Coefficient of Variation Spectral Analysis: An Application to Underwater Acoustics

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Preface

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Acoustic noise in the ocean is often described in terms of its power spectral density. Just as in other media, this noise consists of both narrowband and broadband frequency components. A major problem in the analysis of power spectral density measurements is distinguishing between narrowband spectral components of interest and contaminating narrowband components. In this paper, the use of coefficient of variation (C_V) spectrum is examined as an adjunct to the conventional power spectrum to distinguish narrowband components of interest

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20. Continued:

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from contaminating components. The theory of the C_V is presented. Coefficients for several classical input distributions are developed. It is shown that C_V spectra can be easily implemented as an adjunct procedure during the computation of the ensemble of averaged power spectra. Power and C_V spectra derived from actual at-sea sonobuoy measurements of deep ocean ambient noise are shown. Whereas, it is virutally impossible to separate narrowband contaminating components from narrowband lines of interest in the ensemble of averaged power spectra, these acoustic components of interest can be distinguished in the C_V spectra.

Coefficient of Variation Spectral Analysis:
An Application to Underwater Acoustics

Power spectral analysis using the fast Fourier transform (FFT) has become a routine method for the detection of narrowband components of interest in broadband data. Unfortunately, numerous contaminating narrowband components of no interest are often present.

