frequency-dependent model in which $Q$ varies with frequency as $u^{0.5}$ provides a better match in both cases.
Figure 3. Observed and synthetic vertical seismograms for station ATL (Atlanta, GA). The uppermost trace is the observed seismogram, the second trace is a synthetic seismogram for a frequency-dependent model, the third is a synthetic seismogram for a frequency-dependent model where $Q$ varies as $\omega^{0.5}$, and the bottom trace is for the same model as trace c, but with a layer of low-$Q$ sediments.
Figure 4. Observed and synthetic SH seismograms for station RCD (Rapid City, SD). See caption for Figure 3.
Figure 5. Observed Rayleigh amplitudes at 1 Hz versus theoretical amplitudes predicted by frequency-independent ($\xi = 0.0$) and frequency-dependent ($\xi = 0.5$) $Q_\delta$ models of Mitchell (1980).
Figure 6. Observed Love amplitude at 1 Hz versus theoretical amplitudes predicted by frequency-independent ($\zeta = 0.5$) $Q_B$ models of Mitchell (1980).
References


