INTRODUCTION

This report summarizes progress made during the period 1 December 1981 to 30th November 1982 on grant AFOSR-81-0003 entitled "High Angular Resolution Stellar Interferometry". The AFOSR Program Manager is Dr. H. Radoski and the Principal Investigator is J.C. Dainty.
1. **Space-time Analysis of Images**

**Aim**

The aim of this project is to understand the nature of the spatio-temporal behaviour of stellar images produced in large, Earth-based telescopes. Possible applications include the improvement of signal-to-noise ratio in speckle interferometry and the solution of the phase problem.

**Progress in 1982** (K. O'Donnell, T. Gonsiorowski and J.C. Dainty)

In unpublished work, O'Donnell has investigated a space-time extension of a phase-averaging method first suggested by McGlamery. The process of phase-unwrapping, central to this technique, is very sensitive to noise; O'Donnell has shown that the phase error, in radians, of the Fourier transform of a photon noise limited image is given by

\[ \varepsilon = \frac{1}{2N \cdot |\Lambda(u)|^2} \]  

(1)

where \( N \) is the mean number of detected photons in the image and \( |\Lambda(u)| \) in the Fourier transform of the random intensity of the image, normalized such that \( \Lambda(0) = 0 \). Figure 1 shows this error and also illustrates that the
Figure 1 Upper: Computer simulations of the atmospheric transfer function. Lower: Phase unwrapping error probability as a function of the average # photons per picture, $\bar{N}$, and the power spectrum $|\Lambda_1|$.
phase changes most rapidly when \( |\Lambda(u)| \ll 1 \), thus promoting large errors.

Construction of a space-time analysis camera (STAC) has been commenced with support from both AFOSR and ARO. This camera is to be used both for laboratory studies of speckle and on telescopes for speckle interferometry. Figure 2 shows a diagram of the proposed camera head which was designed during 1982 by Gonsiorowski. Construction should be completed by December 1983.

2. **Phase Problem: One Dimensional**

   **Aim**

   The aim of this project is to apply the theory of complex polynomials and that of entire functions to the phase problem in stellar interferometry. The basic, one-dimensional, phase problem has no unique solution; however, in stellar interferometry, we usually have additional information and by using complex polynomial theory we hope to be able to incorporate this information in an optimum way.

   **Progress in 1982 (B.J. Brames and J.C. Dainty)**

   We have published a paper describing a method of phase retrieval using the cross-power spectrum. This method is illustrated in Figures 3 and 4. Figure 3(a) shows the intensity distribution of a hypothetical one dimensional object (this particular shape, a negative exponential with...
Figure 2 The space-time analysis camera (STAC).
Figure 3. (a) object (b) complex zeros of object (c) complex zeros of power spectrum (d) complex zeros of the cross power spectrum.
Figure 4. + ambiguous zeros of power spectrum
x unambiguous zeros of cross power spectrum
The upper part shows that the noisy, but unambiguous zeros of the cross power spectrum can be used to locate the correct zero in the less noisy power spectrum. This sometimes fails to locate the correct zero as in the lower diagram.
noise, was chosen for clarity of the subsequent diagrams. It can be shown that this object can be represented by its zeros, which are shown in Fig. 3(b); given these zeros, it is possible to compute the object intensity. In conventional stellar speckle interferometry we only have the power spectrum, i.e. the squared modulus of the Fourier transform of the object intensity; this is equivalent to knowing the positions of all the zeros representing the object plus their inverses, as in Fig. 3(c). It is not possible to tell the difference between the two sets of zeros and consequently $2^N$ possible objects can be reconstructed from the power spectrum, one of which is "correct", one of which is the correct solution rotated by $180^\circ$, and $2^N - 2$ of which are incorrect; $N$ is the number of zeros, typically 50.

As Knox and Thompson discovered, additional object information is also available in the cross power spectrum $\phi(u, \Delta u)$; this is defined by

$$\phi(u, \Delta u) = O^*(u) O(u+\Delta u) \quad (2)$$

where $\phi(u)$ is the Fourier transform of the object intensity. The complex zeros of the cross power spectrum are shown in Fig. 3(d); they consist of all of the correct zeros plus the inverses rotated by a known amount. This rotation leads to unambiguous identification of the correct zeros and hence an unambiguous reconstruction of the object.
In practice, the ambiguous zeros of the power spectrum have less statistical error than the unambiguous ones of the cross power spectrum. Therefore we suggest using the noisy but unambiguous zeros of the cross power spectrum to identify the correct, less noisy, zeros in the power spectrum; this is illustrated in Fig. 4(a). Statistical noise can cause incorrect zeros to be identified, as in Fig. 4(b).

3. Phase Problem: Two Dimensions

Aim (see also 2)

Our specific aim in this project is to investigate the phase problem in two dimensions through analytical studies and the development of algorithms.