NARROWBAND ANALYSIS OF UNDERWATER ACOUSTIC DATA

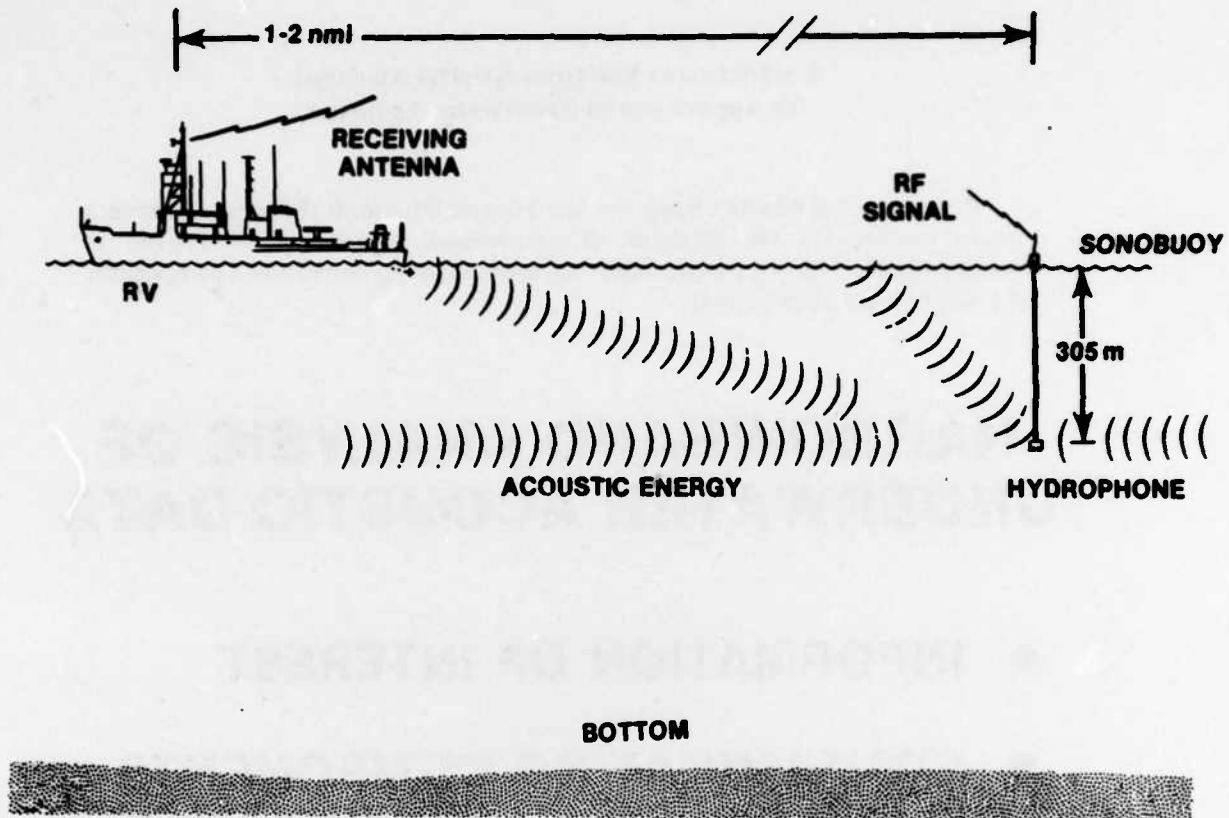## • INFORMATION OF INTEREST

• CONTAMINATING COMPONENTS

Slide 1

Today, we will examine the use of the coefficient of variation (C_V) spectrum as an adjunct to the conventional power spectrum to distinguish narrowband components of interest from the contaminating components. First, we will present a problem in underwater acoustics requiring the identification of narrowband components of interest in the presence of numerous contaminating components. Next, the mathematics of the C_V spectrum will be defined. The C_V spectrum will be developed for several types of input processes. Finally, results obtained from at-sea measurements will be examined.

1

ACOUSTIC EXPERIMENT

1-2 nmi

RECEIVING
ANTENNA

RF
SIGNAL

SONOBUOY

RV

305 m

ACOUSTIC ENERGY

HYDROPHONE

BOTTOM

Slide 2

As an example of the problem, consider an experiment designed to measure the narrowband components of the acoustic energy radiated by a research vessel. To conduct this experiment, a sonobuoy is deployed 1 to 2 nmi from the research vessel. Acoustic energy (radiating from the ship's machinery, propellers, etc.) travels underwater to the sonobuoy's underwater microphone. The acoustic data are then telemetered back to the research vessel via a radio frequency link where the data are recorded on magnetic tape. As can be seen in this slide, acoustic energy from directions other than the research vessel is also arriving at the sonobuoy. As the open ocean is a noisy environment, this energy could contain narrowband components generated by distant ships or other sources. However, the only information of interest is the acoustic energy emanating from the research vessel When the sonobuoy data are played back for power spectral analysis, additional narrowband contaminents (such as, ac line voltage interference and tape recorder artifacts) can also be introduced. Consider the situation when there is no *a priori* information about the number and frequencies of the narrowband components associated with the research vessel's acoustic energy. It would then be extremely difficult to identify these components from all the other contributing narrowband components observed in the power spectrum of the sonobuoy data.

2

POWER SPECTRUM ANALYSIS

- **MAGNITUDE SQUARED PROCESS**

- **CLASSIFICATION BY SNR**

- **BANDWIDTH/FLUCTUATION LIMITATIONS**

Slide 3

The problem is that power spectral analysis provides enough information to distinguish narrowband components from broadband data, but not enough information to distinguish the narrowband information of interest from the contaminants. Since power spectral estimation is essentially a magnitude squared process, narrowband components are distinguished from broadband components by the observation that the level associated with a particular FFT bin is higher than the level in adjacent bins. Calling the contents of these nearby bins "noise," one can detect narrowband components in the noise by use of a signal-to-noise threshold. Any narrowband line, regardless of source, will be classified as a narrowband component if it exceeds the specified signal-to-noise ratio (SNR).

In order to obtain a smooth estimate of the broadband components, individual power spectra are often averaged to provide an ensemble of averaged power spectra. Unless the FFT resolution is sufficiently narrow, it is impossible to determine from the ensembled averaged spectra whether the power level of the narrowband component was constant over the ensemble estimate or varied significantly from spectrum to spectrum. Yet, this information could serve as an additional classification tool.

MOMENT CHARACTERIZATION OF SPECTRAL DATA

- **ENSEMBLE OF SPECTRAL ESTIMATES**
- **CHARACTERIZE BY STANDARD DEVIATION TO MEAN RATIO**
- **WEAKLY FLUCTUATING PROCESSES WILL HAVE LOW RATIOS**
- **HIGHLY FLUCTUATING PROCESSES WILL HAVE HIGH RATIOS**

Slide 4

Consider statistical moment characterization of spectral estimates. If the ensemble of spectral estimates is thought of as a sequence of power values, then the statistical moments of that sequence can be computed. In fact, the ensemble of averaged spectra is the first moment, or mean, of the sequence. If the second moment is computed, then the standard deviation of the sequence can be derived. The standard deviation provides a measure of the fluctuation of the sequence. If the standard deviation is divided by the mean, this normalized measure of the fluctuations is called the C_V (coefficent of variation). A sequence of narrowband power spectral values with a standard deviation less than the mean will have a C_V of less than one. Conversely, a sequence with a standard deviation greater than the mean will have a C_V greater than one.

