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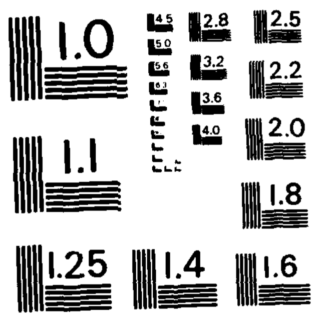

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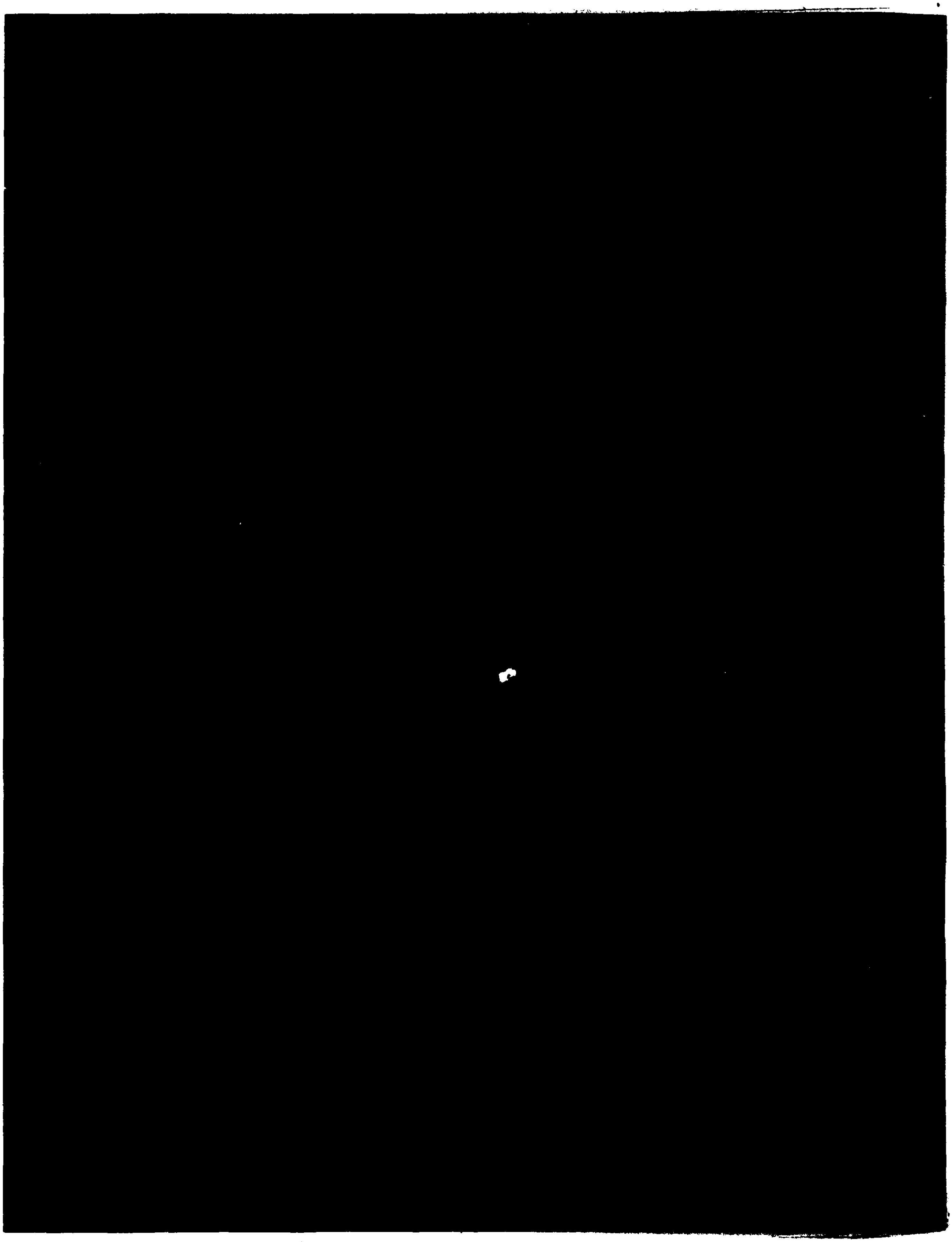

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1. INTRODUCTION

Motions of a ship advancing with constant average speed in regular waves are predicted theoretically by using approximate theories based on the slenderness of ship forms. These theories are the strip theory, most useful in the short-wavelength regime, and the complementary low-frequency slender-body theory; these complementary slender-ship approximations have recently been united and extended in a unified slender-ship theory valid for all frequencies. A detailed mathematical presentation and historical account, including extensive references to the relevant literature, of these slender-ship theories may be found in Newman.[1*]

Agreement between strip-theory predictions and experimental measurements has been found, in a large number of cases, to be sufficient for many practical purposes. This, and the relative mathematical and computational simplicity of the strip theory, have made that theory the most widely used method for predicting ship motions. Indeed, the theory, with the improvements of the recently proposed unified slender-ship theory, seems likely to continue to provide a very useful and practical tool in the future, even if significant improvements in computer performance are made and calculations based on a three-dimensional theory become more practical.

Notwithstanding its many merits, the strip theory evidently has limitation, and in some cases there is a need for potentially more accurate calculations based on a fully three-dimensional theory. For instance, three-dimensional calculations would be useful for predicting the pressure distribution on a complex bow shape, such as one equipped with a bulb or a sonar dome.

At present, fully three-dimensional calculations represent a difficult task, and indeed only a very limited number of numerical results have been obtained by only a few authors: Chang,[2] Guevel and Bougis,[3] and Inglis and Price.[4-7] Furthermore, these sets of numerical results are not entirely consistent: while agreement is good for some hydrodynamic coefficients, discrepancies are very large for other coefficients. This lack of consistency suggests that the accuracy of three-dimensional calculations may be difficult to control, as is the case for the problem of wave resistance.[8]

Three-dimensional calculation methods are based either on numerical solution of an integral equation for the velocity potential or on related assumed distributions of singularities (sources or/and dipoles) on the ship surface. Different integral

*A complete listing of references is given on page 27.

equations can thus be formulated, and these can be solved in several ways, in particular by using an iterative solution procedure or inverting a matrix of influence coefficients. The performance of a three-dimensional calculation method (measured in terms of accuracy control, computing times, and complexity of implementation) must obviously depend, to a large extent, upon the form and mathematical properties of the integral equation and upon the solution procedure selected as the basis of the calculation method. It thus may be useful to consider various alternative integral equations and solution procedures.

