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NAVAL RESEARCH LAB WASHINGTON DC E O HULBURT
CENTER FOR SPACE... N R SHEELEY ET AL. 15 FEB 83
UNCLASSIFIED AFGL-TR-83-0024 NIPR-FY71218203062 F/G 3/2
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Publication 41-82-58

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Abstract: A computational model, based on diffusion, differential rotation, and meridional flow, has been developed to simulate the transport of magnetic flux on the Sun. Using Kitt Peak magnetograms as input, we have determined a best-fit diffusion constant by comparing the computed and observed fields at later times. Our value of $730 \pm 250 \text{ km}^2/\text{s}$ is consistent with Leighton's (1964) estimate of $770-1540 \text{ km}^2/\text{s}$ and is significantly larger than Mosher's (1977) estimate of $200-400 \text{ km}^2/\text{s}$. This suggests that diffusion may be fast enough to account for the observed polar magnetic field reversal without requiring a significant assist from meridional currents.

This paper presents the initial results of a project to simulate the transport of solar magnetic flux using diffusion, differential rotation, and meridional flow. The study concerns the evolution of large-scale fields on a time scale of weeks to years, and ignores the rapid changes that accompany the emergence of new magnetic regions and the day-to-day changes of the supergranular network itself.

Our initial objective was to determine the value of an effective diffusion constant that would provide the best fit between the computed and observed fields. To our knowledge, no fully quantitative determination has been attempted using modern computational techniques and high-quality observations. A resolution of the discrepancy between Leighton's (1964) model-dependent estimate of $770-1540 \text{ km}^2/\text{sec}$ and Mosher's (1977) semi-quantitative estimate of $200-400$
km$^2$/sec) would help to determine the relative importance of diffusion and meridional flow in transporting flux to the Sun's poles. With a value as low as Mosher's, diffusion would require a substantial assist from meridional flow to produce a timely reversal of the polar magnetic field, whereas with a value as high as Leighton's, diffusion could accomplish the reversal alone.

We assume that the photospheric field's radial component satisfies a continuity equation containing the transport effects of Leighton diffusion, differential rotation, and meridional flow. In spherical coordinates this equation is:

$$\frac{3B}{\dot{t}} = \frac{1}{r_1} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial B}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 B}{\partial \phi^2} \right] + \frac{1}{r_2} \left[ \cos^2 \theta \frac{\partial B}{\partial \phi} \right]$$

$$+ \frac{1}{r_3} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \sin 2\theta \right) \right],$$

where the time constants are $r_1 = R^2/K$ ($K$ is the diffusion constant), $r_2 = 1/\omega_0$ ($\omega_0$ is the differential rotation rate), and $r_3 = R/V_0$ ($V_0$ is the amplitude of the meridional flow speed). Typical values are $r_1 = 20$ years (for $K = 800$ km$^2$/sec), $r_2 = 21$ days (for a Newton and Nunn rate of 2.77 deg/day), and $r_3 = 1$ year (for a meridional flow amplitude of 20 m/sec (Duvall, 1977)).

Our study has progressed through several increasingly difficult and realistic phases. First, to test our modeling procedures, we used photographic prints of Kitt Peak magnetograms to estimate the location, pole separation and strength of new bipolar magnetic regions during 1976-1981 as the present sunspot cycle evolved. Using these doublet sources, and an assumed initial global dipole field of 1 Gauss, we calculated the evolution of the large-scale fields and displayed the fields in Carrington synoptic charts which we updated at 27-day intervals to include the contributions of flux from newly emerging regions.

Second, to estimate the diffusion constant, we selected four relatively isolated magnetic regions which were observed on several consecutive solar rotations. For each of these regions, our objective was to deposit the Kitt Peak digital magnetic measurements into the simulator during the initial solar rotation, and to compute the evolving large-scale magnetic fields at elapsed times of 1, 2, and 3 rotation periods for a range of values of the diffusion time $r_1$. In practice, we also considered other combinations such as depositing the flux during its second disk appearance and comparing the computed and observed fields on the third rotation. This procedure not only provided additional tests of our measurement consistency but also helped to avoid possible magnetic field calibration errors associated with the presence of sunspots during the initial rotation. In this second phase of our study, we deposited digital data for only a limited area including the source region. The surrounding area contained the typical, but unrealistic, background field computed from the doublet sources of phase one. To minimize the unknown errors
associated with this unrealistic background field we limited our comparisons to the immediate area of the expanding bipolar region.