COEFFICIENT OF VARIATION SPECTRUM

1) G(f) = TRUE SPECTRAL DENSITY

2) $\widehat{G}(f)$ = ESTIMATED SPECTRAL DENSITY

3) $C_V(f) = \dfrac{\sqrt{E[\widehat{G}(f)^2] - E^2[\widehat{G}(f)]}}{G(f)}$

4) $C_V(f) = \dfrac{\sigma(\widehat{G}(f))}{G(f)}$

5) $\widehat{C}_V(f) = \dfrac{\sigma(\widehat{G}(f))}{E[\widehat{G}(f)]}$

Slide 5

The C_V spectrum can be developed as follows. Equation 1 defines G as the true power spectral density of an input process. It is a function of frequency. In equation 2, $\widehat{G}$ is the *estimated* spectral density. The C_V is then defined as a ratio given in equation 3. The numerator is the square root of the difference between the estimated power spectral density's second moment and first moment squared. The denominator is the true power spectral density. The C_V can be more simply defined, in equation 4, as the ratio of the standard deviation of the estimated power density to the true spectral density. In actual practice, the true spectral density is seldom known. Thus, we must use an estimate of equation 4. As shown in equation 5, the estimated coefficient of variation ($\widehat{C}_V$) is now purely in terms of the estimated power spectral density.

ESTIMATION OF C$_V$(f)

CONSIDER AN N ELEMENT ENSEMBLE OF $\widehat{G}(f)$

6) $\widehat{G}_i(f) \qquad i = 1, 2, \ldots N$

7) $E(\widehat{G}(f)) = \dfrac{1}{N} \sum\limits_{i=1}^{N} \widehat{G}_i(f)$

8) $\sigma(\widehat{G}(f)) = \left[\left(\dfrac{1}{N-1}\right) \sum\limits_{i=1}^{N} (\widehat{G}_i(f) - E(\widehat{G}(f))^2 \right]^{1/2}$

9) $\sigma(\widehat{G}(f)) = \left[\left(\dfrac{1}{N-1}\right) \left| \sum\limits_{i=1}^{N} \widehat{G}_i^2(f) - 2E(\widehat{G}(f) \sum\limits_{i=1}^{N} \widehat{G}_i(f) + E^2(\widehat{G}(f)) \right| \right]^{1/2}$

10) $\widehat{C}_V(f) = \dfrac{\sigma(\widehat{G}(f))}{E(\widehat{G}(f))}$

Slide 6

The $\widehat{C}_v$ can be readily computed provided that the individual elements, $\widehat{G}_i$, comprising the ensemble of N spectral estimates are available, as defined in equation 6. In equation 7, the first moment of the ensemble is computed. This result is the ensemble of averaged power densities. Equation 8 defines the estimate of the unbiased standard deviation. Equation 9, an expansion of equation 8, is shown because the computer implementation of this form of the standard deviation can be accomplished without using matrices. The ratio of equation 9 to equation 7 is the $\widehat{C}_v$ spectrum, as shown in equation 10. Since $\widehat{C}_v$ is so dependent on the ensemble of averaged power spectral densities, it is important to choose a sufficiently large value of N so that the ensemble averaged power is a good estimate of the true power. However, N should be less than the value that would make the sequence of spectral estimates nonstationary.