The object of this study is to present a new integro-differential equation and a related recurrence relation for determining the velocity potential. The results given in this study generalize those obtained previously for the particular problems of wave radiation and diffraction at zero forward speed,[9] ship wave resistance,[10] and potential flow about a body in an unbounded fluid.[11]

The integro-differential equation, defined by Equations (5.1)-(5.4), is an equation for the velocity potential ϕ, rather than for the density of a related distribution of sources or dipoles. This equation involves both a waterline integral (i.e., a line integral around the intersection curve between the mean hull surface and the mean sea plane), as in the problem of ship wave resistance,[10] and a waterplane integral (i.e., an integral of the Green function over the portion of the mean sea plane inside the mean hull surface), as in the problem of radiation and diffraction at zero forward speed.[9] The highly singular dipole terms $\phi(\vec{x})\partial G(\vec{\xi},\vec{x})/\partial n$ and $\phi(\vec{x})\partial G(\vec{\xi},\vec{x})/\partial x$ in the hull and waterline integrals, respectively, in the classical integro-differential equation defined by Equations (4.10c), (4.8), and (4.9), take the forms $[\phi(\vec{x})-\phi(\vec{\xi})]\partial G(\vec{\xi},\vec{x})/\partial n$ and $[\phi(\vec{x})-\phi(\vec{\xi})]\partial G(\vec{\xi},\vec{x})/\partial x$, respectively, in the modified integro-differential equation obtained in this study. These modified dipole terms are nonsingular, i.e., remain finite, as the integration point $\vec{x}$ approaches any field point $\vec{\xi}$ where the hull is smooth (i.e., has a tangent plane).

A recurrence relation is proposed for solving the integro-differential Equation (5.1) iteratively. This recurrence relation is defined by Equation (5.7), where the initial (zeroth) approximation is taken as the nonhomogeneous term $\psi(\vec{\xi})$ in the integro-differential Equation (5.1). In the particular case of potential flow about an ellipsoid (with arbitrary beam-to-length and draft-to-length ratios) in translatory motion, along any direction, in an unbounded fluid, the first approximation, given by Equation (5.6), actually is exact, as is proved in Reference 11; this first approximation was also shown to provide a good approximation to the exact potential for arbitrary translatory motions in an unbounded fluid of a cylinder in the shape of an ogive with arbitrary thickness ratio.

The plan of the study is as follows. The basic potential-flow problem of the three-dimensional theory of flow about a ship moving with constant speed in regular waves is briefly formulated in Section 2; a more detailed formulation of the problem may be found elsewhere, for instance in Reference 1. The basic equations satisfied by the Green function associated with the free-surface boundary condition (2.5) are given in Section 3. Specifically, the Green function, G, satisfies Equations (3.3a and b) or (3.4a and b), depending upon whether the singularity is fully submerged ($\zeta<0$) or exactly at the mean sea surface ($\zeta=0$), respectively. Equations (3.3a and b) for a fully submerged source are well known. However, Equations (3.4a and b), corresponding to a flux across the mean sea surface, are proper in the limiting case when the singularity is exactly at the mean sea surface. Equations (3.3a and b) and (3.4a and b) generalize the corresponding equations obtained previously for the particular cases of ship wave resistance[12] and of wave radiation and diffraction at zero forward speed.[13] Equations (3.3a and b) and (3.4a and b) are used in Section 4 for obtaining basic integral identities satisfied by the velocity potential. The three classical identities (4.10a, b, and c)--valid strictly outside, inside, and on the ship surface, respectively--are obtained first. However, the main new result of Section 4 is identity (4.13). This identity is valid outside, inside, and exactly on the hull surface, and indeed is equivalent to the set of the three usual identities (4.10a, b, and c). The integral identity (4.13) yields an integro-differential equation for determining the velocity potential on the surface of a ship moving at constant speed in regular waves. This equation is examined in Section 5. Finally, an approach to the numerical evaluation of the iterative approximations defined in Section 5 is presented in Section 6.

2. THE BASIC POTENTIAL-FLOW PROBLEM

The basic potential-flow problem of the linearized theory of ship motions in a regular sea is briefly formulated in this section. The sea is assumed to be of infinite depth and horizontal extent. Water is regarded as homogeneous and incompressible, with density ρ. Viscosity effects are ignored, and irrotational flow is assumed. Surface tension, wavebreaking, spray formation at the ship bow, and nonlinearities in the sea-surface boundary condition are neglected. A moving system of coordinates (X,Y,Z) in steady translation with the mean forward velocity U of the ship is defined. Specifically, the mean (undisturbed) sea surface and the centerplane of the ship in its mean position are taken as the planes Z = 0 and Y = 0, respectively; the Z axis is directed vertically upwards, and the X axis is directed toward the ship bow.

In the above-defined translating system of coordinates, the linearized sea-surface boundary condition takes the form

$$[g\partial_Z+(U\partial_X-\partial_T)^2]\Phi' = (U\partial_X-\partial_T)P'/\rho - gQ' \text{ on } Z = 0 \tag{2.1}$$

where g is the acceleration of gravity, T is the time, $\Phi' \equiv \Phi'(\vec{X},T)$ is the velocity potential, $P' \equiv P'(X,Y,T)$ and $Q' \equiv Q'(X,Y,T)$ correspond to distributions of pressure and flux, respectively, at the sea surface (we have $Q' \equiv 0$ for all practical applications, and $P' \equiv 0$ except for surface-effect ships), and the notation ∂_Z, ∂_X, ∂_T is meant for the differential operators $\partial/\partial Z$, $\partial/\partial X$, $\partial/\partial T$, respectively.

The present study is concerned with flows simple-harmonic in time, with radiant frequency ω where ω is the frequency of encounter. However, such free-surface flows are not completely (or uniquely) determined unless an appropriate "radiation condition" is imposed, as is well known and is discussed by Stoker,[14] for instance. A convenient alternative approach, employed previously in Lighthill[15] and Noblesse,[12,13] to the use of such a "radiation condition" consists in defining a time-harmonic flow as the limit, as the small positive auxiliary parameter σ vanishes, of a flow defined by a velocity potential of the form

$$\Phi'(\vec{X},T) = \mathrm{Re}\,\Phi(\vec{X})\exp[(\sigma-i\omega)T] \tag{2.2}$$

where Re represents the real part of the function on the right side. The eventual sea-surface distribution of pressure $P'(X,Y,T)$ and flux $Q'(X,Y,T)$ similarly are assumed to be of the form

$$P'(X,Y,T) = \text{Re } P(X,Y)\exp[(\sigma-i\omega)T] \tag{2.2a}$$

$$Q'(X,Y,T) = \text{Re } Q(X,Y)\exp[(\sigma-i\omega)T] \tag{2.2b}$$

In this alternative approach, one is then faced with an initial-value problem, with the obvious initial conditions $\Phi' = 0$ and $\partial\Phi'/\partial T = 0$ for $T = -\infty$. Use of Equations (2.2) and (2.2a and b) in Equation (2.1) then yields the sea-surface boundary condition

$$[g\partial_Z-(\omega-iU\partial_X+i\sigma)^2]\Phi = i(\omega-iU\partial_X+i\sigma)P/\rho - gQ \text{ on } Z = 0 \tag{2.3}$$

for the "spatial component" $\Phi(\vec{X})$ of the actual potential $\Phi'(\vec{X},T)$.