For each region, the best fit was obtained numerically by minimizing the sum of the squared errors between the computed and observed fields and by allowing for a possible bias in the zero level of the Kitt Peak measurements. A relatively broad minimum was always obtained probably due to the unknown influence of the weak background fields. However, in half of these cases the minimum solution also provided close agreement between the observed and computed values of the centroids and the total surviving flux of the magnetic region.

The results for the case of \( V_0 = 0 \) are shown in the following table:

<table>
<thead>
<tr>
<th>Region</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposition Rotation</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Comparison Rotation</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Diffusion Constant</td>
<td>470*</td>
<td>680*</td>
<td>700*</td>
<td>1160</td>
</tr>
<tr>
<td>Summary: ( K = 1050 \pm 400 \text{ km}^2/\text{s} ) (all 8 measurements)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1140</td>
<td>1220</td>
<td>1690</td>
<td>1070*</td>
</tr>
<tr>
<td>Summary: ( K = 730 \pm 250 \text{ km}^2/\text{s} ) (4 best measurements)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* most reliable determinations, based on considerations of the location and total flux of the evolving region.

The eight determinations in Table 1 yield an average diffusion constant of \( 1050 \pm 400 \text{ km}^2/\text{sec} \), but the four most reliable measurements give the smaller value of \( 730 \pm 250 \text{ km}^2/\text{sec} \). Both are significantly larger than Mosher's (1977) \( 200-400 \text{ km}^2/\text{sec} \), and are consistent with Leighton's (1964) \( 770-1540 \text{ km}^2/\text{sec} \). In future phases, we plan to study the effects of including the observed background field as well as a non-zero value of meridional flow speed.

Figure 1 illustrates some of our procedures graphically. The upper left frame shows a Kitt Peak magnetogram on December 30, 1976. A large bipolar magnetic region (BMR) is visible in the southern hemisphere on this otherwise quiet day near sunspot minimum. The upper middle frame shows these fields after suppressing the short wavelength components of their Fourier representation. Obviously on this day the Kitt Peak zero level was biased in the negative (blue) direction and the positive-polarity north polar field is not visible. The upper right frame, displaying the normal component rather than the line-of-sight component of field, shows the same BMR deposited within the artificial background field. Relatively few BMRs have contributed to this background field at this early stage of the new sunspot cycle.

The lower frames in Figure 1 show the situation 28 days later on January 27, 1977. The lower left and center frames show the raw and smoothed Kitt Peak measurements, respectively. The lower right frame shows the BMR and its surroundings as they would have evolved with a diffusion constant of \( 680 \text{ km}^2/\text{sec} \). At this time, a newly emerged BMR, visible in the magnetogram's northern hemisphere, has
not yet been deposited in the simulator. A recently deposited doublet is visible west of the large BMR in the southern hemisphere. Our measurements are based on a comparison of the flux distributions only in the immediate vicinity of the evolving BMR and not in the surrounding areas where disagreement is both obvious and expected.

Fig. 1: Observed BMR (upper left and center) is deposited into simulated field (upper right), evolved (lower right), and compared with later observed field (lower left and center).

In conclusion, we should like to emphasize that our study is just reaching the point where most of the technical problems have been overcome. While our results to date are tentative, they are encouraging and in particular give a diffusion constant that is consistent with Leighton's original estimate.

We are grateful to C. Keppel for technical support, and to J.W. Harvey for Kitt Peak magnetograms. This work was supported by NASA (DPRs W 14, 429 and W 14, 777), and by AFGL (FY7121-82-03062).

REFERENCES