COEFFICIENT OF VARIATION FOR GAUSSIAN INPUT

11) $G_X(f) = 2 \lim\limits_{T \to \infty} \dfrac{1}{T} E[|X(f,T)|^2]$

12) $\widehat{G}_X(f, T) = \dfrac{2}{T}(X_R^2(f, T) + X_I^2(f, T))$

13) $C_V(f) = \dfrac{\sigma(\widehat{G}(f))}{G(f)}$

14) $C_V(f) = \dfrac{\sqrt{2n}}{n}$; $n = 2$

15) $C_V(f) = 1$

16) $10 \, LOG_{10} \, C_V(f) = 0 \, dB$

Slide 7

Consider an acoustic sequence called X that is Gaussian with zero mean. Its true spectral density is arbitrary, but can be strongly frequency dependent. Equation 11 presents the classical definition of the power spectral density, where X is the Fourier transform of the pressure sequence. Recall that the Fourier transform of a Gaussian process is Gaussian. The variable T is integration time. Equation 12 defines the estimated power density as the sum of the squares at the real and imaginary terms. Thus, G has a Chi-squared distribution of order 2. The C_V is the ratio in equation 13. Equation 14 presents the ratio for the general case of n degrees of freedom. When $n = 2$, the C_V reduces to a value of 1. The corresponding logarithmic result is 0 dB. Note that this result is independent of frequency. Thus, regardless of the power spectral density associated with the input signal, as long as the input has a Gaussian distribution, it will have a true C_V value of 1, independent of frequency.

C_V FOR SPECTRAL LEVEL OF DATA CONTAINING A SINUSOID IN GAUSSIAN NOISE
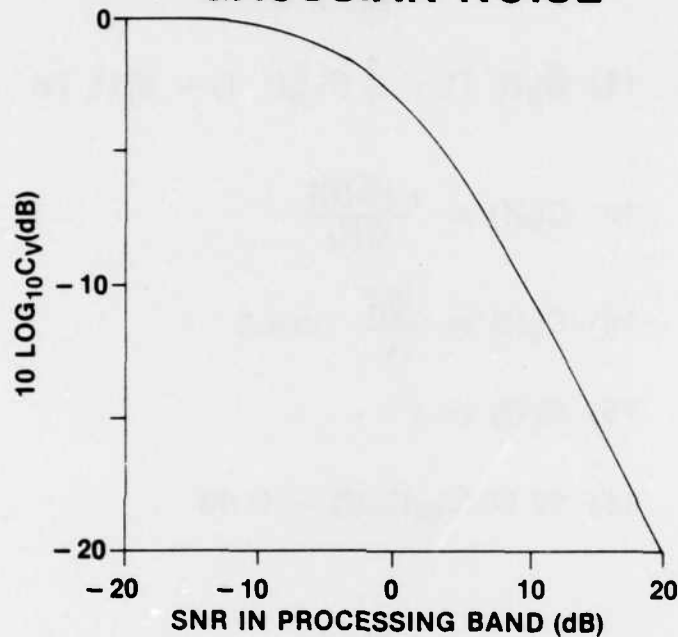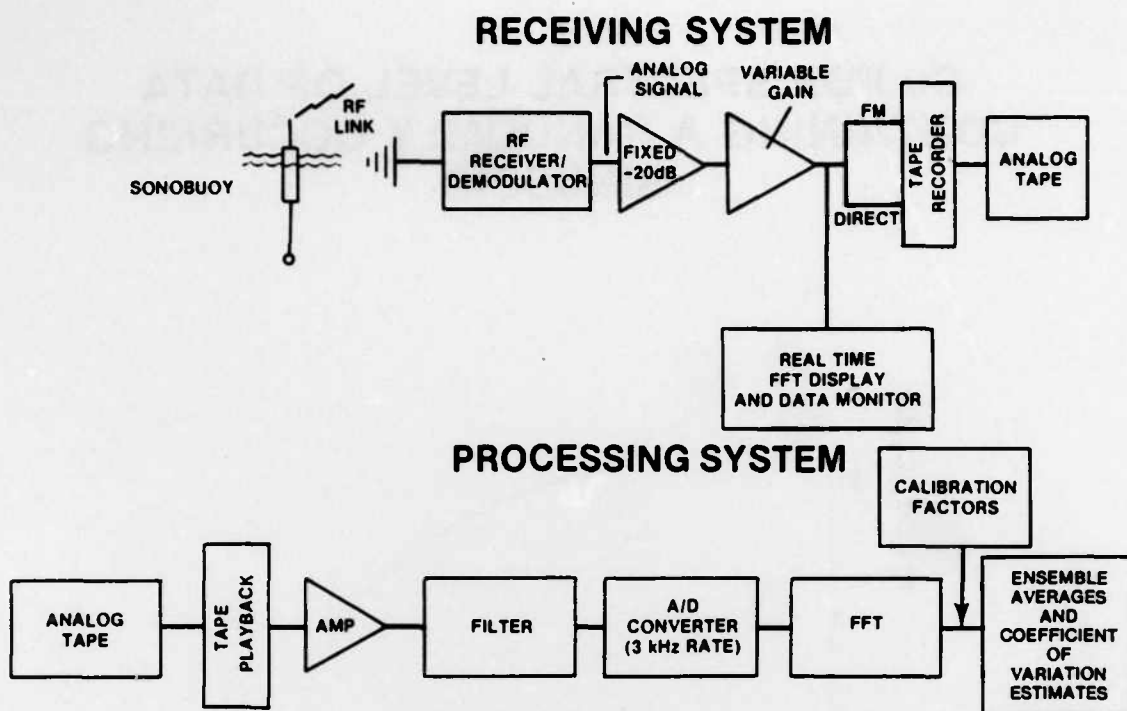