Nondimensional variables are defined in terms of $1/\omega$ as reference time, the ship length L as reference length, and the acceleration of gravity g as reference acceleration, from which the reference velocity $(gL)^{1/2}$, potential $(gL)^{1/2}L$, and pressure ρgL can be formed. The nondimensional variables

$$t = \omega T,\ \vec{x} = \vec{X}/L,\ \phi = \Phi/(gL)^{1/2}L,\ p = P/\rho gL,\ q = Q/(gL)^{1/2} \tag{2.4}$$

are then defined. In terms of these nondimensional variables, the sea-surface boundary condition (2.3) can be shown to take the form

$$[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi = i(f-iF\partial_x+i\varepsilon)p - q \text{ on } z = 0 \tag{2.5}$$

where f is the frequency parameter, F is the Froude number, and ε is the time-growth parameter defined as

$$f = \omega(L/g)^{1/2} \tag{2.6a}$$

$$F = U/(gL)^{1/2} \tag{2.6b}$$

$$\varepsilon = \sigma(L/g)^{1/2} \tag{2.6c}$$

The basic potential-flow problem of the linearized theory of ship motions in a regular sea may now be stated. As is well known, the problem consists in solving the Laplace equation

$$\nabla^2\phi = 0 \text{ in } d \tag{2.7}$$

subject to the boundary conditions specified below. The solution domain d in Equation (2.7) is the domain exterior to the ship hull and bounded upwards by the mean sea surface σ. On the mean sea surface σ, the boundary condition (2.5) must be *satisfied:*

$$[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi = i(f-iF\partial_x+i\varepsilon)p - q \text{ on } \sigma \tag{2.8}$$

where we generally have $p = 0 = q$. The potential $\phi(\vec{x})$ vanishes as $||\vec{x}||\to\infty$ at least as fast as $1/||\vec{x}||$; that is, we have

$$\phi = O(1/||\vec{x}||) \text{ as } ||\vec{x}||\to\infty \tag{2.9}$$

Finally, on the mean position of the ship hull surface h the potential must satisfy the usual Neumann condition

$$\partial\phi/\partial n \text{ given on } h \tag{2.10}$$

where $\partial\phi/\partial n \equiv \nabla\phi\cdot\vec{n}$ is the derivative of ϕ in the direction of the unit normal vector $\vec{n}$ to h, taken to be pointing inside the fluid. The precise form taken by the expression for $\partial\phi/\partial n$ on h in the usual "radiation" and "diffraction" problems may be found in Newman,[1] for instance.

A classical technique for solving a potential-flow problem such as that defined by Equations (2.7) through (2.10), in the general case of an arbitrary ship form, consists in formulating an integral equation for the potential based on the use of a Green function satisfying all the equations of the problem except the "hull boundary condition," which is to be satisfied by means of the integral equation. The required Green function is defined in the following section.

3. THE GREEN FUNCTION

The Green function, $G(\vec{\xi},\vec{x})$, associated with the sea-surface boundary condition (2.5) satisfies the equations

$$\nabla_\xi^2 G = \delta(\xi-x)\delta(\eta-y)\delta(\zeta-z) \text{ in } \zeta < 0 \tag{3.1a}$$

$$[\partial_\zeta-(f-iF\partial_\xi+i\varepsilon)^2]G = 0 \text{ on } \zeta = 0 \tag{3.1b}$$

where $\delta(\;)$ is the usual Dirac "delta function," and ∇_ξ represents the differential operator $(\partial_\xi,\partial_\eta,\partial_\zeta)$. Physically, the Green function $G(\vec{\xi},\vec{x})$ is the "spatial component" of the velocity potential $\mathrm{Re}G(\vec{\xi},\vec{x})\exp[(\varepsilon/f-i)t]$ of the flow created at the field point $\vec{\xi}(\xi,\eta,\zeta\leq 0)$ by a moving source of pulsating strength $\mathrm{Re}\exp[(\varepsilon/f-i)t]$ located at point $\vec{x}(x,y,z<0)$. In the limiting case, $z = 0$, the source at point $\vec{x}$ evidently is no longer fully submerged, so that this physical interpretation of the Green function becomes ambiguous. A complementary physical interpretation for this limiting case is that the pulsating flow created at point $\vec{x}(x,y,z=0)$ stems from a flux across the plane $z = 0$ of the mean sea surface. In the limit $z = 0$, the Green function $G(\vec{\xi},\vec{x})$ must then satisfy the equations

$$\nabla_\xi^2 G = 0 \text{ in } \zeta < 0 \tag{3.2a}$$

$$[\partial_\zeta-(f-iF\partial_\xi+i\varepsilon)^2]G = -\delta(\xi-x)\delta(\eta-y) \text{ on } \zeta = 0 \tag{3.2b}$$

as may be seen from the sea-surface boundary condition (2.5). Equations (3.2a and b), justified above on physical grounds, can be justified mathematically in the manner shown in Noblesse[12,13] for the particular problems of wave radiation and diffraction at zero mean forward speed (F=0) and of steady flow about a ship advancing at constant speed in calm water (f=0).

