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ELECTROMAGNETIC SCATTERING

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This report summarizes research results produced by the investigator under support of the Air Force grant. The work has been mainly concerned with the scattering of electromagnetic waves by bounded bodies of small electrical dimensions, but with some consideration also of acoustic scattering by soft, hard and homogeneous penetrable shapes.
The work has been mainly concerned with the scattering of electromagnetic waves by bounded bodies of small electrical dimensions, but with some consideration also of acoustic scattering by soft, hard and homogeneous penetrable shapes. A major accomplishment has been the preparation (in collaboration with R. E. Kleinman) of a review article [1] summarizing the results available for the low frequency expansions through terms $O(k^3)$ where $k$ is the external wavenumber. The article will constitute Chapter 1 of Volume 1 of the "Handbook on Acoustic, Electromagnetic and Elastic Wave Scattering--Theory and Experiment" edited by V. K. Varadan and V. V. Varadan, to be published by North-Holland, and includes the most recent results for plates as well as for solid bodies. A copy is included herewith.

For an incident electromagnetic wave, the leading term(s) in the low frequency expansion are attributable to induced electric and/or magnetic dipoles, and these can, in turn, be expressed in terms of the corresponding polarizability tensors. In the case of a homogeneous lossy dielectric, the manner in which the body's shape affects the scattering and absorption has been examined [2] by computing the tensor elements using the general purpose program DIELCOM (Senior and Willis, 1982) valid for any body of revolution. The results for a number of generic shapes were compared with those for the corresponding spheroids. When the real part of the dielectric constant is positive, the data for convex bodies are in generally good agreement with the spheroid results, supporting the contention that the tensor elements are only weakly dependent on the body's shape. However,
this is not necessarily true when the real part is negative, due to a bulk resonance of the material. The location and magnitude of the observed resonances are now strongly influenced by the precise details of the body's shape, and the resonances attributable to volume and surface polariton modes have been tracked [2,6] as functions of the length-to-width ratio of the body and the bulk dielectric constant.

For a conducting body there are applications such as remote sensing where it is important to obtain more information about the body than is provided by the dipole terms alone, and this leads naturally to a consideration of the higher order terms in the low frequency expansion. The next one or two terms are of particular interest in this regard.

To judge from the general procedure developed by Stevenson (1953) and modified by Kleinman (1967) it might appear that the determination of the succeeding terms is a relatively straightforward task, involving at each stage the solution of classical potential problems comparable to those entailed in the computation of the leading (zeroth order) terms. Unfortunately, this is not entirely true, and difficulties arise even with the first order terms. One difficulty is the need to find a particular solution \( F(r) \) of the equation

\[
\nabla \cdot F = \nabla \phi
\]

where \( \phi \) is the scattered electrostatic potential attributable to the charge distribution on the surface. In the case of a body
with non-zero interior volume, the standard method (see Stevenson, 1954) for the determination of \( F \) requires the solution of an additional interior Neumann problem; and, in addition, to obtain the first order electric field other than in the far field alone, it is also necessary to solve an integro differential equation for the corresponding potential [3]. Worse still, for a shell or plate of zero interior volume, there does not appear to be any method available for the construction of a solution of (1). The net result is that even for a flat plate it is not yet possible to go beyond the leading term in the low frequency expansion using a potential theory approach.

The problems posed by a flat plate, perfectly conducting or otherwise, were discussed in a series of presentations [4-7]. It is, of course, true that even in the dynamic case it is possible to formulate coupled integral equations for the total (first order) currents induced in the plate and, by solving these numerically, obtain numerical values for our function \( F \). But the whole purpose of a low frequency expansion is to simplify the numerical tasks involved, and if the determination of \( F \) alone requires us to solve integral equations which are just as complicated for the entire dynamic case, there is no longer any rationale for considering a low frequency expansion beyond the first (zeroth order) term.