Slide 8

Now, consider a more complicated pressure sequence consisting of a sinusoidal signal of *fixed* amplitude in the presence of zero mean Gaussian noise. Consider the C_V in dB space as a function of SNR. Note that for very low SNR, C_V is 0 dB. This is consistent with the previous result of 0 dB for a purely Gaussian input. As the SNR increases, C_V decreases. This is intuitively logical, since the power spectrum of a sinusoid of constant level will itself be constant; hence, its second moment will be zero, and the resulting logarithmic C_V will be minus infinity.

C$_V$ FOR SPECTRAL LEVEL OF DATA CONTAINING A RANDOMLY OCCURRING SINUSOID

**PERCENTAGE OF OCCURRENCE OF THE SINUSOID (%)**
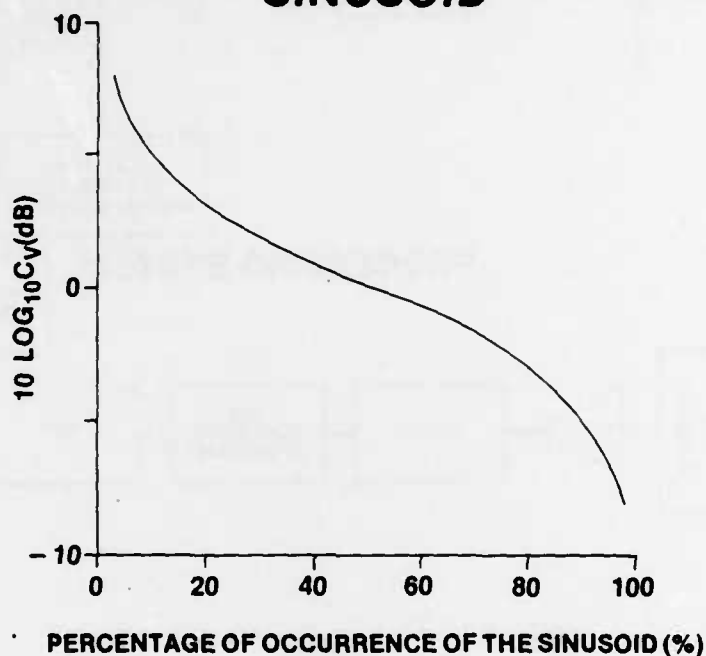
Slide 9

In the previous example, C$_V$ could never be greater than 0 dB, since the standard deviation of the power density was always less than the mean. Now, consider a sequence in which a sinusoid is occurring a specified percentage of the time, and the remaining percentage is reduced to zero level input. As can be seen, when the occurrence of the sinusoid is less than 50 percent, C$_V$ will be greater than 0 dB. As the percentage of occurrence of the sine wave increases, C$_V$ decreases. Again, this is intuitively logical since when the sinusoid is occurring 100 percent of the time, the C$_V$ will be minus infinity dB, i.e., infinitely small in linear space.