The Green function $G(\vec{\xi},\vec{x})$ actually is a function of the four variables $\xi-x$, $\eta-y$, $\zeta+z$, and $(\zeta-z)^2$, and thus is invariant under the substitutions $\xi \leftrightarrow -x$, $\eta \leftrightarrow -y$, $\zeta \leftrightarrow z$. By performing these changes of variables in Equations (3.1a and b) and (3.2a and b), it then may be seen that the Green function $G(\vec{\xi},\vec{x})$ also satisfies the following equations

$$\left.\begin{aligned} \nabla^2 G &= \delta(x-\xi)\delta(y-\eta)\delta(z-\zeta) \text{ in } z<0 \qquad &(3.3a)\\ [\partial_z-(f+iF\partial_x+i\varepsilon)^2]G &= 0 \text{ on } z=0 \qquad &(3.3b)\end{aligned}\right\} \text{ for } \zeta<0$$

$$\left.\begin{aligned} \nabla^2 G &= 0 \text{ in } z<0 \qquad &(3.4a)\\ [\partial_z-(f+iF\partial_x+i\varepsilon)^2]G &= -\delta(x-\xi)\delta(y-\eta) \text{ on } z=0 \qquad &(3.4b)\end{aligned}\right\} \text{ for } \zeta=0$$

where ∇ is the differential operator $(\partial_x,\partial_y,\partial_z)$. Equations (3.3a and b) and (3.4a and b) will be used in the next section for obtaining integral identities satisfied by the velocity potential.

A well known expression for the Green function, in terms of a double integral, can be obtained by using a double Fourier transformation of Equations (3.1a and b) with respect to the horizontal coordinates ξ and η. This Fourier representation of the Green function is given by

$$4\pi G(\vec{\xi},\vec{x}) = -[(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]^{-1/2} + [(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2]^{-1/2}$$

$$-\frac{1}{\pi}\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \, \frac{\exp[(z+\zeta)(\mu^2+\nu^2)^{1/2}+i\{(x-\xi)\mu+(y-\eta)\nu\}]}{(\mu^2+\nu^2)^{1/2} - (f-F\mu+i\varepsilon)^2} \qquad (3.5)$$

The "Cartesian Fourier integral representation" (3.5) can also be expressed in the form of a "polar Fourier representation" by performing the change of variables $\mu = \lambda\cos\theta$ and $\nu = \lambda\sin\theta$, which express the Cartesian Fourier variable μ and ν in terms of the polar variables λ and θ. These equivalent double-integral representations were first obtained by Haskind[16] and Brard,[17] and later by Hanaoka,[18] Stretenski,[19] Eggers,[20] Havelock,[21] and Wehausen,[22] and are therefore well known.

More recently, one-fold integral representations (involving the exponential integral in the integrand) have been obtained and used by Inglis and Price[23] and Guevel and Bougis.[3] These single-integral representations are modifications of the double-integral Fourier representation in terms of the polar coordinates (λ,θ). Single-integral representations associated with the Cartesian Fourier representation (3.5), in the manner shown in Reference 12 for the particular problem of ship wave resistance ($f=0$, $F\neq 0$), have not been obtained to the author's knowledge. However, such single-integral representations are considerably more complex than the corresponding integral representations for the ship wave resistance problem and the series representations obtained in Reference 13 for the particular case of wave radiation and diffraction at zero forward speed ($F=0$, $f\neq 0$). For the practical purpose of numerically evaluating the velocity potential defined by a surface (or line) distribution of singularities (sources or dipoles) with known strength, it may actually be preferable to use a double-integral Fourier representation, such as that given by Equation (3.5), together with an interchange in the order of integration between the Fourier variables (μ,ν) and the space variables (x,y,z), as is shown in Section 6.

4. FUNDAMENTAL INTEGRAL IDENTITIES

In this section, basic integral identities for the velocity potential are obtained by applying a classical Green identity to the potential $\phi \equiv \phi(\vec{x})$ and the previously defined Green function $G \equiv G(\vec{\xi},\vec{x})$. The Green identity is

$$\int_{d'} (\phi\nabla^2 G - G\nabla^2\phi)dv = \int_{\sigma'} (\phi\partial G/\partial z - G\partial\phi/\partial z)dxdy$$

$$+ \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n)da + \int_{h_\infty} (\phi\partial G/\partial n - G\partial\phi/\partial n)da \tag{4.1}$$

where d' is the finite domain bounded by the ship hull surface h, the mean sea plane $z = 0$, and some arbitrary, but sufficiently large, exterior surface h_∞ surrounding the ship surface h, as is shown in Figure 1; furthermore, σ' is the portion of the plane $z = 0$ between the intersection curves c and c_∞ of the plane $z = 0$ with the ship surface h and the exterior surface h_∞, respectively. On the surfaces h and h_∞ we have $\partial\phi/\partial n \equiv \nabla\phi\cdot\vec{n}$ and $\partial G/\partial n \equiv \nabla G\cdot\vec{n}$ where $\vec{n}$ is the unit outward normal vector to h or h_∞, as is shown in Figure 1. Finally, dv and da represent the differential elements of volume and area at the integration point $\vec{x}$ of the domain d' and the surfaces h or h_∞, respectively, and dxdy is the differential element of area of the mean sea surface σ'.

Let the integrand $\phi\partial G/\partial z - G\partial\phi/\partial z$ of the sea-surface integral in Equation (4.1) be expressed in the form $\phi[\partial_z-(f+iF\partial_x+i\varepsilon)^2]G - G[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi +$ $2iF(f+i\varepsilon)\partial(G\phi)/\partial x + F^2\partial(G\partial\phi/\partial x-\phi\partial G/\partial x)/\partial x$. Furthermore, we may use the relation

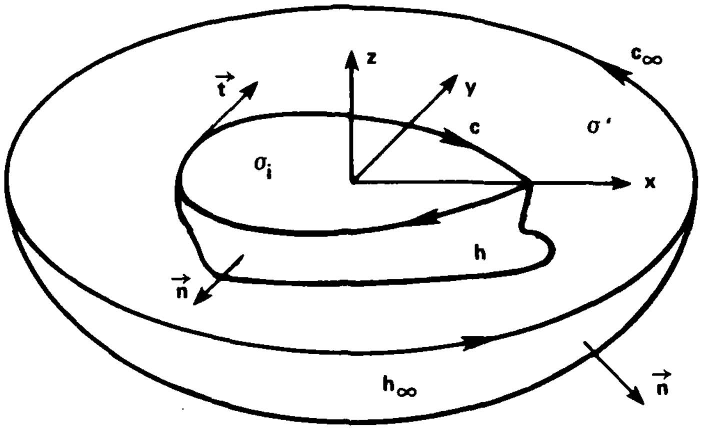

Figure 1 - Definition Sketch

$$\int_{\sigma'} [2i(f+i\varepsilon)\partial(G\phi)/\partial x+F\partial(G\partial\phi/\partial x-\phi\partial G/\partial x)/\partial x]dxdy$$

$$= \int_{c} [2i(f+i\varepsilon)G\phi+F(G\partial\phi/\partial x-\phi\partial G/\partial x)]dy$$