In addition to the above, it is appropriate to remark that two students who received financial support from the previous AFOSR Grant 77-3188 completed their Ph.D. theses under the direction of
the Project Director during the past year. The theses are available from University Microfilms and their abstracts are included in Appendix A.

References


Stevenson, A. F. (1953), Solutions of electromagnetic scattering problems as power series in the ratio (dimension of scatterers/wavelength), J. Appl. Phys. 24, 1134-1142.


Publications Supported by the Grant

Articles


Oral Presentations

Appendix A

ABSTRACT

THE APPLICATION OF COUPLED WIENER-HOPF INTEGRAL EQUATIONS
IN DIFFRACTION PROBLEMS

by

Jeffrey Michael Pond

Co-chairmen: Albert E. Heins, Thomas B.A. Senior

Two diffraction problems involving infinite stacks of half planes with combinations of the Neumann and Dirichlet boundary conditions have been solved using the Wiener-Hopf technique. In both cases the problem is reduced to solving a pair of coupled integral equations of the Wiener-Hopf type for the discontinuities of the field components across the half planes. The matrix factorization technique suggested by Daniele is used to solve the transform equations.

The first problem consists of an infinite stack of half planes with the Neumann and Dirichlet boundary conditions on the tops and bottoms, respectively, of each half plane. The resultant system is solved, using Daniele's method, for normal incidence of the illumination and no stagger of the stack. Simple expressions are found for the reflection and transmission coefficients.

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The second problem considers the diffraction of a plane wave by an infinite stack of interleaving Neumann and Dirichlet half planes. The matrix factorization is obtained, using Daniele's method, for arbitrary angles of incidence and stagger. The transform equations are solved and simple expressions are obtained for the reflection and transmission coefficients.

Daniele's matrix factorization technique, based on the work of Heins, requires a commuting factorization of the matrix. This places severe restrictions on the elements of the matrix for a commuting factorization to exist. Necessary conditions are established for the existence of a commuting factorization of a matrix in Daniele's form.

It is shown that commuting factorizations exist for a much larger class of matrices than are factorable by Daniele's technique. This new method allows for less stringent relationships between the elements of the matrices. The matrix considered in the first problem mentioned above is factorable, using this new technique, for arbitrary angles of incidence and stagger. The matrix factorization is given in terms of the decomposition and factorization of some scalar functions, which are shown to exist. Thus, in principle, the general case of this problem can be solved.
ABSTRACT

FINITE ELEMENT SOLUTION FOR ELECTROMAGNETIC
SCATTERING FROM TWO-DIMENSIONAL BODIES

by

John Lawrance Mason

Co-Chairmen: Thomas B.A. Senior, William J. Anderson

The problem of two-dimensional electromagnetic scattering from bodies in free space has been formulated using a differential equation finite element method. An incident plane wave polarized so as to excite only E-wave fields has been assumed. A weak integral statement was obtained using an approach equivalent to the weighted residuals method, the weighting functions being selected by Galerkin's method. The finite element mesh of general first-order quadrilaterals extended outward into the far field region of the scattering body, where the boundary condition at the outer boundary was obtained from the asymptotic expression for the scattered field.

In order to compare the finite element method with an integral equation moment method, a special case was solved numerically, namely that of a thin, infinitely long resistive strip. Accurate numerical results from a moment method solution were available to serve as a check on the accuracy of the finite element method results. For a perfectly conducting strip (the limiting case of a resistive strip) the finite element mesh contained special triangular elements at the edges to model the singularities in the magnetic field.
which occur there. Two important disadvantages of the finite element solution became apparent. First, the boundary condition used at the outer boundary leads to the presence of standing waves in the computed scattered field, a non-physical result. Second, the computer memory required increased much more rapidly with strip width than in the moment method solution.

The finite element method used was thus unable to compete with the moment method solution for the thin strip problem. However, to demonstrate a problem for which a moment method solution may be less attractive because of its increased memory requirements, numerical results were obtained using the finite element method for scattering from a body consisting of a thin resistive strip embedded in a dielectric cylinder.