RECEIVING SYSTEM

PROCESSING SYSTEM

Slide 10

Now consider C_v applied to a set of measured underwater acoustic data. Recall the experiment described earlier. The goal of that experiment was to measure the narrowband components of the acoustic energy radiated by a research vessel. To accomplish this, a sonobuoy containing a hydrophone was deployed and operated at about 1 to 2 nmi from the vessel. Shown in the upper portion of this slide is a block diagram of the receiving system onboard the research vessel. The telemetered data were demodulated, amplified, and recorded on tape. The data were also monitored with a real time FFT power spectrum analyzer. When this exercise was conducted, many more narrowband lines were observed than originally anticipated. After the sea-test, the data were then processed as shown in the lower portion of this slide. Data from the analog tapes were filtered, A/D converted, and then FFT analyzed. Both ensemble of averaged power spectra and C_v spectra were computed.

POWER SPECTRUM
SONOBUOY UNDERWATER ACOUSTIC DATA
# 5 MIN AVG, 1.1 Hz RESOLUTION
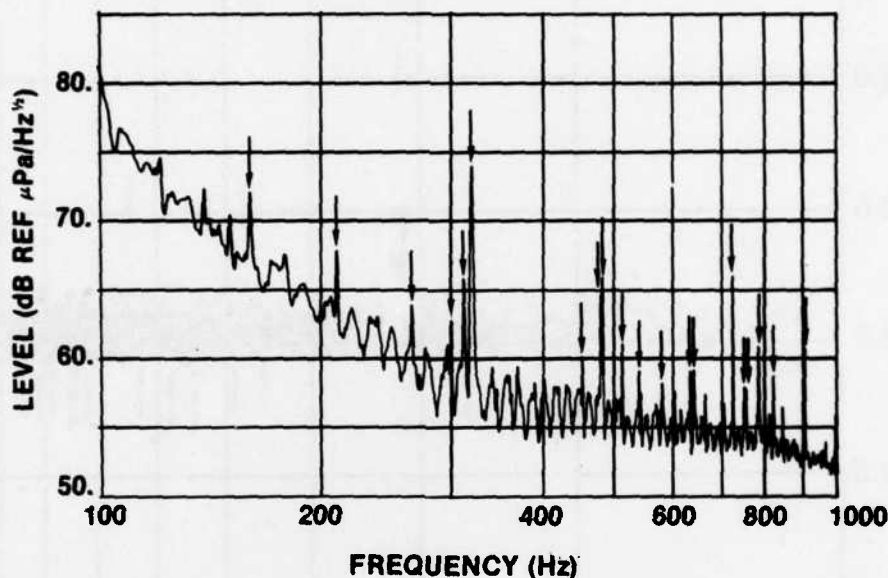
Slide 11

Here is the ensemble of averaged power spectra for 5 minutes of received data. It was computed from Hanning weighted 50 percent overlapped input records. The effective resolution is 1.1 Hz. Although somwhat difficult to see, note the numerous narrowband components rising above a broadband background that decreases as frequency increases. The arrows indicate the narrowband components that were at least 3 dB above the broadband background. Could some or all of these components have been generated by the research vessel?