$$+ \int_{c_\infty} [2i(f+i\varepsilon)G\phi+F(G\partial\phi/\partial x-\phi\partial G/\partial x)]dy \qquad (4.2)$$

where the curves c and c_∞ are oriented clockwise and counterclockwise, respectively, as is shown in Figure 1. The Green identity (4.1) can then be expressed in the form

$$\int_{d'} \phi\nabla^2 G dv - \int_{\sigma'} \phi[\partial_z-(f+iF\partial_x+i\varepsilon)^2]G dxdy$$

$$= \int_{d'} G\nabla^2\phi dv - \int_{\sigma'} G[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi dxdy$$

$$+ \int_h (G\partial\phi/\partial n-\phi\partial G/\partial n)da$$

$$+ F\int_c [2i(f+i\varepsilon)G\phi+F(G\partial\phi/\partial x-\phi\partial G/\partial x)]dy + I_\infty \tag{4.3}$$

where the term I_∞ is given by the integrals

$$I_\infty = \int_{h_\infty} (\phi\partial G/\partial n-G\partial\phi/\partial n)da + F\int_{c_\infty} [2i(f+i\varepsilon)G\phi+F(G\partial\phi/\partial x-\phi\partial G/\partial x)]dy$$

We have $G = O(1/r)$ and $\phi = O(1/r)$ as $r \equiv (\xi^2+\eta^2+\zeta^2)^{1/2} \to \infty$, so that the term I_∞ vanishes as the large surrounding surface h_∞ is made ever larger. The term I_∞ can then be ignored if the finite domain d' and the finite region σ' of the mean sea plane are replaced by the unbounded mean flow domain d and the unbounded mean sea surface σ outside the mean hull surface h and its intersection curve c with the plane $z = 0$, respectively.

By expressing the potential ϕ in the integrands of the two integrals on the left side of Equation (4.3) in the form $\phi = \phi_* + (\phi-\phi_*)$, where $\phi \equiv \phi(\vec{x})$ as was defined previously, and ϕ_* represents the potential at the field point $\vec{\xi}$, i.e., $\phi_* \equiv \phi(\vec{\xi})$, we may obtain

$$\int_d \phi\nabla^2 G dv - \int_\sigma \phi[\partial_z-(f+iF\partial_x+i\varepsilon)^2]G dxdy = C\phi_* + C' \tag{4.4}$$

where C and C´ are defined as

$$C = \int_d \nabla^2 G dv - \int_\sigma [\partial_z - (f+iF\partial_x+i\varepsilon)^2] G dx dy$$

$$C' = \int_d (\phi-\phi_*)\nabla^2 G dv - \int_\sigma (\phi-\phi_*)[\partial_z-(f+iF\partial_x+i\varepsilon)^2] G dx dy \tag{4.5}$$

It may be seen from Equations (3.3) and (3.4) that we have $C' \equiv 0$ if $\phi - \phi_* \equiv \phi(\vec{x}) - \phi(\vec{\xi}) \to 0$ as $\vec{x} \to \vec{\xi}$, that is if the potential is continuous everywhere in the solution domain d and on its boundary $\sigma + h + c$, as is assumed here. Use of Equation (4.4), with $C' = 0$, in Equation (4.3) then yields

$$C\phi_* = \int_d G\nabla^2\phi dv - \int_\sigma G[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi dx dy$$

$$+ \int_h (G\partial\phi/\partial n - \phi\partial G/\partial n) da$$

$$+ F\int_c [2i(f+i\varepsilon)G\phi + F(G\partial\phi/\partial x - \phi\partial G/\partial x)] dy \tag{4.6}$$

Let $\vec{t}(t_x, t_y, 0)$ represent the unit vector tangent to the curve c oriented in the clockwise direction, as is shown in Figure 1. On the mean waterline c, we have $dy = t_y d\ell$, where $d\ell$ is the differential element of arc length of c. Furthermore, we have $\partial\phi/\partial x \equiv \nabla\phi\cdot\vec{i}$, where $\vec{i}$ (1,0,0) is the unit positive vector along the x axis. This then yields $\partial\phi/\partial x = [\vec{n}\partial\phi/\partial n + \vec{t}\partial\phi/\partial\ell + (\vec{n}\times\vec{t})\partial\phi/\partial d]\cdot\vec{i} = n_x\partial\phi/\partial n + t_x\partial\phi/\partial\ell - n_z t_y\partial\phi/\partial d$, where (n_x, n_y, n_z) are the components of the unit outward normal vector $\vec{n}$ to the hull surface h, $\partial\phi/\partial\ell$ is the derivative of ϕ in the direction of the tangent vector $\vec{t}$ to c, and $\partial\phi/\partial d$ is the derivative of ϕ in the direction of the unit vector $\vec{n} \times \vec{t}$, which is tangent to h and pointing downwards.

Equation (4.6) can then be expressed in the form

$$C\phi(\vec{\xi}) = \psi(\vec{\xi}) - L(\vec{\xi};\phi) \tag{4.7}$$

where C is given by Equation (4.5), $\psi(\vec{\xi})$ is the potential defined as

$$\psi(\vec{\xi}) = \int_d G\nabla^2\phi dv - \int_\sigma G[\partial_z-(f-iF\partial_x+i\varepsilon)^2]\phi dxdy$$

$$+ \int_h G\partial\phi/\partial n da + F^2 \int_c Gn_x t_y \partial\phi/\partial n d\ell \tag{4.8}$$

and $L(\vec{\xi};\phi)$ is the linear transform of ϕ defined as

$$L(\vec{\xi};\phi) = \int_h \phi\partial G/\partial n da - 2i(f+i\varepsilon)F \int_c G\phi t_y d\ell$$

$$+ F^2 \int_c [\phi\partial G/\partial x - G(t_x\partial\phi/\partial\ell - n_z t_y \partial\phi/\partial d)]t_y d\ell \tag{4.9}$$

Use of Equations (3.3a and b) and (3.4a and b) in expression (4.5) for C then shows that we have $C \equiv 1$ if the field point $\vec{\xi}$ is strictly outside the hull surface h, in d or on σ, whereas we have $C \equiv 0$ if $\vec{\xi}$ is strictly inside the ship surface h. It can also be seen from Equations (3.3) and (3.4) that we have $C = 1/2$ if the point $\vec{\xi}$ is exactly on the hull surface h or on its intersection c with the plane $z = 0$, at least for points $\vec{\xi}$ where the hull h + c is smooth; more generally, the value of $4\pi C$ (or $2\pi C$) at a point $\vec{\xi}$ of h (or c) is equal to the angle at which d (or σ) is viewed from the point $\vec{\xi}$. We thus have

$$\left\{\begin{array}{c} \phi(\vec{\xi}) \\ \\ 0 \\ \\ \phi(\vec{\xi})/2 \end{array}\right\} = \psi(\vec{\xi}) - L(\vec{\xi};\phi) \text{ for } \vec{\xi} \left\{\begin{array}{l} \text{in } d + \sigma - h - c \\ \\ \text{in } d_i + \sigma_i - h - c \\ \\ \text{exactly on } h + c \end{array}\right\} \begin{array}{r} (4.10a) \\ \\ (4.10b) \\ \\ (4.10c) \end{array}$$

where d_i and σ_i represent the domain and the portion of the plane $z = 0$, respectively, strictly inside the ship surface h, as is shown in Figure 1.