Progress in 1982 (B.J. Brames, M.A. Fiddy and J.C. Dainty)

The three-month visit by M.A. Fiddy was exceedingly productive and yielded an important first paper on the classes of two dimensional objects which provide unique solutions to the two dimensional phase problems. We found a sufficient condition, based on Eisenstein's criterion for irreducibility of polynomials, for an object which guarantees uniqueness of the phase retrieval problem. The sufficient condition requires that the object have two non-zero points in specified positions, one of them being outside the support of the rest of the object. This condition has been investigated using Fienup's algorithm.
(now fully implemented on our LS1 11/23 + Skymnk array processor) and results are shown in Figs. 5-7. In each Figure we show (A) the original object, (B) the Fienup restoration on the original object modulus after 75-250 iterations, (C) the Fienup restoration on the object after only 10 iterations, with a reference containing 5% of the total energy, and (D) with the reference containing .5% of the total energy. Note that in Figs. 5 and 7, which are complicated scenes, the Fienup algorithm applied to data with a rectangular support and no reference points does not converge to the original object; this was independently confirmed by Fienup who has also processed our Fig. 5. This sufficient condition is an important first step in defining the classes of objects which provide a unique solution to the phase problem.

4. Laboratory Simulations

Aim

The aim of this project is to provide a facility for the laboratory simulation of stellar speckle interferometry and related techniques. This facility is primarily for use in investigations of the phase problem and new interferometric methods.

Progress in 1982 (B.J. Brames)

The system shown in Fig. 8 has been constructed. Simulation experiments are run automatically by the LS1 11/23
Figure 6
Figure 8  Laboratory simulation system for speckle interferometry.
computer system. The main peripherals that are used are
(i) the SKYMNK array processor which does a 128x128 Fourier
transform in 5s, (ii) the TV digitizer for input of data,
(iii) the HP display oscilloscope for off-line display of
high-quality images and (iv) the 20MB "Winchester" disc
and cartridge tape for off-line storage of data.

Simulation experiments are currently beginning on
this system, with most of the software already written.
In a typical experiment, a known object (which we are
attempting to reconstruct) is placed in the object generator
and a sequence of approximately 300 speckle images recorded
and analyzed over a period of about 2 hrs. The "atmosphere"
is moved between exposures and all data collected under
computer control. All of the data reduction is carried
out on-line, so that at the end of the period, and after
a similar set of "point source" data are collected, the
reconstructed object can be compared to the original.

5. Observational Speckle Interferometry

Aim

The aim of this project is to undertake speckle obser-
vations, in collaboration with astronomers, in both the
infrared and visible parts of the spectrum.

Progress in 1982 (J.C. Dainty, J.L. Pipher, S. Ridgeway)

In collaboration with Professor J.L. Pipher (University
of Rochester) and Dr. S. Ridgeway (Kitt Peak National
Observatory), infrared speckle observations have been carried out using the 4m telescope at Kitt Peak National Observatory. The first observing session in December 1981 resolved GL915 (the Red Rectangle) as a gaussian-shaped object of dimension 0''8 (N-S) and 0''4 (EW). The observed power spectra at the K wavelength are shown in Fig. 9. A paper on this unusual bipolar nebula is in preparation.

6. Properties of Atmospheric Turbulence

Aim

The aim of this project is to perform experimental studies on the optical effects of atmospheric turbulence: possible topics include the measurement of long-exposure (conventional) atmospheric modulation transfer functions, speckle transfer functions, and the statistics of atmospheric scintillation (twinkling). The studies are made because of their possible importance in different types of interferometric imaging systems.


During the summer of 1981, measurements of the spatio-temporal properties of speckle images were completed and data reduction was completed in 1982. Of particular relevance to speckle interferometry was the short time-scale of seeing at Mauna Kea Observatory. At the same time we measured the wavelength dependence of atmospheric
RED RECTANGLE (GL915)

λ = 2.2 μm (K)

Figure 9  Power spectra of GL915, showing on elongated gaussian-like profile of dimensions 0'.8 (N-S) x 0'.4 (E-W)
scintillation, which obeyed the theoretical predictions of Tatarski.

J. Dugan constructed a prototype anamorphic shearing interferometer for the measurement of the modulation transfer function of atmospheric seeing. The principle of operation is shown in Fig. 10. The interferometer is a modified Mach-Zehnder, with a pair of prisms in one arm and a compensator plate in the other; the effect is to produce two superimposed images of the telescope pupil, one compressed (or elongated) along one axis. Thus at one edge of the pupil, the light is interfered with itself, whereas at the other edge the light is interfered with a point sheared from itself. With tilt fringes introduced perpendicular to the direction of shear, the atmospheric MTF can be found from the fall-off in fringe contrast as a function of shear distance.

Figure 10 also shows the complete optical system. This was used at Mees Observatory in May 1982 and worked qualitatively, although a more engineered version will be required for quantitative measurements.
Figure 10  Complete optical system of the anamorphic shearing interferometer.
LIST OF PUBLICATIONS (research supported by AFOSR)

1. Published during period


2. Submitted during period

(a) 1(e) above


3. Conference Presentations

Three papers were presented at the Annual Conference of the Optical Society of America, Tucson, October 18-22, 1982: papers TuR2, WG1 (invited) and TLM8.
**SCIENTIFIC PERSONNEL**

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<th>Name</th>
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