COEFFICIENT OF VARIATION SPECTRUM
SONOBUOY UNDERWATER ACOUSTIC DATA
5 MIN AVG, 1.1 Hz RESOLUTION
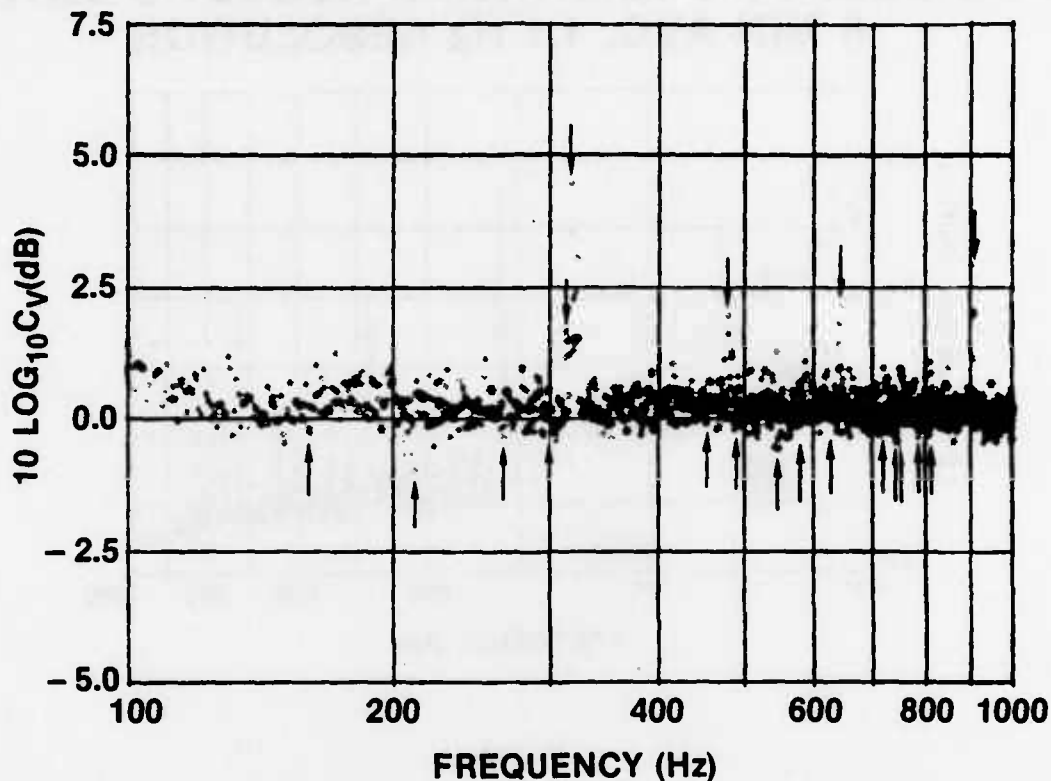
Slide 12

This is the C_V spectrum derived from the same spectra used to compute the ensemble of averaged power spectra just shown. Striking differences can be seen. First, the broadband background now is essentially independent of frequency. Second, the number of observable narrowband components has been drastically reduced. The arrows indicate the narrowband components identified in the previous slide. Only five lines are observable above approximately 2 dB, and no lines are observed below −2 dB. How can these results be interpreted?

Recall that the logarithmic C_V of a Gaussian process is 0 dB. This is true whether the process is narrowband or broadband. Thus, those lines previously observable in the power spectrum, but not observable here, may actually be narrowband Gaussian processes. Distant narrowband lines of near constant source level can be amplitude modulated by the ocean. If this modulation is Gaussian-like, then it would be expected that distant source narrowband components would have C_V on the order of 0 dB, regardless of power spectral level.

What about the five lines with large positive coefficients? Recall the case of a sinusoid randomly occurring a given percentage of the time. During the 5 minutes this data were acquired, the research vessel's engines were being constantly adjusted in an attempt to station keep. In fact, a separate examination of C_V in the frequency regime of 20-30 Hz, where ship's propeller energy is expected, showed C_V values greater than 3 dB. Thus, it is very likely that all five lines shown here are associated with the research vessel. The observation that there are no C_V values less than –2 dB is also significant. Based on the model of a constant amplitude sinusoid in the presence of Gaussian noise, when the SNR is greater than 0 dB, corresponding C_V values should be less than –3 dB. The absence of any substantially negative C_V values would seem to preclude any constant level sinusoides in the data, such as 60 hz and related harmonic contamination.

COEFFICIENT OF VARIATION
SPECTRAL ANALYSIS SUMMARY

- ## USEFUL TECHNIQUE FOR CHARACTERIZATION BY STATISTICAL PROCESS

- ## ADJUNCT TO POWER SPECTRAL ANALYSIS, REQUIRING SLIGHTLY ADDITIONAL PROCESSING AND MEMORY

- ## ESPECIALLY APPROPRIATE TO UNDERWATER ACOUSTICS DUE TO EXPECTED FLUCTUATIONS RELATED TO THE OCEAN ENVIRONMENT

Slide 13

We have examined the use of C_V spectral analysis. The C_V is the ratio of the standard deviation of the power spectral density to the mean. It is a technique for characterizing a process by its statistical behavior. It is a useful adjunct to conventional FFT power spectral analysis, since additional information is gained with only slightly additional processing and memory required. Finally, this technique, appropriate in underwater acoustics due to inherent ocean environment fluctuations, should be equally appropriate to other acoustic environments also containing fluctuating narrowband components.

Thank you. Are there any questions?

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