The value of the constant C on the left side of Equation (4.7) is discontinuous across the ship hull surface h; C being equal to 1 outside h and to 0 inside, as is explicitly indicated in Equations (4.10a, b, and c). This discontinuity in the value of C evidently is accompanied by a corresponding discontinuity on the right side of Equation (4.7). Specifically, the latter discontinuity stems from the dipole-distribution integrals $\int_h \phi\partial G/\partial n da$ and $\int_c \phi\partial G/\partial x t_y d\ell$ in the potential $L(\vec{\xi};\phi)$ defined by Equation (4.9). An identity valid for any point $\vec{\xi}$--outside, inside, or exactly on the ship surface h--can be obtained by eliminating the discontinuity in the value of C in Equation (4.7). This can be done by adding the term $C_i\phi_*$ on both sides of Equation (4.7), with C_i given by

$$C_i = \int_{d_i} \nabla^2 G dv - \int_{\sigma_i} [\partial_z - (f + iF\partial_x + i\varepsilon)^2] G dx dy \tag{4.11}$$

Use of the divergence theorem

$$\int_{d_i} \nabla^2 G dv = \int_{\sigma_i} \partial G/\partial z dx dy + \int_h \partial G/\partial n da$$

yields

$$C_i = \int_{\sigma_i} (f+iF\partial_x+i\varepsilon)^2 G\,dxdy + \int_h \partial G/\partial n\,da$$

Furthermore, by using the relation

$$\int_{\sigma_i} [2i(f+i\varepsilon)\partial G/\partial x - F\partial^2 G/\partial x^2]dxdy = -\int_c [2i(f+i\varepsilon)G - F\partial G/\partial x]t_y\,d\ell$$

we may obtain the following alternative expression for C_i:

$$C_i = (f+i\varepsilon)^2 \int_{\sigma_i} G\,dxdy - 2i(f+i\varepsilon)F\int_c G t_y\,d\ell$$

$$+ F^2\int_c \partial G/\partial x\,t_y\,d\ell + \int_h \partial G/\partial n\,da \qquad (4.12)$$

By adding the term $C_i\phi_*$ on the left and right sides of Equation (4.7), with C_i given by Equation (4.11) on the left side and Equation (4.12) on the right side, we may obtain

$$[1-w(\vec{\xi})]\phi(\vec{\xi}) = \psi(\vec{\xi}) - L'(\vec{\xi};\phi) \qquad (4.13)$$

where $w(\vec{\xi})$ is the "waterplane integral" defined as

$$w(\vec{\xi}) = (f+i\varepsilon)^2 \int_{\sigma_i} G(\vec{\xi},\vec{x})dxdy \qquad (4.14)$$

and $L'(\vec{\xi};\phi)$ is the linear transform of ϕ defined as

$$L'(\vec{\xi};\phi) = \int_h (\phi-\phi_*)\partial G/\partial n da - 2i(f+i\varepsilon)F \int_c G(\phi-\phi_*)t_y d\ell$$

$$+ F^2 \int_c [(\phi-\phi_*)\partial G/\partial x - G(t_x \partial\phi/\partial\ell - n_z t_y \partial\phi/\partial d)]t_y d\ell \qquad (4.15)$$

in which we have $\phi \equiv \phi(\vec{x})$ and $\phi_* \equiv \phi(\vec{\xi})$ as was defined previously. In obtaining Equation (4.13), the relation $C + C_i \equiv 1$ was used. This relation can be obtained by using Equations (3.3a and b) and (3.4a and b) in Equations (4.5) and (4.11), which yield

$$C + C_i = \int_{z<0} \nabla^2 G dv - \int_{z=0} [\partial_z - (f+iF\partial_x+i\varepsilon)^2]G dx dy$$

Identity (4.13) is valid for any point $\vec{\xi}$, whether outside, inside, or exactly on the ship surface h. This identity thus is essentially equivalent to the set of the three classical identities (4.10a, b, and c), which are exclusively valid for $\vec{\xi}$ outside, inside, and on the hull surface h, respectively.

Identities (4.10a, b, and c) and (4.13) correspond to the case of an open hull surface piercing the sea surface. For a closed, fully submerged surface h, the waterplane integral $w(\vec{\xi})$ defined by Equation (4.14) and the integrals around the mean waterline c in Equation (4.8) for the potential $\psi(\vec{\xi})$ and in Equations (4.9) and (4.15) for the potentials $L(\vec{\xi};\phi)$ and $L'(\vec{\xi};\phi)$ are evidently not present. Two other important particular cases of identities (4.10a, b, and c) and (4.13) are obtained in the limiting cases when the Froude number F vanishes, corresponding to wave radiation and diffraction by a body with zero mean forward speed,[9] and when the frequency parameter f vanishes, corresponding to steady flow about a ship advancing in calm water.[10]

5. INTEGRAL EQUATION AND RELATED ITERATIVE APPROXIMATIONS

Identities (4.10a, b, and c) and (4.13) hold for any function ϕ continuous in the domain d and on its boundary $\sigma + h + c$. If the function ϕ is taken as the velocity potential of flow about a ship advancing at constant mean speed in a regular sea, then the normal derivative $\partial\phi/\partial n$ of ϕ is given on the hull surface h, and ϕ satisfies the Laplace Equation (2.7) in the mean flow domain d and the sea-surface boundary condition (2.8), with $p = 0 = q$, at the mean sea surface σ. Identities (4.10c) and (4.13) then yield integro-differential equations for determining the potential ϕ on the ship surface. Specifically, Equation (4.13) becomes

$$[1-w(\vec{\xi})]\phi(\vec{\xi}) = \psi(\vec{\xi}) - L'(\vec{\xi};\phi) \tag{5.1}$$

where the waterplane integral $w(\vec{\xi})$ is given by

$$w(\vec{\xi}) = (f+i\varepsilon)^2 \int_{\sigma_i} G(\vec{\xi},\vec{x})dxdy \tag{5.2}$$

the potential $\psi(\vec{\xi})$ takes the form

$$\psi(\vec{\xi}) = \int_h G\partial\phi/\partial n da + F^2 \int_c G n_x t_y \partial\phi/\partial n d\ell \tag{5.3}$$

and the potential $L'(\vec{\xi};\phi)$ is given by

$$L'(\vec{\xi};\phi) = \int_h (\phi-\phi_*)\partial G/\partial n da - 2i(f+i\varepsilon)F \int_c G(\phi-\phi_*)t_y d\ell$$

$$+ F^2 \int_c [(\phi-\phi_*)\partial G/\partial x - G(t_x \partial\phi/\partial\ell - n_z t_y \partial\phi/\partial d)] t_y d\ell \tag{5.4}$$

The potential $\psi(\vec{\xi})$ is known since $\partial\phi/\partial n$ is given on the ship surface h. The potential $L'(\vec{\xi};\phi)$, on the other hand, is evidently not known.

An approximate solution of the integro-differential Equation (5.1) may be obtained by seeking a solution of Equation (5.1) of the form $\phi(\vec{\xi}) = k(\vec{\xi})\psi(\vec{\xi})$, where the function $k(\vec{\xi}) \equiv \phi(\vec{\xi})/\psi(\vec{\xi})$ is assumed to be slowly varying. Specifically, by adding the term $k(\vec{\xi})L'(\vec{\xi};\psi)$ to both sides of Equation (5.1) and multiplying the resulting equation by $\psi(\vec{\xi})$, we may obtain

$$\phi(\vec{\xi})\{[1-w(\vec{\xi})]\psi(\vec{\xi})+L'(\vec{\xi};\psi)\} = \psi^2(\vec{\xi}) + \phi(\vec{\xi})L'(\vec{\xi};\psi) - \psi(\vec{\xi})L'(\vec{\xi};\phi) \qquad (5.5)$$

If the potential ϕ were actually proportional to the potential ψ, the term $\phi(\vec{\xi})L'(\vec{\xi};\psi) - \psi(\vec{\xi})L'(\vec{\xi};\phi)$ would vanish, and the modified integro-differential Equation (5.5) would yield the solution

$$\phi(\vec{\xi}) = \psi^2(\vec{\xi})/\{[1-w(\vec{\xi})]\psi(\vec{\xi}) + L'(\vec{\xi};\psi)\} \qquad (5.6)$$

More generally, the above expression for the potential can be regarded as the first approximation in the sequence of iterative approximations $\phi^{(n)}$ associated with the recurrence relation

$$\phi^{(n+1)}(\vec{\xi}) = \psi(\vec{\xi})\phi^{(n)}(\vec{\xi})/\{[1-w(\vec{\xi})]\phi^{(n)}(\vec{\xi}) + L'(\vec{\xi};\phi^{(n)})\} \quad \text{for } n \geq 0 \qquad (5.7)$$

and the initial (zeroth) approximation $\phi^{(0)}(\vec{\xi}) = \psi(\vec{\xi})$. An approach to the numerical evaluation of these iterative approximations is presented in the following section.

6. NUMERICAL EVALUATION OF ITERATIVE APPROXIMATIONS

Equation (3.5) for the Green function may be written in the form

$$4\pi G(\vec{\xi},\vec{x}) = -(1/r-1/r') - (1/\pi)\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \overline{E}(\mu,\nu;\vec{\xi})E(\mu,\nu;\vec{x})/D(\mu,\nu) \tag{6.1}$$

where r, r′, $E(\mu,\nu;\vec{x})$, $\overline{E}(\mu,\nu;\vec{\xi})$, and $D(\mu,\nu)$ are defined as

$$r = [(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]^{1/2} \tag{6.1a}$$

$$r' = [(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2]^{1/2} \tag{6.1b}$$

$$E(\mu,\nu;\vec{x}) = \exp[z(\mu^2+\nu^2)^{1/2}+i(x\mu+y\nu)] \tag{6.1c}$$

$$\overline{E}(\mu,\nu;\vec{\xi}) = \exp[\zeta(\mu^2+\nu^2)^{1/2}-i(\xi\mu+\eta\nu)] \tag{6.1d}$$

$$D(\mu,\nu) = (\mu^2+\nu^2)^{1/2}-(f-F\mu+i\varepsilon)^2 \tag{6.1e}$$

The potential $\psi(\vec{\xi})$ defined by Equation (5.3) can then be expressed in the form

$$\psi(\vec{\xi}) = \psi^S(\vec{\xi}) + \psi^R(\vec{\xi}) \tag{6.2}$$

where the potentials ψ^S and ψ^R correspond to the singular terms $1/r - 1/r'$ and the regular term defined by the double integral, respectively, in Equation (6.1) for the Green function. Specifically, the potential ψ^S is given by the hull-surface integral

$$\psi^S(\vec{\xi}) = -(1/4\pi) \int_h (1/r-1/r')\partial\phi/\partial n da \tag{6.2a}$$

and the potential ψ^R may be expressed in the form

$$\psi^R(\vec{\xi}) = -(1/4\pi^2) \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \overline{E}(\mu,\nu;\vec{\xi})A(\mu,\nu)/D(\mu,\nu) \tag{6.2b}$$

where $A(\mu,\nu)$ is defined as

$$A(\mu,\nu) = \int_h E(\mu,\nu;\vec{x})\partial\phi/\partial n da + F^2 \int_c E(\mu,\nu;\vec{x})n_x t_y \partial\phi/\partial n d\ell \tag{6.2c}$$

The waterplane integral $w(\vec{\xi})$ defined by Equation (5.2) takes the form

$$w(\vec{\xi}) = -\frac{(f+i\varepsilon)^2}{4\pi^2} \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \frac{\overline{E}(\mu,\nu;\vec{\xi})}{D(\mu,\nu)} \int_{\sigma_i} \exp[i(x\mu+y\nu)]dxdy \tag{6.3}$$

The potential $L'(\vec{\xi};\psi)$ defined by Equation (5.4) can then be expressed in the form

$$L'(\vec{\xi};\psi) = L_S'(\vec{\xi};\psi) + L_R'(\vec{\xi};\psi) \tag{6.4}$$

where the potentials L_S' and L_R' are defined as

$$L'_S(\vec{\xi};\psi) = -(1/4\pi)\int_h [(\psi^S+\psi^R)-(\psi^S_*+\psi^R_*)]\partial(1/r-1/r')/\partial n da \tag{6.4a}$$

$$L'_R(\vec{\xi};\psi) = -(1/4\pi^2)\int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \bar{E}(\mu,\nu;\vec{\xi})A'(\mu,\nu)/D(\mu,\nu) \tag{6.4b}$$

with $A'(\mu,\nu)$ given by

$$A'(\mu,\nu) = \int_h [(\psi^S+\psi^R)-(\psi^S_*+\psi^R_*)]\partial E(\mu,\nu;\vec{x})/\partial n da$$

$$- 2i(f+i\varepsilon)F\int_c (\psi^R-\psi^R_*)E(\mu,\nu;\vec{x})t_y d\ell$$

$$+ F^2\int_c \{(\psi^R-\psi^R_*)\partial E/\partial x-[t_x\partial\psi^R/\partial\ell-n_z t_y\partial(\psi^S+\psi^R)/\partial d]E\}t_y d\ell \tag{6.4c}$$

The first iterative approximation $\phi^{(1)}(\vec{\xi})$ can then be determined by using Equations (6.2), (6.3), and (6.4) in Equation (5.6). The potential $L'(\vec{\xi},\phi^{(n)})$, $n \geq 1$, defined by Equation (5.7), for the second and subsequent iterative approximations $\phi^{(n+1)}(n\geq 1)$, can be expressed in a form almost identical to that given above by Equations (6.4) and (6.4a, b, and c) for the potential $L'(\vec{\xi};\psi)$.

The basic computational task common to Equations (6.2b and c), (6.3), and (6.4b and c) consists in evaluating a double Fourier integral, $I(\vec{\xi})$ say, of the form

$$I(\vec{\xi}) = \int_{-\infty}^{\infty} d\nu \int_{-\infty}^{\infty} d\mu \exp[\zeta(\mu^2+\nu^2)^{1/2}-i(\xi\mu+\eta\nu)]N(\mu,\nu)/D(\mu,\nu) \tag{6.5}$$

where $D(\mu,\nu)$ is given by Equation (6.1e), and $N(\mu,\nu)$ is defined by a surface or a line integral of the type

$$N(\mu,\nu) = \int_{h,\sigma_i,c} \exp[z(\mu^2+\nu^2)^{1/2}+i(x\mu+y\nu)]A(\vec{x})da,dxdy,d\ell \tag{6.5a}$$

If a small positive number is used for the parameter ε, the function $D(\mu,\nu)$ has no real root, and the integral (6.5) can be evaluated without difficulty in principle.

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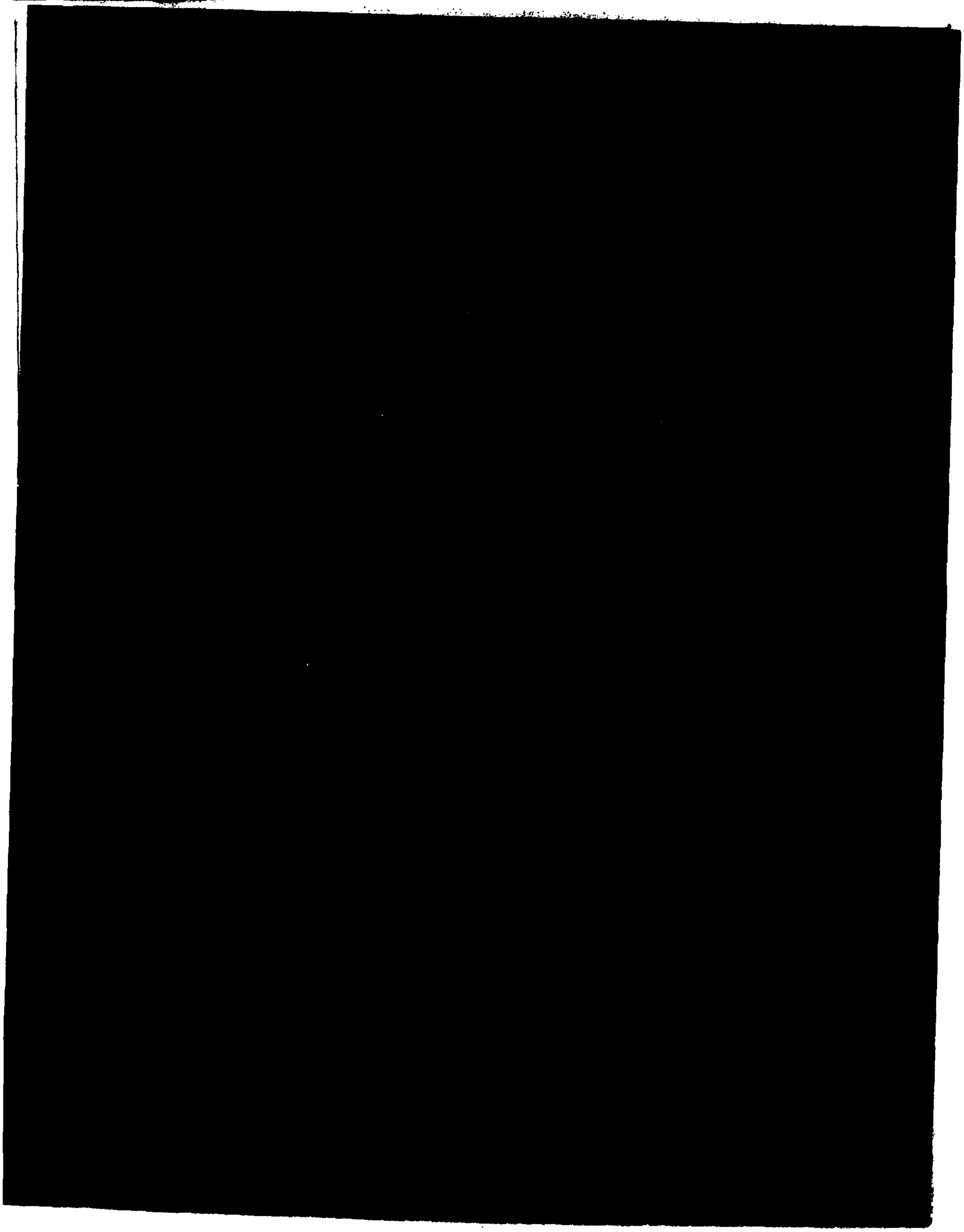

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